Who are you?

• What experience have you with Structural Equation Modeling (SEM) so far?
  – 12% no experience; 36% beginners; 40% occasional users; 12% experienced users

• Have you used Mplus before?
  – 13% no; 87% yes

• Have you used Mplus before to analyze longitudinal data?
  – 48% no; 52% yes

• What is your main learning objective today?
  – 21% curious about SEM and Mplus; 74% learning SEM and Mplus for longitudinal data; 10% my advisor requires me; 36% complex data; 10% latent variable stuff

With special thanks to...

Luc Goossens       Tod Little       Patrick Curran

Karl Jöreskog       Bengt & Linda Muthén
(no pics available)
Change is inevitable. Change is constant.
(Benjamin Disraeli)

Change is the nursery of music, joy, life, and eternity.
(John Donne)

When you're finished changing, you're finished.
(Benjamin Franklin)

We are restless because of incessant change, but we would be frightened if change were stopped.
(Lyman Bryson)

Change is a measure of time.
(Edwin Way Teale)
**General measurement and design issues**
- Time and intervals
- Differential growth
- Missing values

**Classic methods and disadvantages**
- Difference score
- Repeated ANOVA
- Autoregression
- Cross-lagged models

**New(er) methods**
- LCM
- LGCM
- LGMM
- LTA

**Exercise 1**

---

**The example DATA**

- **N** = 405 adolescents + mothers
- From three cohorts

**Classic methods and disadvantages**
- Difference score
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**Exercise 1**

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**What measurement intervals should we take?**

- How fast is the developmental process?
- Intervals must be equal to or less than expected processes of change (e.g., schooling studies at half-year intervals)
- If too short: too sensitive to measurement error
- If too long: insensitive to change and variability in change

A great example article:

---

**When measuring CHANGE, how can we define TIME?**

- Age in years, months, days
- Experiential time: Amount of time something is experienced
- Years of schooling (grade), length of relationship, amount of practice
- Calibrate an beginning of event, measure time experienced
- Time since onset or time from normative or expected occurrence.

**What measurement intervals should we take?**

- How fast is the developmental process?
- Intervals must be equal to or less than expected processes of change (e.g., schooling studies at half-year intervals)
- If too short: too sensitive to measurement error
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A great example article:
• General measurement and design issues
  – Time & Intervals
  – Differential growth
  – Missing values
• Classic methods and disadvantages
  – Difference score
  – Repeated ANOVA
  – Autoregression
  – Cross-lagged models
• New(er) methods
  – LCM
  – LGM
  – LCA
  – LCGM
  – LTA
• Exercise 1

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• How to test for?
  – Not really possible. But…
  – MVA (SPSS)
• References:
  – Little & Rubin (2002). Statistical analysis with missing data. Wiley
Table with Separate Variance t Test in output, with in rows all variables with (+5%) missings, and in columns all variables in dataset. Cells contain a t-value (+ p) indicating whether or not missings in the row variable is correlated significantly with the values of the column variable, and therefore is selective. Check patterns of significant t-values. If not clear pattern, MAR is very likely!

When selecting EM in the 'Estimation' menu, Little's MCAR test is provided (a summary of t-tests above). If not significant: MCAR! If $\chi^2/df$ (normed $\chi^2$) < 2: MAR.

PREVENTING!

- Dillman (1978)
  - Intensive follow-up and tracking of subjects
  - Repeated invitations to participation, reminders
  - Repeated sending of the measurements
  - Do everything to prevent large dropout!

- Planned missingness
  - Do not measure all variables in all participants at all times.

- Cohort-sequential design!!
  - Let new persons come in at each wave of the study,
  - This way you create different patterns of missingness, not only dropout!

- Different ways to increase the chances of MAR or MCAR!

CURING!

- Purpose is NOT to fill in empty cells in the data!

- Purpose IS to estimate the population parameters as good as possible, using a sample with missing data!

Which methods can help us in this challenging task?
BAD ways to deal with missing data

• List-wise Deletion
  – Variances biased, means biased
  – Acceptable only if power is not an issue and the incomplete data is MCAR

• Pair-wise Deletion
  – Variances biased, means biased
  – Acceptable only if power is not an issue and the incomplete data is MCAR

• Sample-wise Mean Substitution
  – Variances reduced, correlations biased
  – Never acceptable!

• Subject-wise Mean Substitution
  – Depends on homogeneity of the items used
  – Acceptable only if set of items is homogeneous and only few missings!

QUESTIONABLE ways to deal with missing data

• Regression Imputation
  – All subjects with same values on IV get the same estimated value on the DV.
  – Variances reduced
  – Assumes MCAR

• Stochastic Regression Imputation
  – Same as above but a random error component is added to reduce the loss in variance
  – Still assumes MCAR

GOOD ways to deal with missing data

• But only if enough variables related to missingness are included in analysis (MAR), or missingness is MCAR

• EM Imputation
  – Imputes the missing data values in an iterative way, starting with the E step
  – The E(estimate)-step is a stochastic regression-based imputation for each variable.
  – The M(aximization)-step is to calculate a complete covariance matrix based on the estimated values.
  – The E-step is repeated for each variable but the regression is now on the covariance matrix estimated in the previous M-step.
  – The EM-steps are repeated until the imputed estimates don’t differ from one iteration to the other
GOOD ways to deal with missing data

- But only if enough variables related to missingness are included in analysis (MAR), or missingness is MCAR

- Multiple Imputation
  - Estimate N (e.g., 5) datasets using the EM algorithm
  - Each dataset is based on a kind of resampling of the original sample (equivalent to a random selection of a different sample from the population)
  - Possible way 1:
    - Run the analyses N times
    - Summarize the results of these N analyses using the formulas of Rubin (1987)
  - Possible way 2:
    - Collapse the N samples to one dataset and do the analyses.

- Full Information Maximum Likelihood
  - Sufficient statistics (means, covariances) are estimated with the Expectation Maximization (EM) algorithm
  - Those estimates then serve as the start values for the Maximum Likelihood model estimation
  - Does not impute the missing values.
  - Can only be used when testing a SEM-model.
  - Available in Lisrel, AMOS, Mplus, EQS, etc.

EXAMPLES comes with LGC and other models.

### Exercise 1

```plaintext
DEFINE: change14 = anti4-anti1;
MODEL: change14 ON support;
```

CHANGE14 = ANTI4 – ANTI1

- Though many problems with it, still popular (e.g., intervention, pretest-posttest, or clinical studies)
- Most cited problem: Unreliability of the difference score
  - when the measures comprising the difference are only modestly reliable and positively correlated
  - which is typically the case in longitudinal research!
- And therefore also: lack of validity
- Change is measured without taking level into account!

```plaintext
difference score inp
DEFINE: changel4 = anti4-anti1;
MODEL: changel4 ON support;
```

### Exercise 1
\[ \text{CHANGE}_{14} = \text{ANTI}_4 - \text{ANTI}_1 \]

- Though many problems with it, still popular (e.g., intervention, pretest-posttest, or clinical studies)
- Most cited problem: Unreliability of the difference score
  - when the measures comprising the difference are only modestly reliable and positively correlated
  - which is typically the case in longitudinal research!
- And therefore also: lack of validity
- Change is measured without taking level into account!

**Difference score.inp**

```plaintext
DEFINE: changel4 = anti4-anti1;
MODEL: changel4 ON support;
```

**Exercise!**

- **SPSS output**: \( \text{R ANOVA.spv} \)
- **So, a good method**
  - To describe and test an overall mean change function, and test for the form of it (linear, quadratic, cubic, etc.)
  - To test for the effect of covariates on the change function (e.g., support): Time x Support interactions!
  - To test for the effect of between-subject factors on the change function (e.g., gender): Time x Gender interactions!
- **But**
  - Only tests mean change over time in the whole sample and not deviations from that mean change
  - And... group statistics (e.g., mean) represent everyone, and no one!
  - Equal intervals between measurements are necessary!
  - Change is an outcome of the repeated measures (time) and cannot be used as a predictor of outcomes.
• With observed variables: autoregression.inp
• With latent variables: autoregression latent.inp

• General measurement and design issues
  - Measurement
  - Time & Intervals
  - Differential growth
  - Missing values

• Classic methods and disadvantages
  - Difference score
  - Repeated ANOVA
  - Autoregression

• New(er) methods
  - LCM
  - LCGA
  - LGMM
  - LTA

• Exercise 1

---

• But...
  - Implicit assumption of decreasing correlations across time!
  - Indifferent to the functional form of change!
  - Only RELATIVE change in terms of rank order!
  - Interpretation?
    - Does ANT1 predict (.86) an increase in ANT4?
    - Increase only relative to others in the sample!
    - Even if everyone decreases!
  - No trajectories of individual change over time!

Extensions: mediation: autoregressionb.inp
moderation: autoregression-multigroup.inp

---

• A multivariate extension of the autoregressive model of change: crosslagged.inp

• General measurement and design issues
  - Measurement
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• Classic methods and disadvantages
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• Exercise 1

---

• Interesting for examining direction of effects, by comparing cross-lagged coefficients (easily done in SEM)

• But
  - Same problem with interpretation of change as in AR model!
  - Also shares all other problems of the AR model!

+ Statistical drawbacks:
  - If X is (much) more stable than Y, then Y will have a stronger effect on X than vice versa!
  - And, the less reliable X is, the less Y can explain it.

---

So,

What do we want exactly when we try to assess change?

RELIEABLE estimates of change!
Estimates of ABSOLUTE change!
INDIVIDUAL estimates of change!
With 2 WAVES of data

Latent Change Models (LCM)

- Change in latent variables, using (CFA)
- First step = Longitudinal invariant factor model (multiple indicators), resulting in reliable scores for Anti-1 and 4.
- Second step = Restructuring Anti-1 and Anti-4 in latent Level and Change factors, using a very simple equation:
  \[ \text{Anti-1} = \alpha \text{Anti-Level} \]
  \[ \text{Anti-4} = \alpha \text{Anti-Level} + \beta \text{Anti-Change} \]
- As a consequence:
  \[ \text{Anti-Change} = \text{Anti-4} - \text{Anti-1} \]
  \[ \text{Anti-Level} = \text{Anti-1} \]

- Observed variables?
The problem of measurement error

\[ \text{DATA} = \text{MODEL} + \text{ERROR} \]

- True variance: correlated
- Error variance: not correlated

\[ \Rightarrow \text{Total covariance: underestimated!} \]

Latent variables!
Solution for measurement error!

\[ \text{SEM} = \text{analysis with latent variables!} \]
With 2 WAVES of data

Latent Change Models (LCM)

• Change in latent variables, using (CFA)
  • First step = Longitudinal invariant factor model (multiple indicators), resulting in reliable scores for Anti-1 and Anti-4.
  • Second step = Restructuring Anti-1 and Anti-4 in latent Level and Change factors, using a very simple equation:
    \[ \text{Anti-1} = 1 \times \text{Anti-Level} \]
    \[ \text{Anti-4} = 1 \times \text{Anti-Level} + 1 \times \text{Anti-Change} \]
    \[ \text{Anti-Change} = \text{Anti-4} - \text{Anti-1} \]
    \[ \text{Anti-Level} = \text{Anti-1} \]

• New(er) methods
  • LCM
  • LGCM
  • LCGA
  • LGMM
  • LTA

• Exercise 1
  • lcm_ANTI.inp

With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2003; Willet & Sayer, 1994)

• Questions:
  - Does an individual characteristic (e.g., antisocial behavior) change over time?
  - Which trajectory is followed?
  - Interindividual differences?

• Step 1: Within-Person
  - Equation for every subject in the sample:
    \[ \text{anti} = \text{intercept} + (\text{slope} \times \text{Time}) + \text{error} \] (regression)
    - Growth can be non-linear too!
    - Assumption: Individuals share the shape of the change function (e.g., linear), but can differ in the amount or rate of change (individual growth parameters: intercept, slope, etc.)

• Step 2: Between-Person
  - Means (fixed) & variances (random) of intercepts, slopes
  - Predictors of change (conditional growth models).

• General measurement and design issues
  - Measurement
  - Time & Intervals
  - Differential growth
  - Missing values

• Classic methods and disadvantages
  - Difference score
  - Repeated ANOVA
  - Autoregression
  - Cross-lagged models

• New(er) methods
  • LCM
  • LGCM
  • LCGA
  • LGMM
  • LTA

• Exercise 1

• Main advantages:
  - Only two waves of data needed!
  - Reliable estimates of change!
  - Change is assessed as a latent factor!
  - With a mean: Mean change in the total sample!
  - With a variance: Individual differences in change!
  - That can be predicted and used as a predictor!
  - Latent factor scores can be estimated!

• But
  - Only linear change function!
• General measurement and design issues
  - Measurement
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  - Missing values

• Classic methods and disadvantages
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• New(er) methods
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• Exercise 1

With 3+ WAVES of data
Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

\[ \text{anti} = 2 + (1.5 \times \text{time}) + \text{error} \]

Intercept
Initial level
Rate of change

With 3+ WAVES of data
Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)
• General measurement and design issues
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• Exercise 1

---

With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

\[ \text{anti} = 1 + (1.5 \times \text{time}) + (0.5 \times \text{time}^2) + \text{error} \]

---

Two Approaches to GCM

<table>
<thead>
<tr>
<th>Multilevel</th>
<th>SEM: LGCM</th>
</tr>
</thead>
<tbody>
<tr>
<td>- regression approach</td>
<td>- factor approach</td>
</tr>
<tr>
<td>- fixed &amp; random effects</td>
<td>- means &amp; variances</td>
</tr>
<tr>
<td>- flexible with missing values &amp; individual-varying intervals</td>
<td>- not very flexible with individual-varying intervals</td>
</tr>
<tr>
<td>- less flexible in prediction</td>
<td>- very flexible in prediction</td>
</tr>
<tr>
<td>[ \text{length} = b_{00} + b_{01} \times \text{gender} + \text{error} ]</td>
<td>- little attention for model fit</td>
</tr>
<tr>
<td>- overload of fit indices</td>
<td>- no overload of fit indices</td>
</tr>
</tbody>
</table>

---
Latent Growth Curve Models (LGCM)

(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

PARAMETERS in the model

- Mean intercept / fixed effect intercept
  = Mean initial level of all individuals
- Variance intercept / random effect intercept
  = Interindividual differences in initial level
- Mean slope / fixed effect slope
  = Mean rate of growth across individuals
- Variance slope / random effect slope
  = Interindividual differences in rate of change

With 3+ WAVES of data

lgcm_anti.inp (with missing data, and FIML)

lgcm_anti + predictors.inp (gender and support as predictors)

lgcm_anti + predictors + interaction.inp (support X level anti)

lgcm anti - piecewise.inp (piecewise model with 2 slopes)
With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

• Can be extended to a multivariate LGCM!

lgcm_anti + gpa.inp
– Correlated intercepts = cross-sectional association
– Correlated intercept & slope = level of IV is predicting rates of change in DV!
– Correlated slopes = common underlying growth in two constructs = change associated with change (causality?)

• But
– Assumption: same shape of the growth function for all subjects; interindividual differences in change are modeled as deviations from that overall mean.

Exercise 1

With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

Extension 1: Measurement invariance - curve of factors!

New(er) methods
– LCM
– LGCA
– LGM
– LTA

Exercise 1

With 3+ WAVES of data

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Exercise 1
With 3+ WAVES of data
Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)
Extension 1: Measurement invariance - curve of factors!
- Measurement invariance (e.g. Boy-girl)
- Longitudinal: is the ruler the same over time?
- if not: difficult to disentangle growth from change in the ruler!

General measurement and design issues
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New(er) methods
- LGCM
- LCGA
- LCMM
- LTA

Excercise 1

Note: Equal errors not required (ð MI; boy-girl) correlated over time
With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

Extension 1: Measurement invariance - curve of factors!

• Models
  - Antisocial baseline
  - Antisocial FL invariance
  - Antisocial FL+ I invariance

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(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

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Exercise 1
With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

Extension 1: Measurement invariance - curve of factors!

- Measurement level of likert scales?
  - Nominal
  - Ordinal
  - Interval
  - Ratio

- Up to now: assumed interval
- Debate whether ordinal?!
  ➔ More serious with lower number of scale points (e.g. 3 or less)

General measurement and design issues
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Excercise!
With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

Extension 1: Measurement invariance - curve of factors!

- Models
  - Antisocial_baselineWLSMV
    - Models
  - Antisocial_FL invariance WLSMV
    - Models
  - Antisocial_FL+Threshold invariance WLSMV
    - Models
- General measurement and design issues
  - Measurement
  - Time & Intervals
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- Exercise 1

With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

Extension 2: Cohort sequential design!

- Mixing cross-sectional & longitudinal design
- Remember: CS-design = MAR
- Many longitudinal studies have multiple cohorts.
- Example: PhD of 4 years, but wanting to measure antisocial behavior from age 13 to 20 (8 years)

- Growth curve for each cohort, but:
  - Fix mean and variance of intercept and slope equal across cohorts
  - Fix int-slp correlation equal across cohorts
  - Adjust slope factor loadings of anti in line with cohort or birthyear

Cohort 1 (1985):  0   1    2   3
Cohort 2 (1983):  2   3    4   5
Cohort 3 (1981):  4   5    7   8

cohort-sequential growth model anti.inp (linear)
cohort-sequential growth model anti.inp (curvilinear)

With 3+ WAVES of data

Latent Growth Curve Models (LGCM)
(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

Extension 2: Cohort sequential design!

Method 1: Multigroup modeling

<table>
<thead>
<tr>
<th>Cohort</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (1985)</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>2 (1983)</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>3 (1981)</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
</tbody>
</table>

- Linking adjacent segments of change from different cohorts to estimate a common growth curve
With 3+ WAVES of data

Latent Growth Curve Models (LGCM)

(Duncan et al., 1994; McArdle & Nesselroade, 2002; Willet & Sayer, 1994)

Extension 2: Cohort sequential design!

Method 2: Data rearrangement (DATA COHORT)

<table>
<thead>
<tr>
<th>Cohort</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (1985)</td>
<td>X   X   X   MCAR    MCAR    MCAR</td>
</tr>
<tr>
<td>2 (1983)</td>
<td>MCAR MCAR MCAR MCAR   X   X   X</td>
</tr>
<tr>
<td>3 (1981)</td>
<td>MCAR MCAR MCAR MCAR X   X   X</td>
</tr>
</tbody>
</table>

- Estimate 1 intercept and 1 slope, using all available data
- Works out the time score based on birth and measurement year
- Idea = rearrange our cohort & time data to age data
- Then a growth curve for the complete age span!
- Only works with continuous variables!

cohort-sequential growth model anti_2.inp (linear)
cohort-sequential growth model anti_2b.inp (curvilinear)
cohort-sequential growth model anti_2_piecewise.inp

With 3/4+ WAVES of data

Latent Growth Mixture Modeling (LGMM)

(Muthén, 2001, 2004)

and a special case of it:

Latent Class Growth Analysis (LCGA)

(Nagin, 1999, 2001)

- LGMM:
  - relaxes this single population assumption to allow for parameter differences across unobserved subpopulations
  - by using a combination of continuous and categorical latent variables (mixed)
  - by introducing a latent class variable C, a trajectory class variable, representing k unobserved subpopulations in the sample (note that in LGCM k = 1)
  - by estimating a separate growth model for each of the latent classes
  - and estimating latent class conditional probabilities (membership)
With 3/4+ WAVES of data

Latent Growth Mixture Modeling (LGMM)
Latent Class Growth Analysis (LCGA)

- LGMM: Class differences
  - Typically between mean intercepts between classes
  - Mean slopes in the classes
  - Variance in intercept and slope
  - Shape of the growth function!
  - Influence of covariates!

- LGMM: Extensions
  - Including covariates of change
  - Including outcomes that are predicted from growth

- LCGA (Nagin)
  - Very similar, only no variances within classes are estimated
  (therefore a semi-parametric approach)
  - Individuals within classes are treated as homogeneous

- Estimation: Using the EM algorithm
  - Estimation of each individual’s probability of membership in each class (conditional probabilities)
  - Measures of fit and classification quality:
    - BIC
      - Small values correspond to a good model with a large likelihood and not too many parameters
    - Look at the big drops in BIC from one solution to another!
    - Sensitive to the number of classes!
  - LMRT test
    - Test of a solution with k-1 classes against a solution with k classes
    - Low p-value indicates that solution k-1 should be rejected in favor of the solution with k classes.
  - Entropy
    - Measure of classification quality based on the individual class probabilities.
    - High values (closer to 1) indicate good classification.

- Examples with 2 classes:
  - lcga_anti2.inp (Nagin approach)
  - lgmm_anti2.inp (with equal variances across classes)
  - lgmm_anti2free.inp (with free variances across classes)

---

With 4+ WAVES of data

Latent Growth Mixture Modeling (LGMM)
Latent Class Growth Analysis (LCGA)

- Classical methods
  - Autoregression
  - Cross-lagged models

- New(er) methods
  - LCM
  - LCGM
  - LCGA
  - LGMM
  - LTA

- Exercise 1

---

Antisocial data: fit statistics

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- Exercise 1

---
• General measurement and design issues
  – Measurement
  – Time & Intervals
  – Differential growth
  – Missing values

• Classic methods and disadvantages
  – Difference score
  – Repeated ANOVA
  – Autoregression
  – Cross-lagged models

• New(er) methods
  – LCM
  – LCGA
  – LGMM

• Exercise 1

With 4+ WAVES of data
Latent Growth Mixture Modeling (LGMM)
Latent Class Growth Analysis (LCGA)

Antisocial data: Best solution: 5 classes, LCGA, curvilinear

With 4+ WAVES of data
Latent Growth Mixture Modeling (LGMM)

Antisocial data: Best solution: lca_antis5c.dat

With 3/4+ WAVES of data
Latent Transition Analysis (LTA)
(Kaplan, 2008; Meeus et al., 2010)

• A longitudinal extension of LCA or LPA
  – first, uses class-specific parameters (the continuous/ categorical observed multidimensional scores) as measurement parameters (~ LCA or cluster analysis)
  – invariant across time!
  – and, uses class probabilities as structural parameters to estimate the number of participants in each of the classes
  – then using latent transition probabilities to calculate patterns of stability and change over time in the movement or transition between classes (~ mover-stayer model)

An ideal model for testing typological or person-oriented developmental theories!
• General measurement and design issues
  – Measurement
  – Time & Intervals
  – Differential growth
  – Missing values
• Classic methods and disadvantages
  – Difference score
  – Repeated ANOVA
  – Autoregression
  – Cross-lagged models
• New(er) methods
  – LCM
  – LGCM
  – LCGL
  – LGFMM
  – LTA
• Exercise!
  – estimated using Robust Maximum Likelihood estimation
  – models with different # latent classes fitted and compared
  – decision based on BIC, entropy, and interpretability

With 3/4+ WAVES of data
Latent Transition Analysis (LTA)
(Kaplan, 2008; Meeus et al., 2010)

• Example: data on parenting across three waves
  – 659 adolescents, measured 3 times: 13, 15 and 17 years
  – psychological control, support/warmth, firm control (continuous)
  – scores standardized (z) to ease interpretation
  – hypotheses: at least 4 latent classes (parenting styles) and gradually better parenting from 13 (peak of puberty years, reactive parenting) to 17 years
  – estimated using Robust Maximum Likelihood estimation
  – models with different # latent classes fitted and compared
  – decision based on BIC, entropy, and interpretability

Autoregression – Cross-lagged models

With 3/4+ WAVES of data
Latent Transition Analysis (LTA)
(Kaplan, 2008; Meeus et al., 2010)

• Selected output
  – Measurement part + latent class probabilities

With 3/4+ WAVES of data
Latent Transition Analysis (LTA)
(Kaplan, 2008; Meeus et al., 2010)

• Selected output k = 5 solution
  – Transition probabilities into latent classes at T+1
### General measurement and design issues
- Measurement
- Time & Intervals
- Differential growth
- Missing values

### Classic methods and disadvantages
- Difference score
- Repeated ANOVA
- Autoregression
- Cross-lagged models

### New(er) methods
- LCM
- LGCM
- LCGA
- LGMM
- LTA

## To conclude
- Since two decades, we have interesting new methods to analyze change and development!
- **Mplus** provides a powerful tool to analyze change and development, and is constantly improving!
- And, as said before…

#### Change is inevitable. Change is constant.
*(Benjamin Disraeli)*

#### Change is the nursery of music, joy, life, and eternity.
*(John Donne)*

#### When you’re finished changing, you’re finished.
*(Benjamin Franklin)*

#### We are restless because of incessant change, but we would be frightened if change were stopped.
*(Lyman Bryson)*

#### Change is a measure of time.
*(Edwin Way Teale)*

### Exercise 1
1. Check the direction of effects (using a cross-lagged model) between GPA and antisocial behavior, using data of Times 1 and 3. Check whether results are the same for boys and girls.
   - **DATA** are SEMDATA.DAT; **software = Mplus 7.3**
2. Estimate a LGCM of GPA, using the FIML approach for missing data. Check models with linear and curvilinear change. Interpret the parameters that are found.
   - **DATA** are SEMDATA.DAT; **software = Mplus 7.3**
3. Find the optimal LCGM/LGMM solution of the GPA data. Explain why this solution was chosen and interpret the different classes.
   - **DATA** are SEMDATA.DAT; **software = Mplus 7.3**
4. Evaluate the effect of mother support on development of GPA, using a conditional growth model and FIML.
   - **DATA** are SEMDATA.DAT; **software = Mplus 7.3**
5. Test a measurement invariant LGCM on the antisocial data, and evaluate whether this is better than a constant trend. If time allows, also experiment with factor loadings of time to model different intervals between waves.
   - **DATA** are ANTISOCIAL.CSV; **software = Mplus 7.3**

### Exercise 2
- Setup and do the analyses using Mplus!
- Ask for help while doing the analyses!
- Present the results to the audience, using a single or two slides and explain the effects in words!
Informative websites

- www.statmodel.com: the Mplus site!
- http://davidakenny.net/cm/causalm.htm: great SEM page!
- http://www.ats.ucla.edu/stat/seminars/: online examples and videos on Mplus

An introduction to SEM


Defining time and choosing intervals


Missing data and how to handle them


Latent Change Model (LCM)


Latent Growth Curve modeling (LGM)

Literature

Latent Class Growth Analysis (LCGA)

Latent Growth Mixture Modeling (LGMM)

Literature

Latent Transition Analysis
Further literature

• Model fit

• MLR (and related Chi-square test)

• Reporting about SEM