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WORKING PAPER

Rationing : dynamic considerations, equivalent sacrifice and links between the two approaches

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Abstract

Suppose a quantity t of a given resource is divided among two agents. If an additional quantity Δt becomes available, how shall we share it among the agents ? By looking at the way we can share this increment (or decrement), it is possible to derive some existing rationing methods but also some new ones. Three new methods seem particularly interesting. They can also be derived following an Equivalent Sacrifice approach.

1 Introduction

Consider the following problem where 2 agents must share a non-negative amount t of a resource. Each agent has his own non-negative demand : a for agent 1 and b for agent 2. A rationing method r will assign to each agent his non-negative share : r for agent 1 and $t - r$ for agent 2. Formally, a rationing method is a mapping $r : \mathbb{R}^3_+ \to \mathbb{R}_+ : (a, b, t) \to r(a, b, t)$ satisfying the following conditions : for all t such that $t \leq a + b$,

- $r(a, b, t) \geq 0$ and $t - r(a, b, t) \geq 0$;
- $r(a, b, t) \leq a$ and $t - r(a, b, t) \leq b$.

The first conditions guarantees that the share of each agent be non-negative and the second one imposes that nobody gets more than his demand.

Many of the axioms that have been proposed in the literature (see e.g. Moulin, 2002; Thomson, 2003, for a review) to characterize rationing methods are "static" with respect to t : they do not consider changes in t. For example, Equal Treatment of Equals, Symmetry, Ranking, Progressivity, Regressivity, Lower Bound, Upper Bound, Sustainability, Preeminence, Irrelevance of Reallocations, Independence of Merging and Splitting, Decomposition, Zero consistency, Consistency, Scale Invariance[1]. Nevertheless, some axioms consider what happens when the available quantity t of the resource varies, namely,

[1]Note that some of the axioms in this list are concerned with changes in the number of agents and are not really relevant to the 2 agents problem.

Continuity, Additivity, Lower Composition, Upper Composition, Monotonicity. For example, given three quantities t, t' and t'', Additivity, Lower and Upper Composition tell us how $r(a, b, t), r(a, b, t')$ and $r(a, b, t'')$ should be related. To the best of my knowledge, these "dynamic" axioms have always been combined with static axioms to axiomatically characterize some rationing methods.

In the first part of this paper, I focus on a dynamic approach to the rationing problem. I derive some rationing methods by only imposing some dynamic conditions, in the following way.

Suppose that the first agent's share is x and the second agent's is $t - x$. If an additional unit of the shared resource becomes available, what shall we do? We can consider a new problem in terms of a, b and $t + 1$ and compute the new solution or we can have a more local approach and just share this additional unit among the agents. Then, in the new situation, the agents have what they had previously plus their share of the additional unit. What then characterizes a rationing method is the way the additional unit is shared among the agents.

Similarly, when t decreases by 1 unit, we may just look how to share this loss, each agent loosing only his share of the missing unit. Obviously, if we face a resource whose quantity can vary continuously, the right way to do this is to consider infinitesimal increments in t and to look how to share each infinitesimal increment. Each way of dividing the infinitesimal increment will define a differential equation, which, in turn, will uniquely define a rationing method.

In Section 2, I propose some ways of dividing each increment, write the corresponding differential equation and solve them. Some well-known rationing methods are obtained. Some new ones also appear. Section 3 presents another derivation, based on an Equivalent Sacrifice principle, for the new methods obtained in Section 2. Section 4 is devoted to the analysis of the link between the two approaches used in the two preceding sections. In the last section, I generalize the approach to any number of agents. The generalization is straightforward and doesn't bring any new issue. I choose to present this dynamic approach in the special case of two agents because it allows me to use a simple notation and to better carry the important ideas.

2 Some dynamic characterizations

In this section, I examine several principles according to which we could divide the infinitesimal change in t among the agents. I begin with some trivial results and go on with some (hopefully) less trivial ones.

2.1 Uniform Gains, Uniform Losses and Proportional method

An obvious way of dividing each increment is *according to the ratio of their demands.* Formally,

$$\frac{\partial r / \partial t}{\partial (t - r) / \partial t} = \frac{a}{b}, \quad 0 \leq t \leq a + b. \tag{1}$$

As $r(a,b,0)=0$, the unique solution is easily found to be the proportional method, i.e.

$$r(a,b,t) = \frac{a}{a+b}t. \tag{2}$$

The reader will easily write the differential equations corresponding to Uniform Gains, Uniform Losses or the Contested Garment method (here, the differential equation holds only for some subdomain of $[0,a+b]$). These equations can easily be solved.

2.2 Loss Proportional method

At each point (if $t \leq a+b$), each agent suffers a relative loss: $(a - r(a,b,t))/a$ for the first agent and $(b - t + r(a,b,t))/b$ for the second agent. These losses might be considered as possible measures of the need or desire for additional increments. It might then be reasonable to share each increment between the agents *according to the ratio of their losses.* Formally,

$$\frac{\partial r/\partial t}{\partial(t-r)/\partial t} = \frac{\frac{a-r(a,b,t)}{a}}{\frac{b-t+r(a,b,t)}{b}}, \quad 0 \leq t \leq a+b. \tag{3}$$

In order to solve this equation, let us introduce the mapping y defined by

$$y(a,b,x) = t - r(a,b,t) \text{ for } t \text{ such that } t = x + y(a,b,x), \tag{4}$$

or by

$$x = r(a,b,x+y(a,b,x)). \tag{5}$$

In other words, for any x in $[0,a]$ representing the share of agent 1, the value $y(a,b,x)$ is the share of the second agent. To each rationing method r corresponds one and only one mapping y. Therefore, I will often use y instead of r and I will use the same name *rationing method* for y.

Note that, in general, y is a mapping but not necessarily a function. It can be a one-to-many mapping and it needs not be defined for all values of x between 0 and a. It is a function (a one-to-one mapping) if and only if r is strictly monotonic.

A 1 *Strict Monotonicity.*

$$t' > t \Rightarrow r(a,b,t') > r(a,b,t) \textit{ and } t' - r(a,b,t') > t - r(a,b,t).$$

Because (3) clearly imposes that r be strictly monotonic, y is a function and we can restate (3) in terms of y and we obtain

$$\frac{\partial y(a,b,x)}{\partial x} = \frac{a}{a-x} \; \frac{b-y(a,b,x)}{b}, \quad 0 \leq x \leq a. \tag{6}$$

With the boundary condition $r(a,b,0)=0$, we find the unique solution

$$y(a,b,x) = b\left[1 - \left(\frac{a-x}{a}\right)^{a/b}\right]. \tag{7}$$

Unfortunately, expressing this solution in terms of $r(a,b,t)$ rather than $y(a,b,x)$ requires to solve a polynomial in $x^{a/b}$ which is in general impossible. Nevertheless, a numerical solution can easily be found and (7) defines an interesting rationing method even if not in a closed form. I suggest to call it *Loss Proportional*. The shape of the function y is illustrated in Fig. 1. An interesting feature of this rationing method is how it shares the first units, close to the origin. When $t = 0$, we have $x = 0 = y(a,b,x)$. Therefore, the relative loss of each agent is equal to 1 and the slope of the function y is equal to 1. It can be seen as reasonable to share equally the first increments because both agents suffer such large losses that favouring one of them would be unfair. Because we will use this property later on, I give it a name.

A 2 *Equal Treatment at Full Dissatisfaction.* $\frac{\partial y(a,b,x)}{\partial x}|_{x=0} = 1$.

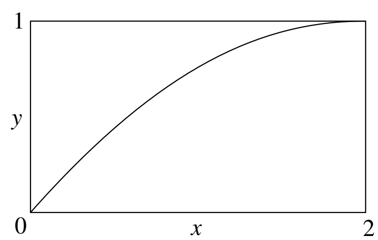

Figure 1: The Loss Proportional method for $a = 2$ and $b = 1$.

2.3 Gain Proportional method

Here, I follow an approach that can be seen as dual to the approach that lead us to the Loss Proportional method. At each point (if $t \leq a+b$), each agent enjoys a relative gain: $r(a,b,t)/a$ for the first agent and $(t - r(a,b,t))/b$ for the second agent. These gains might be considered as possible measures of the satisfaction of the agents. Let us now think in terms of decrement rather than increment.

If $t = a + b$ and if the amount of the shared resource decreases by one infinitesimal decrement, we might think that we have no good reason to favour one of the agents as both of them are fully satisfied. Therefore, each agent should lose one half of the decrement. In other words,

A 3 *Equal Treatment at Full Satisfaction.* $\frac{\partial y(a,b,x)}{\partial x}|_{x=a} = 1$.

If t continues to decrease and if both agents continue to equally share the losses, the satisfaction or relative gain of one of the agents (say 2, if $a > b$) will decrease faster than the satisfaction of the other one. It might then be fair to share the following decrements *according to the ratio of the gains*. Formally,

$$\frac{\partial r/\partial t}{\partial(t-r)/\partial t} = \frac{\frac{r(a,b,t)}{a}}{\frac{t-r(a,b,t)}{b}} \ , \quad 0 \le t \le a+b. \tag{8}$$

Using y and the boundary condition $r(a,b,a+b) = a$, we find the unique solution

$$y(a,b,x) = b\left(\frac{x}{a}\right)^{a/b} . \tag{9}$$

Here again, we cannot obtain a closed form for r. Fig. 2 illustrates how the Gain Proportional method works. Obviously, the Gain Proportional method is the dual of the Loss Proportional one where Duality is defined as follows.

A 4 *Duality. Two rationing methods r and r^* are* dual *to each other if $r^*(a,b,t) = a - r(a,b,a+b-t)$.*

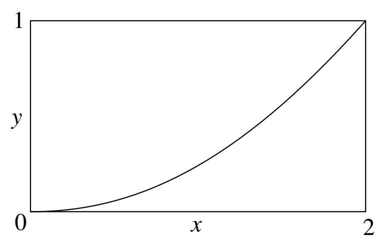

Figure 2: The Gain Proportional method for $a = 2$ and $b = 1$.

2.4 Gain or Loss Proportional method

It might be interesting to follow the Loss Proportional method when close to the origin and the Gain Proportional method when t is close to $a+b$. There are at least two possible ways to do this. The first one is to gradually shift from one principle to the other one, as indicated in the following differential equation.

$$\frac{\partial r/\partial t}{\partial(t-r)/\partial t} = \frac{b}{a}\left[\frac{a-r(a,b,t)}{b-t+r(a,b,t)} + \frac{r(a,b,t)}{t-r(a,b,t)}\right]. \tag{10}$$

Close to the origin, the first term (the loss part) drives the slope while the second one is almost zero. When t approaches $a+b$, the second term becomes dominant. This equation has no solution satisfying the boundary conditions.

Another way to shift from a loss oriented to a gain oriented scheme is to share the resource with the Loss Proportional method up to the point where each agent gets a fixed quantity q_i and, then, to turn to the Gain Proportional method for sharing the rest of the resource. We will assume that q_i is the same function of the demand for both agents, i.e. $q_1 = \varepsilon(a)$ and $q_2 = \varepsilon(b)$. Formally,

$$\frac{\partial r/\partial t}{\partial(t-r)/\partial t} = \begin{cases} \dfrac{\frac{\varepsilon(b)}{\varepsilon(b)-t+r(\varepsilon(a),\varepsilon(b),t)}}{\frac{\varepsilon(a)}{\varepsilon(a)-r(\varepsilon(a),\varepsilon(b),t)}}, & t < \varepsilon(a)+\varepsilon(b), \\ \\ \dfrac{\frac{b-\varepsilon(b)}{t-r(a-\varepsilon(a),b-\varepsilon(b),t-\varepsilon(a)-\varepsilon(b))}}{\frac{a-\varepsilon(a)}{r(a-\varepsilon(a),b-\varepsilon(b),t-\varepsilon(a)-\varepsilon(b))}}, & t \geq \varepsilon(a)+\varepsilon(b). \end{cases} \tag{11}$$

Using the boundary conditions $r(a, b, 0) = 0$ and $r(a, b, a + b) = a$, the solution is found to be

$$y_\varepsilon(a,b,x) = \begin{cases} \varepsilon(b) - \varepsilon(b)\left[\frac{\varepsilon(a)-x}{\varepsilon(a)}\right]^{\varepsilon(a)/\varepsilon(b)}, & x < \varepsilon(a), \\ \\ \varepsilon(b) + [b-\varepsilon(b)]\left[\frac{x-\varepsilon(a)}{a-\varepsilon(a)}\right]^{\frac{a-\varepsilon(a)}{b-\varepsilon(b)}}, & x \geq \varepsilon(a). \end{cases} \tag{12}$$

I call this family of rationing methods, parametrized by ε, the Gain or Loss Proportional method. It is depicted in Fig. 3. This family seems somehow ad

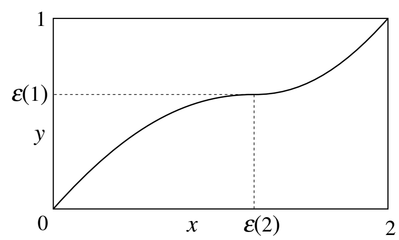

Figure 3: The method defined by (12) for $a = 2$, $b = 1$ and $\varepsilon(x) = 0.6x$.

hoc. It combines the ideas of proportionality to the relative loss and proportionality to the relative gain. But the way these two ideas are combined seems quite arbitrary. Some deeper motivation is required and will be provided, at least partly, in Section 3. This method satisfies Equivalent Treatment at Full Satisfaction and Dissatisfaction.

2.5 Classical axioms

In the next paragraphs, we consider some classical axioms found in the rationing literature (for a survey, see Moulin, 2002 or Thomson, 2003) and see which ones are satisfied by the three new methods introduced in Sections 2.2, 2.3 and 2.4. We use the terminology of Moulin (2002). The Loss Proportional and Gain Proportional methods satisfy Strict Ranking, Strict Ranking*, Invariance, Symmetry, Continuity and Strict Monotonicity. The Loss Proportional method is regressive while, by duality, the Gain Proportional one is progressive. None of them satisfy Lower or Upper Composition nor Equal Sacrifice.

The Gain or Loss Proportional method defined by (12) satisfies Strict Ranking, Strict Ranking*, Invariance, Continuity, Strict Monotonicity and Symmetry. It satisfies Self Duality if $\varepsilon(u) = u/2$ for all $u > 0$. It is not Regressive nor Progressive and doesn't satisfy Lower or Upper Composition nor Equal Sacrifice.

We now consider another property. Let $f(\lambda, z)$ be a real-valued function with $0 \leq \lambda \leq \Lambda$ and $z \geq 0$ and Λ finite or not. Furthermore, $f(0, z) = 0$, $f(\Lambda, z) = z$ and $f(\lambda, z)$ is non-decreasing and continuous in λ over $[0, \Lambda]$. We say that a rationing method r is parametric (Young, 1987) if $r(a, b, t) = f(\lambda, a)$ and $t - r(a, b, t) = f(\lambda, b)$ where λ is a solution of $f(\lambda, a) + f(\lambda, b) = t$.

The Gain Proportional method is parametric, with $f(\lambda, z) = z\lambda^{1/z}$ and $\Lambda = 1$. This is easy to find: starting from

$$x + y(a, b, x) = t,$$

we replace $y(a, b, x)$ by

$$b\left(\frac{x}{a}\right)^{a/b},$$

yielding

$$x + b\left(\frac{x}{a}\right)^{a/b} = t.$$

Letting $\lambda = (x/a)^a$, we find

$$a\lambda^{1/a} + b\lambda^{1/b} = t,$$

and the proof is complete.

The Loss Proportional method is also parametric, with

$$f(\lambda, z) = z(1 - (1 - \lambda)^{1/z})$$

and $\Lambda = 1$.

The Gain or Loss Proportional method $y_\varepsilon(a, b, x)$ is parametric with

$$f(\lambda, z) = \begin{cases} \varepsilon(z) - \varepsilon(z)(1 - 2\lambda)^{1/\varepsilon(z)} & \text{if } 0 \leq \lambda \leq 1/2 \\ \varepsilon(z) + [z - \varepsilon(z)](2\lambda - 1)^{1/[z - \varepsilon(z)]}) & \text{if } 1/2 \leq \lambda \leq 1. \end{cases}$$

3 An Equal Sacrifice approach

The Equal Sacrifice condition has its roots in taxation policy (see e.g. Mill, 1859; Young, 1990). It is usually presented like this:

$$u(a) - u(a - v) = u(b) - u(b - w),$$

where a en b are the income of the agents, v and w are the taxes to be paid, u is a continuous and strictly increasing function. The obvious translation of this condition in our context is:

$$u(a) - u(r(a, b, t)) = u(b) - u(t - r(a, b, t)), \quad 0 \leq t \leq a + b, \tag{13}$$

Because u is strictly increasing, we find that r is Strictly Monotonic and, so, we can restate (13) in terms of y instead of r:

$$u(a) - u(x) = u(b) - u(y(a,b,x)), \quad 0 \leq x \leq a, \tag{14}$$

Because of the Strict Monotonicity of r, $y(a,b,0) = 0$, and (14) boils down to

$$u(x) = u(y(a,b,x))$$

and

$$y(a,b,x) = x$$

which is possible only if $a = b$. This is clearly not interesting; but, in many cases, it is reasonable to suppose that u is not the same for both agents. For example, if food has to be shared among refugees, a father of 4 children will not experiment the same utility as a father of two children if they receive the same quantity. So we can rewrite (14) as

$$u_1(a) - u_1(x) = u_2(b) - u_2(y(a,b,x)), \quad 0 \leq x \leq a. \tag{15}$$

This equation defines a principle that could be called *Equivalent Sacrifice*[2]. Note that, for the same reasons as above, $y(a,b,0) = 0$ and (15) is equivalent to

$$u_1(x) = u_2(y(a,b,x)). \tag{16}$$

Now, if we do not want the rationing method to depend upon the agents' identities (their name, hair color or any irrelevant characteristic), we may impose Symmetry.

A 5 *Symmetry. A rationing method y is* symmetric *iff $y(b,a,y(a,b,x)) = x$.*

Under Symmetry, u cannot depend upon the agent himself but only upon his demand. This can be shown formally: $u_1(z)$ can be written more explicitely as $u_1(a,z)$ because, if the utility depends on the agent, then it also depends on his demand. Similarly, the utility of the second agent can be written as $u_2(b,z)$. Suppose now that the utility of an agent depends on the agent himself and not only on his demand. Then, there is d and w such that $u_1(d,w) \neq u_2(d,w)$. Using then (16) with a demand d for both agents, we find that $y(d,d,w) \neq w$. This contradicts symmetry. So, if Symmetry is satisfied, then $u_1(d,z) = u_2(d,z)$.

So, we can consider u as a function of the demand and (16) becomes

$$u(a,x) = u(b,y(a,b,x)), \quad 0 \leq x \leq a. \tag{17}$$

[2]I use the name Equivalent Sacrifice instead of Equal Sacrifice because Moulin (2002) uses the name Equal Sacrifice for another weakening of (14), namely

$$x > 0 \Rightarrow u(a) - u(x) \geq u(b) - u(y(a,b,x)) \text{ and}$$
$$y(a,b,x) > 0 \Rightarrow u(b) - u(y(a,b,x)) \geq u(a) - u(x).$$

3.1 Another derivation of the Gain Proportional method

I now examine the consequences of adopting one of four *ad hoc* utility function u: the linear utility, the power function, the exponential and the logarithm, because these are so common in the economic literature. I will further give a stronger motivation for the power function.

Linear utility The utility of both agents is linear if it can be put in the form

$$u_1(x) = m_1 x \text{ and } u_2(x) = m_2 x.$$

If the utility depends only on the demand of the player (as imposed by symmetry), then m is a function of the demand and $u(a,x) = m(a)x$. So, (17) becomes

$$m(a)x = m(b)y(a,b,x), \quad 0 \le x \le a. \tag{18}$$

Because u is strictly increasing, we know that $y(a,b,a) = b$ and so, $m(a) = c/a$ where c is a positive real number. Finally, we find that $y(a,b,x) = \frac{b}{a}x$, i.e. y is the proportional method.

Power function The utility of an agent is a power function (for a characterization of this utility function, see e.g. Marchant & Luce, 2003) if it can be written as $u(x) = \phi x^{\psi}$. If the utility of an agent can vary with his demand, then $u(a,x) = \phi(a)x^{\psi(a)}$ and (17) becomes

$$\phi(a)x^{\psi(a)} = \phi(b)(y(a,b,x))^{\psi(b)}, \quad 0 \le x \le a. \tag{19}$$

Without loss of generality, we can choose $u(a,a) = 1$. Consequently, $u(x,x) = 1$. So,

$$\phi(x)x^{\psi(x)} = 1$$

and

$$\phi(x) = x^{-\psi(x)}.$$

Finally, (19) becomes

$$\left(\frac{x}{a}\right)^{\psi(a)} = \left(\frac{y(a,b,x)}{b}\right)^{\psi(b)}. \tag{20}$$

If we now impose Equal Treatment at Full Satisfaction, we find that $\psi(x) = kx$, with $k > 0$ and, therefore, $y(a,b,x)$ is the Gain Proportional method.

Exponential Another popular form for the utility function is the exponential, i.e.

$$u(a,x) = \frac{1 - e^{-h(a)x}}{g(a)}, \quad \text{with } h(a)g(a) > 0.$$

This form has also recently received strong theoretical support (Luce, 2000). Without loss of generality, we can choose $u(a,a) = 1$ and we obtain then

$$g(x) = 1 - e^{-h(x)x}.$$

If we impose Symmetry and Equivalent Sacrifice with an exponential utility function, we then find

$$y(a,b,x) = -\frac{1}{h(b)} \ln\left(1 - (1 - e^{-h(a)x})\frac{1 - e^{-h(b)b}}{1 - e^{-h(a)a}}\right). \tag{21}$$

If we then impose that $h(x)$ be constant for all x, we obtain the Uniform Gains method when that constant tends to infinity. But we see no good reason to impose this.

It is interesting to note that none of the rationing methods satisfying Symmetry and Equivalent Sacrifice with an exponential utility function verifies Equal Treatment at Full Satisfaction. This is actually because we allow zero demands. But if we exclude the case $a = 0$ or $b = 0$, then there is a solution satisfying Equal Treatment at Full Satisfaction : it cannot be put in a closed form and does not seem very interesting. I therefore do not present it here.

Logarithm Let us now turn to the logarithmic utility function (also recently characterized by Marchant and Luce (2003)):

$$u(a,x) = p(a)\ln[q(a)x], \quad 0 < x \leq a.$$

With this utility function, (17) becomes

$$p(a)\ln\frac{x}{a} = p(b)\ln\frac{y(a,b,x)}{b}, \quad 0 < x \leq a.$$

Solving for y, we find

$$y(a,b,x) = b\exp\left(\frac{p(a)}{p(b)}\ln\frac{x}{a}\right), \quad 0 < x \leq a.$$

Imposing Equal Treatment at Full Satisfaction yields $p(a) = \gamma a$ and

$$y(a,b,x) = b\left(\frac{x}{a}\right)^{a/b}$$

which is precisely the Gain Proportional method.

3.2 Another derivation of the Gain or Loss Proportional method

In their characterization of the power function, Marchant and Luce (2003) assumed the existence of a status quo : ε. All values above ε are considered as gains and all values below ε are considered as losses. The way gains and losses are perceived by an agent need not be the same. Since Kahneman and Tversky's paper on prospect theory in 1979, many papers have provided empirical support for such a status quo.

This status quo is very likely to exist and play an important role in many rationing problems. Suppose an agent claims 100 after a bankruptcy. He probably expects to get less than 100 because he suspects t to be smaller than the sum of the claims. But he might hope to get at least 40 for different reasons (if he gets less than 40, then he is in trouble; he thinks that 40 is reasonable taking into account the other demands and t; ...). So, when he finally gets his share, if it is more than 40, he might consider this as good news; it is more than what he expected. It is a gain. But if he gets less than 40, he might be disappointed and consider this as a loss. The status quo corresponds in this case to the expectation of the agent.

Using some of the axioms of extensive measurement (in absence of risk or uncertainty) and an invariance condition, Marchant and Luce (2003) derived the power function used in the preceding section by assuming that $\varepsilon = 0$. But we might as well consider non-zero values for ε. In that case, Marchant and Luce (2003) derived a more general power function, namely :

$$u(x) = \begin{cases} \phi(x-\varepsilon)^{\psi}, & x \geq \varepsilon \\ \\ \phi'(\varepsilon - x)^{\psi}, & x < \varepsilon \end{cases} \tag{22}$$

where $\phi > 0$ and $\phi' < 0$. Note that thanks to the special role played by the status quo, this utility function can not be transformed by a translation but only by a multiplicative factor α (for the gains) or α' (for the losses).

If we now consider the status quo of each agent as being equal to his own demand, then, following the same reasonning as in the preceding section, we easily obtain the Loss Proportional method, by assuming Equivalent Sacrifice, Symmetry and Equal Treatment at Full Dissatisfaction.

But we can also take a more general route and consider the status quo of each agent i as being equal to some value ε_i between 0 and his demand. The next step is obviously to impose Symmetry; then the status quo of each agent depends only on his demand, i.e. $\varepsilon_1 = \varepsilon(a)$ and $\varepsilon_2 = \varepsilon(b)$ and the utility function can be written as

$$u(a,x) = \begin{cases} \phi(a)\,[x - \varepsilon(a)]^{\psi(a)}\,, & x \geq \varepsilon(a) \\ \\ \phi'(a)\,[\varepsilon(a) - x]^{\psi'(a)}\,, & x < \varepsilon(a) \end{cases} \tag{23}$$

where $\phi(a) > 0$ and $\phi'(a) < 0$. A reasonable condition in this context is to impose that the status quo of both agents be commensurable.

A 6 *Status Quo Commensurability.* $y(a, b, \varepsilon_1) = \varepsilon_2$.

This, combined with some weak conditions (for example monotonicity), implies the following : if an agent enjoys a gain (a loss), with respect to his status quo, then the other agent should also enjoy a gain (resp. a loss), with respect to his own status quo.

Imposing then Equivalent Sacrifice (15) defines a very large set of rationing methods in which at least three interesting subsets can be distinguished.

- If $\varepsilon(x) = 0$ and Equal Treatment at Full Satisfaction is satisfied, then we obtain the Gain Proportional method (see p. 9). This is consistent with the fact that we obtained it in Section 2 by reasoning in terms of gains.

- If $\varepsilon(x) = x$ and Equal Treatment at Full Dissatisfaction is satisfied, then we obtain the Loss Proportional method. This is consistent with the fact that we obtained it in Section 2 by reasoning in terms of losses.

- If $0 < \varepsilon(x) < x$ and if Status Quo Commensurability and Equal Treatment at Full Satisfaction and at Full Dissatisfaction are satisfied, then we obtain the Gain or Loss Proportional method defined by (12). If, on top of this, we impose Self Duality, then $\varepsilon(x) = x/2$.

3.3 A complete characterization of the Gain or Loss Proportional method

In this section, we formally restate the results of Section 3.2. We begin with the characterization of the power function by Marchant and Luce (2003). Let $\oplus$ be a binary operation called joint receipt. The joint receipt of two objects ξ and η, written $\xi \oplus \eta$, is the fact of receiving both of them. We denote by $\mathcal{D}$ the set of objects under consideration. The preferences of an agent over $\mathcal{D}$ are described by the binary relation $\succsim$, whose asymmetric (resp. symmetric) part is denoted by $\succ$ (resp. $\sim$). We assume that $\mathcal{D}$ is closed under $\oplus$, i.e. the joint receipt of two objects in $\mathcal{D}$ is also in $\mathcal{D}$. In the context of rationing methods, the elements of $\mathcal{D}$ are quantities of some resource to be divided among two agents. These quantities can be positive or negative. Indeed, even if the share of each agent is always non-negative, we consider positive and negative quantities because the share of an agent can be the joint receipt of a positive and a negative quantity. This is particularly relevant when we consider that t can vary and that the share of an agent is his previous share together with his share (positive or negative) of an increment or decrement. So, we will consider that $\mathcal{D} = \mathbb{R}$. Experimental research (for example, Luce, 2000) has shown that $\xi \oplus \eta$ is not necessarily perceived or valued the same as $\xi + \eta$. Some reasonable conditions can be imposed on $\oplus$. For all ξ, η, ζ in $\mathbb{R}$,

A 7 *Associativity.* $\xi \oplus (\eta \oplus \zeta) \sim (\xi \oplus \eta) \oplus \zeta$.

A 8 *Monotonicity.* $\xi \succsim \eta \Leftrightarrow \zeta \oplus \xi \succsim \zeta \oplus \eta$.

A 9 *Identity. There is ε in $\mathcal{D}$ such that $\varepsilon \oplus \xi \sim \xi$.*

A 10 *Continuity. $\oplus$, considered as a function of two variables, is continuous in each variable.*

A 11 *Displaced Multiplicative Invariance. There exists a function σ from $\mathbb{R}^+$ to $\mathbb{R}^+$ such that, for some real numbers s and s' and for all $\xi, \eta, \zeta \in \mathcal{D}$, with $\zeta > 0$,*

$$[\zeta(\xi - s) + s] \oplus [\zeta(\eta - s) + s] = \sigma(\zeta)[(\xi \oplus \eta) - s'] + s'.$$

Note that Displaced Multiplicative Invariance is a generalization of Homogeneity where the role of the zero (the absorbing element for multiplication) is played by s in the left-hand side and by s' in the right-hand side. Let $s' = s = 0$ and $\sigma(\zeta) = \zeta$; we then obtain:

A 12 *Homogeneity. For all $\zeta > 0$,*

$$(\zeta\xi) \oplus (\zeta\eta) = \zeta(\xi \oplus \eta).$$

Proposition 1 (Marchant & Luce, 2003) *Let $\mathcal{D} = \mathbb{R}$ and $\succsim$ be the usual ordering $\geq$ on the real numbers. Suppose that $\oplus$, defined over $\mathbb{R}$, satisfies Associativity, Identity, Monotonicity, Continuity and Displaced Multiplicative Invariance. Then $s = s' = \varepsilon$ and $\sigma(z) = z$ and, for some constants $\beta > 0$ and $\gamma < 0$, $\xi \oplus \eta$ is given by:*

$$\xi \oplus \eta = \begin{cases} [(\xi-\varepsilon)^\beta + (\eta-\varepsilon)^\beta]^{1/\beta} + \varepsilon, & \xi \geq \varepsilon, \eta \geq \varepsilon \\ [(\xi-\varepsilon)^\beta + \frac{1}{\gamma}(\varepsilon-\eta)^\beta]^{1/\beta} + \varepsilon, & \xi \geq \varepsilon > \eta, \xi \oplus \eta \geq \varepsilon \\ \varepsilon - [\gamma(\xi-\varepsilon)^\beta + (\varepsilon-\eta)^\beta]^{1/\beta}, & \xi \geq \varepsilon > \eta, \xi \oplus \eta < \varepsilon \\ \varepsilon - [(\varepsilon-\xi)^\beta + (\varepsilon-\eta)^\beta]^{1/\beta}, & \xi \leq \varepsilon, \eta \leq \varepsilon. \end{cases} \tag{24}$$

Furthermore, there is a value or utility function u defined by (22), where $\phi' = \phi/\gamma$ and such that $u(\xi \oplus \eta) = u(\xi) + u(\eta)$.

We are now ready to characterize the Gain or Loss Proportional method.

Proposition 2 *Let $\oplus_1$ and $\oplus_2$ be two binary operations defined over $\mathbb{R}$ and representing the joint receipt for agents 1 and 2, with $\mathbb{R}$ closed under $\oplus_1$ and $\oplus_2$. Suppose that $\oplus_1$ and $\oplus_2$ satisfy Associativity, Monotonicity, Continuity, Displaced Multiplicative Invariance and Identity, with ε_1 and ε_2. Let y be a rationing method defined for two agents 1 and 2, with demands a and b. Suppose that y satisfies Symmetry, the Equivalent Sacrifice condition (15) with*

$$u_i(x \oplus_i y) = u_i(x) + u_i(y),\ i \in \{1, 2\},$$

Status Quo Comparability, Equal Treatment at Full Satisfaction and Dissatisfaction. Then, y is the Gain or Loss Proportional method defined by (12).

Proof. By Proposition 1, we know that there are utility functions such that $u_i(x \oplus_i y) = u_i(x) + u_i(y)$, $i \in \{1, 2\}$ and they necessarily have the form given by (22). Because of Symmetry, the utility functions to be used in the Equivalent Sacrifice Condition may not depend on the agent but only on his demand. So, combining Proposition 1 and Symmetry, the utility functions to be used in the Equivalent Sacrifice Condition are defined by (23). Because (23) is strictly increasing w.r.t. x and because of Status Quo Comparability, it is clear that agent 1 receives a gain (a loss) iff agent 2 also receives a gain (resp. a loss). The Equivalent Sacrifice condition can therefore be written as

$$\begin{cases} \phi(a)\,[x-\varepsilon(a)]^{\psi(a)} = \phi(b)\,[y(a,b,x)-\varepsilon(b)]^{\psi(b)}\,, & x \geq \varepsilon(a) \\ \\ \phi'(a)\,[\varepsilon(a)-x]^{\psi'(a)} = \phi'(b)\,[\varepsilon(b)-y(a,b,x)]^{\psi'(b)}\,, & x < \varepsilon(a) \end{cases} \tag{25}$$

If we solve for y, we obtain a family of rationing methods defined by

$$y(a,b,x) = \begin{cases} \varepsilon(b) + \left[\frac{\phi(a)}{\phi(b)}\right]^{1/\psi(b)} [x-\varepsilon(a)]^{\psi(a)/\psi(b)} & x \geq \varepsilon(a) \\ \\ \varepsilon(b) - \left[\frac{\phi'(a)}{\phi'(b)}\right]^{1/\psi'(b)} [\varepsilon(a)-x]^{\psi'(a)/\psi'(b)} & x < \varepsilon(a) \end{cases} \tag{26}$$

Let us now write the partial derivative of y for the gains

$$\frac{\partial y(a,b,x)}{\partial x} = \left[\frac{\phi(a)}{\phi(b)}\right]^{1/\psi(b)} \frac{\psi(a)}{\psi(b)}\,[x-\varepsilon(a)]^{[\psi(a)/\psi(b)]-1}\,, \quad x \geq \varepsilon(a),$$

and impose Equal Treatment at Full Satisfaction:

$$\left[\frac{\phi(a)}{\phi(b)}\right]^{1/\psi(b)} \frac{\psi(a)}{\psi(b)}\,[a-\varepsilon(a)]^{[\psi(a)/\psi(b)]-1} = 1.$$

So,

$$\left[\frac{\phi(a)}{\phi(b)}\right]^{1/\psi(b)} = \frac{\psi(b)}{\psi(a)} \frac{a-\varepsilon(a)}{[a-\varepsilon(a)]^{\psi(a)/\psi(b)}}$$

and we can rewrite (26) for the gains:

$$y(a,b,x) = \varepsilon(b) + \frac{\psi(b)}{\psi(a)}[a-\varepsilon(a)] \left[\frac{x-\varepsilon(a)}{a-\varepsilon(a)}\right]^{\psi(a)/\psi(b)}, \quad x \geq \varepsilon(a). \tag{27}$$

Imposing now the boundary condition $y(a,b,a) = b$, we obtain

$$b = \varepsilon(b) + \frac{\psi(b)}{\psi(a)}[a-\varepsilon(a)]$$

or

$$\frac{\psi(b)}{\psi(a)} = \frac{b-\varepsilon(b)}{a-\varepsilon(a)}.$$

We now replace in (27).

$$y(a,b,x) = \varepsilon(b) + [b-\varepsilon(b)] \left[\frac{x-\varepsilon(a)}{a-\varepsilon(a)}\right]^{\frac{a-\varepsilon(a)}{b-\varepsilon(b)}}, \quad x \geq \varepsilon(a).$$

If we follow the same reasoning for the losses, the proof is complete. □

Similar propositions can easily be written for the Gain Proportional method and for the Loss Proportional method. For the former, we just have to impose $\varepsilon(x) = 0, \forall x \geq 0$, and drop Status Quo Commensurability and Equal Treatment at Full Dissatisfaction. For the latter, we have to impose $\varepsilon(x) = x, \forall x \geq 0$, and drop Status Quo Commensurability and Equal Treatment at Full Satisfaction.

4 Back to differential equations

The Gain Proportional method has been derived in two different ways: in Section 2, I used a differential equation; in Section 3, I used the Equivalent Sacrifice principle. These two approaches were also followed for deriving the Loss Proportional method and the Gain or Loss Proportional method. I now explore the links existing between these two approaches.

Proposition 3 *The rationing method y satisfies the Equivalent Sacrifice condition (16), with u_1 and u_2 almost everywhere differentiable, if and only if y is the solution of a differential equation of the form*

$$\frac{\partial y(a,b,x)}{\partial x} = \frac{f(x)}{g(y(a,b,x))}, \quad x \in [0,a] \ \setminus \{s_1, \ldots, s_p\}, \tag{28}$$

where

- $\{s_1, \ldots, s_p\}$ *is a finite subset of* $[0,a]$,
- $f(x) > 0$ *and* $g(y(a,b,x)) > 0, \forall x \in [0,a] \ \setminus \{s_1, \ldots, s_p\}$.

Proof. As shown higher, the Equivalent Sacrifice condition can be written as

$$u_1(x) = u_2(y(a,b,x)).$$

Because u_2 is continuous and strictly increasing, we can write

$$y(a,b,x) = u_2^{-1}(u_1(x)).$$

Let $\{r_1, \ldots, r_q\}$ be the union of all points in $[0,a]$ where u_1 or u_2 are not differentiable. This set is finite. So, we can write

$$\frac{\partial y(a,b,x)}{\partial x} = \frac{u_1'(x)}{u_2'(y(a,b,x))}, \quad x \in [0,a] \ \setminus \{r_1, \ldots, r_q\}.$$

Let then $f = u_1'$, $g = u_2'$, $p = q$ and $\{s_1, \ldots, s_p\} = \{r_1, \ldots, r_q\}$. Because u_1 and u_2 are strictly increasing, f and g are strictly positive. So, the "only if" part of the proof is complete.

We now turn to the "if" part. Equation (28) can be rewritten as

$$g(y(a,b,x))\, \partial y(a,b,x) = f(x)\, \partial x, \quad x \in [0,a] \ \setminus \{s_1, \ldots, s_p\}$$

which admits, among others, the following solution:

$$G(y(a,b,x)) = F(x),$$

where

$$G(z) = \int_{[0,z] \setminus \{s_1, \ldots, s_p\}} g(w)\, \mathrm{d}w$$

and

$$F(z) = \int_{[0,z]\,\setminus\{s_1,\ldots,s_p\}} f(w)\,\mathrm{d}w.$$

Because f and g are strictly positive, F and G are strictly increasing. They are also clearly continuous and almost everywhere differentiable. Let then $u_1 = F$ and $u_2 = G$, and the proof is complete. □

Roughly speaking, a rationing method can be described by a differential equation if and only if it satisfies the Equivalent Sacrifice condition. In the next proposition, the consequences of Symmetry are analyzed. From now on, I make the assumption that u is everywhere differentiable. The previous assumption (that it is almost everywhere differentiable) does not bring much and complicates the statement of the results. Furthermore, I have shown in Proposition 3 how the case of almost everywhere differentiable utilities can be dealt with.

Proposition 4 *Consider a Symmetric rationing method y. It satisfies the Equivalent Sacrifice condition (16), with $u(a,x)$ differentiable with respect to x, if and only if y is the solution of a differential equation of the form*

$$\frac{\partial y(a,b,x)}{\partial x} = \frac{f(a,x)}{f(b,y(a,b,x))}, \quad 0 \le x \le a,$$

with $f(a,z) > 0$ $\forall z$ such that $0 \le z \le a$.

Proof. Because $y(a,b,0) = 0$, the Equivalent Sacrifice condition and Symmetry yield

$$u(a,x) = u(b,y(a,b,x)).$$

Because u is continuous and strictly increasing in its second argument, there exists a function $u_b^{-1}(y)$ which is the inverse of $u(b,y)$ when b is hold constant. Hence

$$y(a,b,x) = u_b^{-1}(u(a,x))$$

and

$$\frac{\partial y(a,b,x)}{\partial x} = \frac{\frac{\partial u(a,x)}{\partial x}}{\frac{\partial u(b,y(a,b,x))}{\partial x}}.$$

Let then

$$f(a,x) = \frac{\partial u(a,x)}{\partial x}.$$

This proves the "only if" part. The "if" part is simple. □

We now look at the conditions that, together with Equivalent Sacrifice, are equivalent to a differential equation where the function f of Proposition 4 would depend only on the relative gain (as in the Gain Proportional method).

Proposition 5 *Consider a Symmetric rationing method y. It satisfies the Equivalent Sacrifice condition, with $u(a,x)$ homogeneous, i.e. $\lambda u(a,x) = u(\lambda a, \lambda x)$, and differentiable with respect to x, if and only if y is the solution of a differential equation of the form*

$$\frac{\partial y}{\partial x} = \frac{f(x/a)}{f(y/b)}, \quad 0 \leq x \leq a,$$

with $f(z) > 0$ $\forall z$ such that $0 \leq z \leq 1$.

Proof. Note first that $\lambda u(a,x) = u(\lambda a, \lambda x)$ implies $u(a,x) = au(1, x/a)$. Define then $h(x) = u(1,x)$ and we can write $u(a,x) = ah(x/a)$. The rest of the proof is similar to those of Propositions 3 and 4 and is left to the reader. □

Note that the Gain (Loss) Proportional method corresponds to this last proposition, with $f(x) = 1/x$ (resp. $1/(1-x)$). Propositions 4 and 5 can also be generalized to almost everywhere differentiable utility functions. This is left as an exercise.

5 When there are more than two agents

Any of the rationing methods I have presented so far or that could possibly be obtained as a solution of a differential equation like (6), (8) or more generally (28) can trivially be extended to the case of more than 2 agents. Let us slightly modify the notation. Let $\mathbf{a}$ be a n-dimensional vector such that a_i is the demand of agent i. Then $y_i(\mathbf{a}, x)$, $i = 2 \ldots n$, is the share of agent i when agent 1 receives the share x. We can write $n-1$ differential equations, each one linking agent 1 to another agent. By transitivity, each agent will be linked to any other agent. For example, if we want to generalize the Gain Proportional method, we write the following system.

$$\frac{\partial y_i(\mathbf{a}, x)}{\partial x} = \frac{a_1}{x} \; \frac{y_i(\mathbf{a}, x)}{a_i}, \; i = 2 \ldots n. \tag{29}$$

Each equation can be solved independently of the others, yielding

$$y_i(\mathbf{a}, x) = a_i (\frac{x}{a_1})^{a_1/a_i}, \; i = 2 \ldots n. \tag{30}$$

These $n-1$ equations define a curve Γ in the n-dimensional hyper-rectangle with edges of length $a_1, a_2, \ldots, a_n$. Given a quantity t of the shared resource, the shares of the agents will correspond to the intersection of Γ with the hyperplane $x + y_2 + \ldots + y_n = t$. Here again, finding the point corresponding to the intersection might be difficult and require numerical methods.

There might be some other ways to extend the methods presented here to the cases involving more than 2 agents but, as Moulin (1997) showed it, symmetric and resource monotonic methods admit at most one consistent extension.

A 13 *Consistency. Given t, the quantity to be shared, n agents and their demands* $\mathbf{a}$*, their shares are $x, y_2, \ldots, y_n$. If we remove one agent i and accordingly decrease t by his share, the share of each remaining agent remains unchanged.*

It is easy to check that the extension presented in this section is precisely *the* consistent extension.

6 Conclusion

Three new rationing methods were presented in this paper : the Gain Proportional, Loss Proportional, Gain or Loss Proportional methods. They were derived by imposing only *dynamic* conditions, in the form of differential equations. It was also possible to derive them by following an Equivalent Sacrifice approach and using a utility function that was recently characterized by Marchant and Luce (2003). Using their axioms and some axioms specific to the rationing problem, we thus have complete characterizations of the three new methods. Some links have been established between the two approaches (differential equations and Equivalent Sacrifice).

The Equivalent Sacrifice approach seems in some sense superior to the dynamic approach because it yields more "atomic" characterizations. For example, in order to characterize the Gain or Loss Proportional method in Proposition 2 we need a lot of different conditions. If we follow the dynamic approach, we need only one condition to characterize the Gain or Loss Proportional method. It is not so helpful at decomposing a method in its elementary properties.

But, if we look at Proposition 5, we see that the Equivalent Sacrifice approach, with $u(a, x)$ homogeneous, is not very natural or enlightening while the dynamic approach, with

$$\frac{\partial y}{\partial x} = \frac{f(x/a)}{f(y/b)},$$

seems very natural. Both approaches seem thus to have their advantages.

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