

# Consistent bibliometric rankings of authors and of journals

Denis Bouyssou\*      Thierry Marchant†

January 12, 2010

## Abstract

Rankings of journals and rankings of scientists are usually discussed separately. We argue that a consistent approach to both rankings is desirable because both the quality of a journal and the quality of a scientist depend on the papers it/he publishes. We present a pair of consistent rankings (impact factor for the journals and total number of citations for the authors) and we provide an axiomatic characterization thereof.

Keywords: bibliometrics, ranking of journals, ranking of authors, consistent rankings, Impact Factor, citations.

## 1 Introduction

Rankings of academic entities are increasingly popular: rankings of universities, departments, MBA programs, researchers, journals, papers and so on. Yet, there is no consensus about how to rank academic entities; this is reflected in the immense literature about rankings. In this literature, we find papers proposing new ways to rank academic entities (Egghe, 2006; Hirsch, 2005; Pinski and Narin, 1976; Sidiropoulos et al., 2007; Taber, 2005) and supposedly improving on previous ways; papers criticizing specific rankings (Billaut et al., pear; Kokko and Sutherland, 1999) or even the very idea of rankings (Adler et al., 2008; Osterloh and Frey, 2009). We also find papers trying to better understand existing rankings, using for instance a statistical approach or an axiomatic one.

In the latter category, we find axiomatizations of the ranking of journals according to the impact factor (Bouyssou and Marchant, 2009) or according to a recursive impact measure (Palacios-Huerta and Volij, 2004); axiomatizations of the rankings of scientists according to the total number of citations (Marchant, 2009b), the total number of citations weighted by the inverse of the number

---

Authors are listed alphabetically.

\*CNRS and Université Paris Dauphine. Email: bouyssou@lamsade.dauphine.fr

†Ghent University. Dunantlaan 1, 9000 Ghent, Belgium. Email: thierry.marchant@ugent.be. Corresponding author.

of coauthors (Marchant, 2009b), the h-index (Marchant, 2009a; Quesada, 2009; Woeginger, 2008b), the g-index (Woeginger, 2008a)<sup>1</sup>. All these papers have a characteristic in common: they analyze a single ranking, of a single kind of academic entity: either journals or scientists. The present paper is close in spirit to them (it follows an axiomatic approach) but it tries to go further by looking at two kinds of academic entities at a time: journals and scientists. Indeed, it seems to us that we should be consistent in the way we rank scientists and journals, because these entities are linked. Indeed, a journal is a collection of papers written by scientists and we expect, to some extent, that a journal publishing papers written by top scientists be a top journal. We could on the contrary rank scientists and journals according to two completely unrelated criteria (for instance the h-index for authors and the impact factor for journals), not paying attention to consistency. But this may yield paradoxical results. Suppose for instance that authors  $a$  and  $b$  publish all their papers in journals  $j$  and  $j'$  respectively, without coauthors. Suppose also that journals  $j$  and  $j'$  publish no papers except those of authors  $a$  and  $b$ . Suppose finally that  $a$  and  $b$  have the same number of publications. In such a case (although it is not very likely), we would certainly expect that  $a$  and  $b$  be ranked in the same way as  $j$  and  $j'$ , because  $a$  can be identified with  $j$  and  $b$  with  $j'$ . Yet, we can have  $a$  better than  $b$  according to the h-index and  $j$  worse than  $j'$  according to the impact factor. Suppose indeed  $a$  has two publications, each with two citations and  $b$  has two publications, one with one citation and the other one with four citations. Then the h-index of  $a$  is higher than that of  $b$  while the impact factor of  $j$  is lower than that of  $j'$ . This is definitely not acceptable. That is why, we will simultaneously analyze (axiomatize) rankings of scientists and rankings of journals.

On the side of journals, we will focus on the ranking based on the impact factor (Garfield, 1972) and we will combine it, on the side of scientists, with two rankings, according to either the total number of citations (van Raan, 2006) or the total number of citations weighted by the inverse of the number of coauthors (Brown, 1996; Pijpers, 2006). We will show that these two pairs of rankings are consistent (in a sense that we will soon make precise) and we will characterize them by an axiom system in which consistency plays a central role. The reason why we chose these two pairs of rankings is by no means that we consider them as superior but that (i) they form consistent pairs of rankings (in a sense roughly presented in the previous paragraph and formally defined in next section) and (ii) they are very popular and we think it is important to have a deep understanding of tools that are so often used.

In Section 2, we formally present the rankings we want to analyze and the framework in which we consider them. The next section presents a series of axioms that are necessary for our rankings. We then show in Section 4 that these axioms are not only necessary but also sufficient. We then have a short section proving that our axioms are independent and, finally, some concluding remarks.

---

<sup>1</sup>Actually, Palacios-Huerta and Volij (2004), Quesada (2009), Woeginger (2008b) and Woeginger (2008a) do not *stricto sensu* characterize rankings but rather indices.

## 2 Notation, definitions and models

Let  $J = \{j, j', \dots\}$  be a finite set ( $\#J \geq 3$ ) representing the set of all journals we want to consider. It is fixed. The set of all papers published in these journals will be represented by a finite subset of  $\mathcal{P}$ , where  $\mathcal{P}$  is a countably infinite set (for example, and without loss of generality  $\mathbb{N}$ ). We will consider various sets of papers:  $P, P', \dots$ . The typical elements of  $P$  will be  $p, p', q, \dots$ . The set of all authors publishing in the journals in  $J$  is denoted by  $A$ . It is finite, fixed and contains at least three elements. We assume that  $A, P$  and  $J$  are disjoint. We could also consider that  $A$  and  $J$  can vary (various subsets of universal sets  $\mathcal{A}$  and  $\mathcal{J}$ ). Enriching our framework in this way would imply a larger set of axioms in order to characterize the rankings, thereby making our paper cumbersome. That is why we decided to have only one varying set and we chose the set of papers because, in real life, it changes much faster than  $A$  and  $J$ .

A bibliometric profile  $G$  (or just profile, for short) is a digraph whose nodes set is  $A \cup P \cup J$ . Its arcs can be partitioned into three sets:  $S, C$  and  $M$  with the following interpretation. If an author  $a$  publishes a paper  $p$ , then  $(a, p) \in S$ , also written as  $a \underline{S} p$  (author  $a$  Signs paper  $p$ ). If a paper  $p$  cites a paper  $q$ , then  $(p, q) \in C$ , also written as  $p \underline{C}$ ites paper  $q$ ). If a paper  $p$  is published in journal  $j$ , then  $(p, j) \in M$ , also written as  $p \underline{M}$ edium  $j$  (paper  $p$  is published in Medium  $j$ ). For example, Figure 1 represents a profile  $G$  with three authors, four papers and three journals. In this profile, paper  $p$  is published in journal  $j$  and signed by authors  $a$  and  $b$ . It cites paper  $q$  and  $s$  and is not cited.

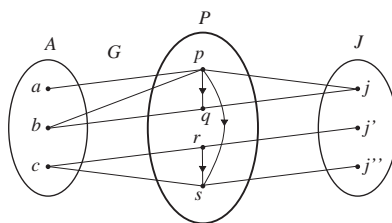


Figure 1: A profile

A bibliometric profile  $G$  is therefore a 6-tuple  $(A, P, J, S, C, M)$  with  $S \subseteq A \times P$ ,  $C \subseteq P \times P$  and  $M \subseteq P \times J$ , satisfying some constraints:

- for all  $p \in P$ , there is  $a \in A : a \underline{S} p$  (a paper has at least one author).
- for all  $p \in P$ , there is a unique  $j \in J : p \underline{M} j$  (a paper is published in one and only one journal).
- there is no cycle of any length in  $C$  (because a paper can only cite older papers).
- each journal publishes at least one paper.

These constraints will not play a major role in the sequel but, every time we will construct new profiles or modify existing ones, we will make sure that they satisfy these constraints.

Since the sets  $A$  and  $J$  will not be allowed to vary in this paper, we will henceforth designate a profile  $G$  by the 4-tuple  $(P, S, C, M)$ , for the sake of brevity. The set of all bibliometric profiles  $G$  (where  $P, S, C$  and  $M$  can vary) is denoted by  $\mathbb{G}$ . Let  $WO(X)$  denote the set of all weak orders<sup>2</sup> on a set  $X$ . A bibliometric ranking system is a mapping  $R : \mathbb{G} \rightarrow WO(A) \times WO(J) : G \rightarrow R(G) = (\succsim_G^A, \succsim_G^J)$ . So, to each bibliometric profile, a ranking system associates two rankings: a ranking of authors and one of journals. The statement  $(a, b) \in \succsim_G^A$  (also written  $a \succsim_G^A b$ ) is interpreted as ‘author  $a$  is at least as good as author  $b$  in the profile  $G$ .’ When  $a \succsim_G^A b$  and NOT  $b \succsim_G^A a$ , we write  $a \succ_G^A b$  ( $a$  is strictly better than  $b$ ). When  $a \succsim_G^A b$  and  $b \succsim_G^A a$ , we write  $a \sim_G^A b$  ( $a$  is equivalent to  $b$ ). Similar interpretations and notation hold for  $\succsim_G^J$ , on the set of journals.

Given a profile  $G = (P, S, C, M)$ , the set of papers signed by author  $a$  is  $S(a)$  and the set of authors signing paper  $p$  is  $S^{-1}(p)$ . The set of papers cited by paper  $p$  is  $C(p)$  and the set of papers citing paper  $p$  is  $C^{-1}(p)$ . The journal publishing paper  $p$  is  $M(j)$  and the set of papers published in journal  $j$  is  $M^{-1}(j)$ .

The impact factor of a journal  $j$  is usually defined as the number of citations in year  $i$  to papers published in  $j$  in years  $i-1$  and  $i-2$ , divided by the number of papers published in  $j$  in years  $i-1$  and  $i-2$ . In our framework, since there is no explicit mention of time, we slightly modify this definition as follows. The impact factor of journal  $j$  in profile  $G$  is denoted by  $IF(j, G)$  and defined by

$$IF(j, G) = \frac{\sum_{p:pMj} \#C^{-1}(p)}{\#M^{-1}(j)}.$$

Although our definition is not equivalent to the traditional one, it is very close in spirit (average number of citations per paper) and, when a research domain is in a steady state, we expect both definition to yield almost the same ranking of journals.

The bibliometric ranking systems we want to axiomatize are defined by

$$j \succsim_G^J j' \iff IF(j, G) \geq IF(j', G) \quad (1)$$

for the journals and by

$$a \succsim_G^A b \iff \sum_{p \in P:aSp} \frac{\#C^{-1}(p)}{\#S^{-1}(p)} \geq \sum_{p \in P:bSp} \frac{\#C^{-1}(p)}{\#S^{-1}(p)} \quad (2)$$

or

$$a \succsim_G^A b \iff \sum_{p \in P:aSp} \#C^{-1}(p) \geq \sum_{p \in P:bSp} \#C^{-1}(p) \quad (3)$$

for the authors. Hence, when using (3), authors are ranked according to the total number of citations of the papers that they have published. The ranking

<sup>2</sup>A weak order on a set  $X$  is a transitive and complete binary relation on  $X$ .

based on (2) does the same but with an adjustment that takes care of the number of co-authors of each paper. So, we actually consider two ranking systems: the first one defined by (1) and (2), the second one by (1) and (3).

### 3 Axioms

In this section, we present a series of axioms that are satisfied by our two ranking systems. Later on, we will prove that they are not only necessary but also sufficient. The first axiom we consider is central in our paper: it links the ranking of journals and the ranking of scientists; it guarantees that they will be consistent in some sense (as in the introduction).

**A 1 Consistency.** *Let  $G = (P, S, C, M)$  be a profile such that there are two authors  $a, b$  and two journals  $j, j'$  such that  $\{a\} = S^{-1}(M^{-1}(j))$ ,  $\{b\} = S^{-1}(M^{-1}(j'))$ ,  $\{j\} = M(S(a))$ ,  $\{j'\} = M(S(b))$  and  $\#S(a) = \#S(b)$ . Then  $a \succ_G^A b$  iff  $j \succ_G^J j'$ .*

Let us analyze the content of this condition in a less formal way. When two authors publish all their papers in two (different) journals without any coauthor and these journals publish no other papers, we want the journals and the authors to be ranked in a consistent way. In particular, if both authors have the same numbers of papers, then the authors should be ranked in the same way as the corresponding journals.

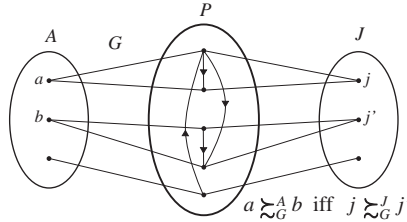


Figure 2: Consistency

We now need to impose some conditions on the ranking of journals and, separately, on the ranking of authors. We begin with the journals.

#### 3.1 Axioms for journals

Our first condition is about the role played by authors in the determination of the ranking of journals.

**A 2 Authors Do Not Matter (ADNM).** *For all profiles  $G, G'$  with  $G = (P, S, C, M)$  and  $G' = (P, S', C, M)$ , we have  $\succ_G^J = \succ_{G'}^J$ .*

In this condition,  $G$  and  $G'$  differ only in  $S'$ . So, they are identical except that some papers have different authors in  $G$  and  $G'$ . This axiom therefore expresses

the fact that the ranking of journals does not depend on the authors publishing in the journal. The only things that matter are the papers and the citations. Most people will find this condition very appealing but some might, on the contrary, consider that a journal in which only top scientists publish, is a good journal, even with few citations.

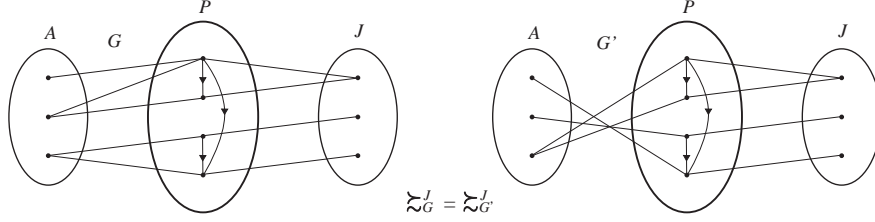


Figure 3: Authors Do Not Matter

**A 3** Independence of  $\lambda_G^J$  wrt the Source (IJS). For all profiles  $G, G'$  with  $G = (P, S, C, M)$  and  $G' = (P, S, C', M)$ , such that  $p C q, p' C' q, p' C' q, p C' q, C \Delta C' = \{(p, q), (p', q)\}^3$ , we have  $\lambda_G^J = \lambda_{G'}^J$ .

According to this condition, all citations have the same impact on the ranking of journals, whatever the citing author, paper or journal. This can in some circumstances be seen as a weakness: one might want to give more weight to a citation by a top paper (in a top journal or written by a top author or with many citations or ...) than to a citation by a low-ranked paper.

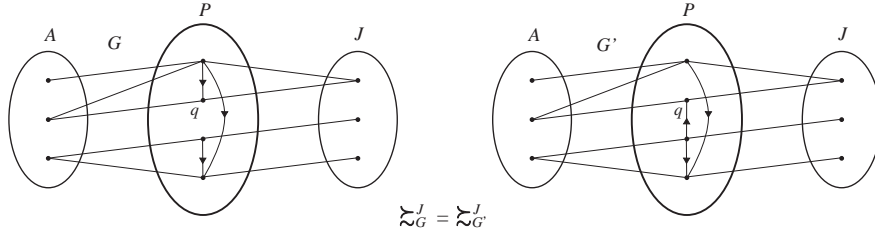


Figure 4: Independence of  $\lambda_G^J$  wrt the Source

**A 4** Homogeneity of  $\lambda_G^J$ . Let  $G = (P \cup Q, S, C, M)$  be a profile and  $I$  a set of journals included in  $J$  such that  $P \cap Q = \emptyset, Q = M^{-1}(I)$  ( $Q$  is the set of all papers published in the journals in  $I$ ) and  $C(Q) \subseteq Q$  (the papers in  $I$  cite only papers in  $I$ ). Consider now another profile  $G'$  identical to  $G$  except that all papers in  $Q$  (and the corresponding citations) have been cloned  $t$  times. Then  $\lambda_G^J = \lambda_{G'}^J$ .

<sup>3</sup> $C \Delta C'$  is the symmetric difference between  $C$  and  $C'$ , i.e.,  $C \Delta C' = C \cup C' \setminus (C \cap C')$

The profile  $G'$  is formally defined as follows. Let  $Q_1, Q_2, \dots, Q_t$  be mutually disjoint sets of papers with the same cardinality as  $Q$  and all disjoint from  $P$ . For  $i \in \{1, \dots, t\}$ , let  $f_i$  be a bijection from  $P \cup Q$  to  $P \cup Q_i$  such that  $f_i(p) = p$  for all  $p \in P$ . Define  $P' = P \cup \bigcup_{i=1}^t Q_i$ ,  $G' = (P', S', C', M')$  and assume, for  $i \in \{1, \dots, t\}$  and all  $p \in P \cup Q$ ,

- $S'^{-1}(f_i(p)) = S^{-1}(p)$  (the paper  $p$  and its eventual clones in  $G'$  have the same authors as  $p$  in  $G$ );
- $C'^{-1}(f_i(p)) = f_i(C^{-1}(p))$  (the paper  $p$  and its eventual clones in  $G'$  are cited by the same papers as  $p$  in  $G$ );
- $M'(f_i(p)) = M(p)$  (the paper  $p$  and its eventual clones in  $G'$  are published in the same journal as  $p$  in  $G$ ).

We can consider the set  $I$  as a research domain (because the papers in  $I$  cite only papers in  $I$ ). Two special cases worth noticing occur when  $I$  is a single journal or  $I = J$ . Put informally, by Homogeneity of  $\succsim^J$ , if a set of papers, corresponding

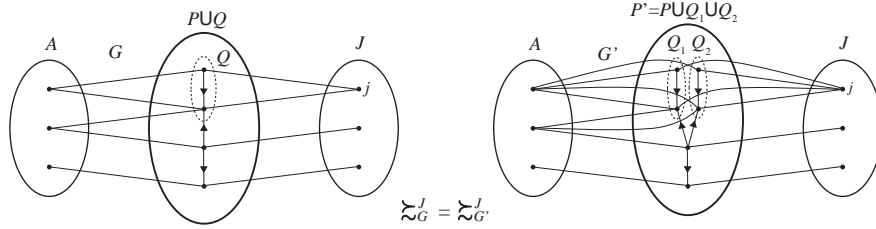


Figure 5: Homogeneity of  $\succsim^J$

to a research domain, is cloned, then the ranking of journals does not change. We see two reasons (not independent) to impose this condition. First, it makes clear that we do not want to reward a journal for its size; the quality of a journal is an intensive property (like density or colour), that is, a property that does not depend on the size of the object being measured (Greiner et al., 1995), contrary to extensive properties (like mass or number of inhabitants). In Figure 5, the size of journal  $j$  on the right-hand side is twice the size on the left-hand side but the “citation structure” is almost the same; it is just duplicated. The same is true for the “signature structure”. We therefore do not see any reason to modify the ranking. Second, the researchers in a domain might try to artificially improve the position of their journals in the ranking by systematically splitting their papers in  $t$  smaller papers (for instance, a theoretical part and an experimental part). This would result in a profile with many more papers in  $j$  and citations to  $j$ . Homogeneity of  $\succsim^J$  clearly expresses that this should not be rewarded. So, this condition can be seen as a guarantee against manipulations. Notice that if papers were to be splitted in two, one of them would most certainly cite the other one. In our statement of Homogeneity of  $\succsim^J$ , this is not the case. But, if we do not take self-citations into account, then citations between clones vanish

and the statement of Homogeneity of  $\succsim^J$  becomes a very good representation of what would occur in case of splitting.

**A 5 Transfer.** For all profiles  $G, G'$  with  $G = (P, S, C, M)$  and  $G' = (P, S, C', M)$ , such that  $q \succsim^M j, q' \succsim^M j, p \succ^C q, p \succ^{C'} q', p \not\succeq^C q', p \not\succeq^{C'} q$  and  $C \Delta C' = \{(p, q), (p, q')\}$ , we have  $\succsim_G^J = \succsim_{G'}^J$ .

In other words, if papers  $q$  and  $q'$  are published in the same journal and we add a citation to one of them, then it does not matter whether we add it to  $q$  or  $q'$ . A consequence of this condition is that 100 papers with one citation each have the same value as 100 papers without citations, except one of them with 100 citations. This might be subject to discussion.

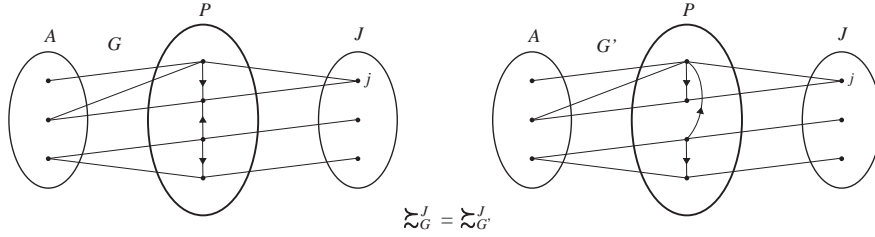


Figure 6: Transfer

**A 6 Monotonicity.** For all profiles  $G, G'$  with  $G = (P, S, C, M)$  and  $G' = (P, S, C', M)$ , such that  $p \succ^M j, C' = C \cup \{(q, p)\}$  and  $C \cap \{(q, p)\} = \emptyset$ , we have  $j \sim_G^J j' \Rightarrow j \succ_{G'}^J j'$ .

The idea underlying this condition is very simple: the more citations, the better. A bit more precisely, if two journals  $j, j'$  are equivalent in the ranking  $\succsim_G^J$  and  $j$  receives an additional citation in  $G'$ , then  $j$  is strictly better than  $j'$  in the ranking  $\succsim_{G'}^J$ . This condition makes sense in many circumstances but not always. For instance, if journal  $j$  receives a citation from paper  $p$  arguing that most papers in  $j$  have been accepted without any reviewing process, then the position of  $j$  should worsen and not improve. But, actually, this argument against Monotonicity is not completely valid because it involves some information that is not available in our framework (as usual in bibliometrics): the reason why a paper cites another paper (Bornmann and Daniel, 2008).

We now turn to some axioms putting constraints on the ranking of authors.

### 3.2 Axioms for authors

Our first condition in this section is analogous to ADNMM but for authors.

**A 7 Journals Do Not Matter (JDNM).** For all profiles  $G, G'$  with  $G = (P, S, C, M)$  and  $G' = (P, S, C, M')$ , we have  $\succsim_G^A = \succsim_{G'}^A$ .



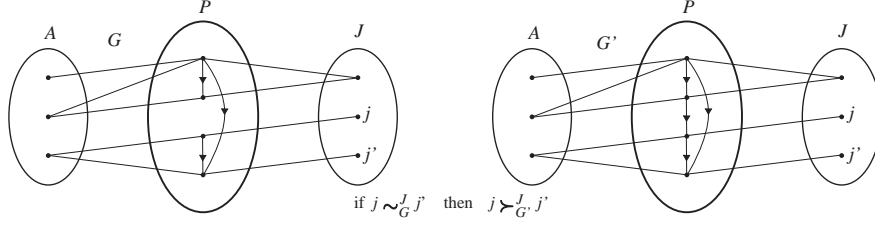


Figure 7: Monotonicity

This axiom expresses the fact that the ranking of authors should not depend on the journals in which they publish. At first sight, this may appear strange because we usually consider that good scientists publish in good journals but this relation is only indirect. It is not because a scientist publishes in good journal that he is a good scientist. Indeed, a weak scientist can publish a bad paper in a good journal (we all know some cases where the reviewing process did not work properly). So, it is very sensible to impose condition JDNM.

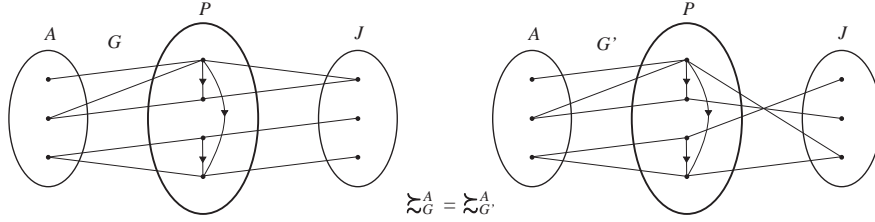


Figure 8: Journals Do Not Matter

**A 8 Dummy Paper.** Let  $G = (P, S, C, M)$  be a profile,  $a$  an author in  $A$ ,  $j$  a journal in  $J$  and  $p$  a paper not in  $P$ . Define  $G' = (P \cup \{p\}, S \cup \{(a, p)\}, C, M \cup \{(p, j)\})$ . Then  $\succ_G^A = \succ_{G'}^A$ .

Put differently, an additional paper that does not cite and is not cited by any paper has no influence on the ranking of authors. But, if we have to compare two scientist with the same publication/citation records except that one of them (say  $a$ ) has 50 additional publications without any citations, we might consider  $a$  as weaker because his 50 uncited publications indicate a lower average quality.

We next present a condition very similar to Homogeneity of  $\succ^J$ . But this one is tailored for the ranking of authors.

**A 9 Homogeneity of  $\succ^A$ .** Let  $P, P_1, P_2, \dots, P_t$  be mutually disjoint sets of papers with the same cardinality. Let  $f_1, \dots, f_t$  be bijections from  $P$  to  $P_1, \dots, P_t$ . Define  $P' = \bigcup_{i=1}^t P_i$ . Let  $G, G'$  be two profiles with  $G = (P, S, C, M)$  and  $G' = (P', S', C', M')$  satisfying, for  $i \in \{1, \dots, t\}$  and all  $p \in P$ ,

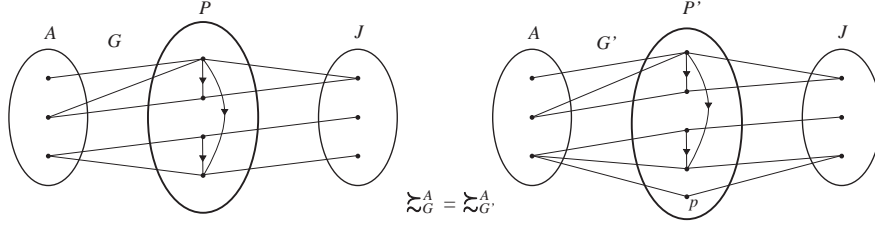


Figure 9: Dummy Paper

- $S'^{-1}(f_i(p)) = S^{-1}(p)$  (the clones of  $p$  in  $G'$  have the same authors as  $p$  in  $G$ );
- $C'^{-1}(f_i(p)) = f_i(C^{-1}(p))$  (the clones of  $p$  in  $G'$  are cited by the same papers as  $p$  in  $G$ );
- $M'(f_i(p)) = M(p)$  (the clones of  $p$  in  $G'$  are published in the same journal as  $p$ ).

Then  $\mathcal{Z}_G^A = \mathcal{Z}_{G'}^A$ .

By Homogeneity of  $\mathcal{Z}^A$ , if each paper in  $P$  is cloned (as well as the corresponding citations), then the ranking of authors does not change. The motivation for this condition is similar to the motivation for Homogeneity of  $\mathcal{Z}^J$

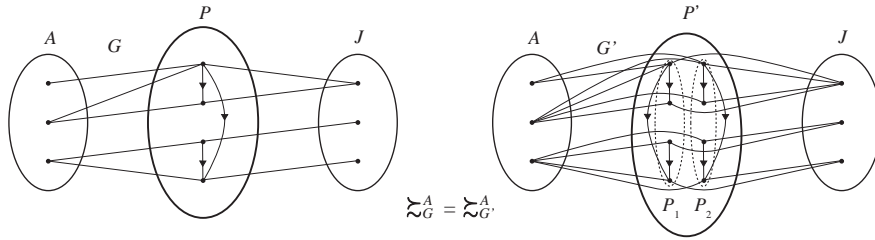


Figure 10: Homogeneity of  $\mathcal{Z}^A$

The next two conditions are about the influence of coauthors on the ranking of authors. The first condition says that we do not care about them while the second one says that authors writing with coauthors should be ‘penalized’.

**A 10 Coauthors Do Not Matter (CDNM).** Let  $G = (P, S, C, M)$  and  $G' = (P', S', C', M')$  be two profiles such that

- $P' = \bigcup_{p \in P} Q_p$ , where  $Q_p \subset P$ ,  $\#Q_p = \#S^{-1}(p)$  and  $Q_p \cap Q_q = \emptyset$  for all  $p, q \in P$ ;
- For every  $p \in P$  and  $q, q' \in Q_p$ ,  $\#S'^{-1}(q) = 1$ ,  $S'^{-1}(q) \neq S'^{-1}(q')$  and  $S'^{-1}(q) \in S^{-1}(p)$ ;

- For every  $p, p' \in P$ ,  $q \in Q_p$ ,  $\#(C'^{-1}(q) \cap Q_{p'}) = 1$  if  $p' \in C(p)$ ; otherwise  $\#(C'^{-1}(q) \cap Q_{p'}) = 0$ ;
- For every  $p \in P$  and  $q \in Q_p$ ,  $M(q) = M(p)$ .

Then  $\succ_G^A = \succ_{G'}^A$ .

Otherwise stated, suppose  $G'$  is a profile similar to  $G$  except that each paper  $p$  has been replaced by as many papers as  $p$  has coauthors and each paper in  $G'$  is signed by only one author. So, each author has the same number of papers in  $G'$  as in  $G$ , in the same journals, receiving the same number of citations. Then the ranking of authors in  $G'$  should be the same as in  $G$ . Still in other words, a scientist publishing alone or with coauthors is rewarded in the same way. This is probably not entirely acceptable to most of us since this might generate an inflation of the number of coauthors. We may therefore want to impose a condition explicitly discounting coauthored papers, like the following condition.

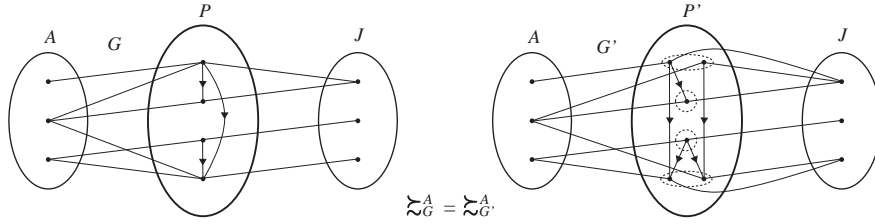


Figure 11: Coauthors Do Not Matter

**A 11** No Reward for Association (NRA). Let  $G = (P, S, C, M)$  be a profile and  $B$  a subset of  $A$  such that, for some  $j \in J$  and some  $Q \in P$ , for all  $b \in B$ ,  $\#S(b) = 1$ ,  $S^{-1}(S(b)) = b$ ,  $M(S(b)) = j$ ,  $C^{-1}(S(b)) = Q \subseteq P$ . Let  $G' = (P, S', C, M)$  be a profile such that  $S' = S \cup \{(b, p) : b \in B, p \in S(B)\}$ . Then  $\succ_G^A = \succ_{G'}^A$ .

The rationale for this condition is the following. Suppose  $a_1, a_2, \dots$  are  $m$  identical authors (clones) with exactly one publication in the same journal, without coauthors and cited by the same papers. Suppose now that, instead of publishing alone, these authors decide to form an association and to put each other's name on their papers. Then every author in this association has  $m$  publications, each with  $m - 1$  coauthors. Condition NRA states that such an 'artificial' inflation of the number of publications should have no effect. Condition NRA is of course incompatible with CDN and we will impose one or the other but not both.

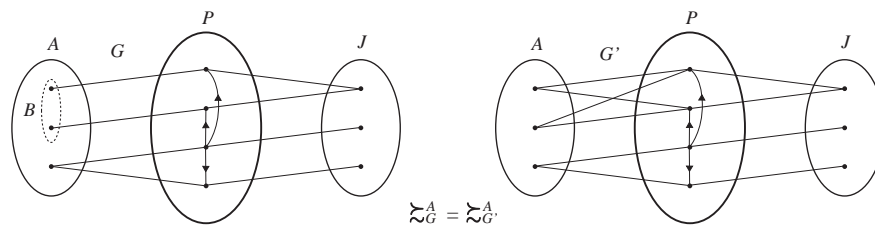


Figure 12: No Reward for Association

## 4 Results and proofs

### 4.1 Results

We are now ready to state our main results. The first one is about rankings of authors according to their total number of citations, weighted by the number of authors, and rankings of journals according to their impact factor.

**Theorem 1** *Assume  $\#J \geq 3$  and  $\#A \geq 3$ . A bibliometric ranking system  $R$  satisfies Conditions ADN $M$  (A2), IJS (A3), Homogeneity of  $\zeta^J$  (A4), Transfer (A5), Monotonicity (A6), JD $NM$  (A7), Dummy Paper (A8), Homogeneity of  $\zeta^A$  (A9), NRA (A11) and Consistency (A1) if and only if the journals are ranked according to (1) and the authors according to (2).*

The next result is about rankings of authors according to their total number of citations, not weighted by the number of authors, and rankings of journals according to their impact factor.

**Theorem 2** *Assume  $\#J \geq 3$  and  $\#A \geq 3$ . A bibliometric ranking system  $R$  satisfies Conditions ADN $M$  (A2), IJS (A3), Homogeneity of  $\zeta^J$  (A4), Transfer (A5), Monotonicity (A6), JD $NM$  (A7), Dummy Paper (A8), Homogeneity of  $\zeta^A$  (A9), CD $NM$  (A10) and Consistency (A1) if and only if the journals are ranked according to (1) and the authors according to (3).*

Both theorems involve ten conditions. This is a lot more than in all axiomatic papers mentioned in the introduction: they all use three to five axioms. There are two reasons for this high number of axioms. First, in each result, we characterize two rankings instead of one; so, on average, we actually use five conditions per ranking. Second, our framework is very rich: with a profile, we have a very detailed description of a situation. We not only know how many citations a paper receives but also from which paper. We also know the (co)authorship structure. We can deduce who cites who, who publishes in the same journal, and so on. This is much richer than the framework in any other axiomatic paper published so far. Indeed, most papers are based on distributions: how many papers with so many citations. Our very rich framework has allowed us to simultaneously characterize rankings of journals and scientists. It

should also allow us to characterize many other rankings even those for which distributions are not enough (as, e.g., in Palacios-Huerta and Volij (2004)). It will then be possible to compare different rankings on the basis of their characterizing axioms. This is why the extra cost (many axioms) associated to our framework seems worth paying.

The only difference between the set of axioms in Theorem 1 and 2 is the condition about coauthors. This makes the comparison between the rankings according to the total number of citations (weighted or not) very simple.

## 4.2 Preliminary Lemmas

We now turn to the proofs, but before proving Theorems 1 and 2, we will prove a few lemmas.

**Lemma 1** *Assume the bibliometric ranking system  $R$  satisfies Transfer (A5), IJS (A3) and Homogeneity of  $\succsim^J$  (A4). For any profile  $G = (P, S, C, M)$  and any journal  $j^* \in J$ , there exists a profile  $G'(P', S', C', M')$  such that*

- $\text{IF}(j, G) = \text{IF}(j, G')$  for all  $j \in J$ ;
- $C'(M'^{-1}(j^*)) \subseteq M'^{-1}(j^*)$  (papers in  $j^*$  cite only papers in  $j^*$ ) and
- $\succsim_G^J = \succsim_{G'}^J$ .

**Proof.** Let  $\beta = \max_{p \notin M^{-1}(j^*)} \#\{q \in C^{-1}(p) \cap M^{-1}(j^*)\}$ . Starting from  $G$ , we construct a new profile  $G_1 = (P_1, S_1, C_1, M_1)$  as in the statement of Homogeneity of  $\succsim^J$  with  $I = J$  and  $t = 2\beta + 1$ . By Homogeneity of  $\succsim^J$ ,  $\succsim_{G_1}^J = \succsim_G^J$ .

Let  $p$  be a paper not in  $M^{-1}(j^*)$ . The  $t$  clones of  $p$  in  $P_1$  are  $f_1(p), \dots, f_t(p)$ . By construction, there are no citations in  $C_1$  among the clones of  $p$ . By construction, there are at most  $t\beta$  citations from all papers in  $M_1^{-1}(j^*)$  to all clones of  $p$ . We are going to replace these citations by citations among the clones of  $p$ . By IJS and Transfer, this will have no effect on the ranking of journals. We must now check that moving these citations will not create a cycle. Among the  $t$  clones of  $p$ , we can have at most  $t(t-1)/2$  citations without creating a cycle. So, we must have  $t\beta \leq t(t-1)/2$ . This is the case since  $t = 2\beta + 1$ . For any other paper  $p'$  not in  $M^{-1}(j^*)$ , we can also replace all citations from all papers in  $M_1^{-1}(j^*)$  to all clones of  $p'$  by citations between the clones of  $p'$ . So, let  $G_2 = (P_1, S_1, C_2, M_1)$  be a new profile identical to  $G_1$  except that the citations from all papers in  $M_1^{-1}(j^*)$  to any clone of a paper not in  $j^*$  have been replaced by citations between papers not in  $j^*$ . By Transfer and IJS,  $\succsim_{G_2}^J = \succsim_{G_1}^J$ . The impact factors in  $G$  and in  $G_2$  are identical because, for each journal, the number of citations and the number of papers are multiplied by the same factor  $t$ . Let  $G' = G_2$  and the proof is done.  $\square$

**Lemma 2** *Assume the bibliometric ranking system  $R$  satisfies ADN M (A2), JDNM (A7), Homogeneity of  $\succsim^A$  (A9) and Consistency (A1). Let  $a, b \in A$ ,  $j, j' \in J$  and  $G = (P, S, C, M)$  be a profile such that*

- $\#S(a) = \#S(b)$  ( $a$  and  $b$  have the same number of papers);
- $S^{-1}(M^{-1}(j)) = \{a\}$  and  $S^{-1}(M^{-1}(j')) = \{b\}$  (All papers published in  $j$  are signed by  $a$  and all papers published in  $j'$  by  $b$ );
- $M(S(a)) = \{j\}$  and  $M(S(b)) = \{j'\}$  (All papers of  $a$  are published in  $j$  and all papers of  $b$  in  $j'$ );
- the restriction of  $C$  to  $M^{-1}(j)$  is isomorphic to the restriction of  $C$  to  $M^{-1}(j')$ .

Then  $j \sim_G^J j'$ .

**Proof.** Suppose for contradiction  $j \succ_G^J j'$ . By Consistency,  $a \succ_G^A b$ . Define  $G_1 = (P, S_1, C, M)$  with  $S_1$  identical to  $S$  except that the papers in  $j$  are now signed by  $b$  and those in  $j'$  by  $a$ . By ADN, we still have  $j \succ_{G_1}^J j'$ . By Consistency,  $b \succ_{G_1}^A a$ . Define  $G_2 = (P, S_1, C, M_2)$  with  $M_2$  identical to  $M$  except that the papers of  $a$  are now published in  $j$  and those of  $b$  in  $j'$ . By JDN, we still have  $b \succ_{G_2}^A a$ . By Consistency,  $j' \succ_{G_2}^J j$ . Let  $P_3$  be a set of papers disjoint from  $P$  and with the same cardinality as  $P$ . Let  $f$  be a bijection from  $P$  to  $P_3$ . Let  $G_3 = (P_3, S_3, C_3, M_3)$  with, for all  $p \in P$ ,  $S_3^{-1}(f(p)) = S^{-1}(p)$ ,  $C_3^{-1}(f(p)) = f(C^{-1}(p))$  and  $M_3(f(p)) = M(p)$ . By Homogeneity of  $\succ^A$  (with  $t = 1$ ),  $a \succ_{G_3}^A b$ .

Let  $f'$  be another permutation from  $P$  to  $P_3$ , such that  $f'(S_1(a)) = f(S_1(b))$ ,  $f'(S_1(b)) = f(S_1(a))$  and  $f'(p) = f(p)$  for every  $p : p \notin S_1(a) \cup S_1(b)$ . Let  $G_4 = (P_3, S_4, C_4, M_4)$  with, for all  $p \in P$ ,  $S_4^{-1}(f'(p)) = S_1^{-1}(p)$ ,  $C_4^{-1}(f'(p)) = f'(C^{-1}(p))$  and  $M_4(f'(p)) = M_2(p)$ . By Homogeneity of  $\succ^A$  (with  $t = 1$ ),  $b \succ_{G_4}^A a$ . But,  $G_3$  and  $G_4$  are actually the same profiles. It is therefore impossible to have  $a \succ_{G_3}^A b$  and  $b \succ_{G_4}^A a$ . This contradiction proves that  $j \sim_G^J j'$ .  $\square$

**Lemma 3** Assume  $\#A \geq 3$ . If a bibliometric ranking system  $R$  satisfies Conditions ADN (A2), IJS (A3), Homogeneity of  $\succ^J$  (A4), Transfer (A5), Monotonicity (A6), JDN (A7), Homogeneity of  $\succ^A$  (A9) and Consistency (A1), then the journals are ranked according to (1).

Notice that this lemma involves conditions about the ranking of authors (JDN and Homogeneity of  $\succ^A$ ) although it says nothing about the ranking of authors. But, thanks to Consistency, conditions about the ranking of authors also have some bite on the ranking of journals.

**Proof of Lemma 3.** Consider a profile  $G = (P, S, C, M) \in \mathbb{G}$  and two journals  $j, j' \in J$ . Choose two authors  $a, b$  and let  $G_1 = (P, S_1, C, M)$  be a profile such that  $S_1(a) = M^{-1}(j)$ ,  $\{a\} = S^{-1}(M^{-1}(j))$ ,  $S_1(b) = M^{-1}(j')$  and  $\{b\} = S^{-1}(M^{-1}(j'))$ . Such a profile exists because  $\#A \geq 3$ . By ADN,  $\succ_{G_1}^J = \succ_G^J$ . Let  $G_2 = (P_2, S_2, C_2, M_2)$  be a profile as in Lemma 1 with  $j^* = j$ . Let then  $G_3 = (P_3, S_3, C_3, M_3)$  be a profile as in Lemma 1 with  $j^* = j'$ . We have  $\succ_{G_3}^J = \succ_{G_1}^J$ , the impact factors in  $G_1$  and  $G_3$  are identical and  $G_3$  does not

contain any citation from papers in  $j$  to any other journal or from papers in  $j'$  to any other journal.

Starting from  $G_3$ , we construct a new profile  $G_4 = (P_4, S_4, C_4, M_4)$  as in the statement of Homogeneity of  $\succsim^J$ , where  $M_3^{-1}(j)$  plays the role of  $Q$  and  $t = \#M_3^{-1}(j')$ . By Homogeneity of  $\succsim^J$ , we have  $\succsim_{G_4}^J = \succsim_{G_3}^J$ . Starting now from  $G_4$ , we construct a new profile  $G_5 = (P_5, S_5, C_5, M_5)$  as in the statement of Homogeneity of  $\succsim^J$ , where  $M_3^{-1}(j')$  plays the role of  $Q$  and  $t = \#M_3^{-1}(j)$ . By Homogeneity of  $\succsim^J$ , we have  $\succsim_{G_5}^J = \succsim_{G_4}^J = \succsim_{G_3}^J$ . Furthermore, the impact factor of every journal in  $J$  is the same in  $G_3, G_4$  and  $G_5$ .

Suppose there are  $m$  journals. Using Lemma 1 ( $m - 1$ ) times, we can construct a profile  $G_6 = (P_6, S_6, C_6, M_6)$  such that  $\succsim_{G_6}^J = \succsim_{G_5}^J$ , the impact factors in  $G_5$  and  $G_6$  are identical and  $G_6$  does not contain any citation to papers in  $j$  from other journals. Using again Lemma 1 ( $m - 1$ ) times, we can construct a profile  $G_7 = (P_7, S_7, C_7, M_7)$  such that  $\succsim_{G_7}^J = \succsim_{G_6}^J$ , the impact factors in  $G_6$  and  $G_7$  are identical and  $G_7$  does not contain any citation to papers in  $j'$  from other journals.

Let  $\pi$  be a bijection (one-to-one mapping) from  $M_7^{-1}(j)$  to  $M_7^{-1}(j')$ . Let  $G_8 = (P_7, S_7, C_8, M_7)$  be a profile such that  $C_8^{-1}(p) = C_7^{-1}(p)$  for all  $p \notin M_7^{-1}(j')$ ,  $\#\{(p, q) \in C_8 : qM_7j'\} = \#\{(p, q) \in C_7 : qM_7j'\}$  and the restriction of  $C_8$  to  $M_7^{-1}(j)$  is a subset (if  $j$  is less cited than  $j'$ ) or a superset (if  $j$  is more cited than  $j'$ ) of  $\{(\pi(p), \pi(q)) : (p, q) \in C_8 \cap (M_7^{-1}(j'))^2\}$ . By Transfer and IJS,  $\succsim_{G_8}^J = \succsim_{G_7}^J$ .

Let us now construct the profile  $G_9 = (P_7, S_7, C_9, M_7)$  as follows. If the restriction of  $C_8$  to  $M_7^{-1}(j)$  is a subset of  $\{(\pi(p), \pi(q)) : (p, q) \in C_8 \cap (M_7^{-1}(j'))^2\}$ , we delete some citations among papers published in  $j'$  so that  $\{(\pi(p), \pi(q)) : (p, q) \in C_9 \cap (M_7^{-1}(j'))^2\}$  is exactly the restriction of  $C_8$  to  $M_7^{-1}(j)$ . If the restriction of  $C_8$  to  $M_7^{-1}(j)$  is a superset of  $\{(\pi(p), \pi(q)) : (p, q) \in C_8 \cap (M_7^{-1}(j'))^2\}$ , we delete some citations among papers published in  $j$  so that  $\{(\pi(p), \pi(q)) : (p, q) \in C_9 \cap (M_7^{-1}(j'))^2\}$  is exactly the restriction of  $C_8$  to  $M_7^{-1}(j)$ .

In profile  $G_9$ ,  $j$  and  $j'$  have the same number of papers. All papers published in  $j$  are signed by  $a$  and all papers published in  $j'$  by  $b$ . Papers in  $j$  (resp.  $j'$ ) are cited only by papers in  $j$  (resp.  $j'$ ). In addition, the restriction of  $C_9$  to papers in  $j$  is isomorphic to the restriction of  $C_9$  to papers in  $j'$ . Hence, by Lemma 2,  $j \sim_{G_9}^J j'$ .

Then, by Monotonicity,

$$\#\{(p, q) \in C_7 : qM_7j\} > \#\{(p, q) \in C_7 : qM_7j'\} \Rightarrow j \succ_{G_8}^J j'.$$

If  $\#\{(p, q) \in C_7 : qM_7j\} = \#\{(p, q) \in C_7 : qM_7j'\}$ , then  $G_8 = G_9$  and  $j \sim_{G_8}^J j'$ . So,  $j \succsim_{G_8}^J j'$  iff

$$\#\{(p, q) \in C_7 : qM_7j\} \geq \#\{(p, q) \in C_7 : qM_7j'\}$$

iff  $\text{IF}(j, G_7) \geq \text{IF}(j', G_7)$  (because  $j$  and  $j'$  have the same number of papers in  $G_7$ ) iff  $\text{IF}(j, G) \geq \text{IF}(j', G)$ . Finally,  $j \succsim_G^J j'$  iff  $\text{IF}(j, G) \geq \text{IF}(j', G)$ .  $\square$

### 4.3 Proofs

**Proof of Theorem 1.** Necessity is clear. We show sufficiency. Consider a profile  $G = (P, S, C, M)$  and two authors  $a, b \in A$ . Suppose first that both authors have the same number of papers, without any coauthor. Since there are at least three journals in  $J$ , we can construct a new profile  $G' = (P, S, C, M')$  such that  $S(a) = M'^{-1}(j)$ ,  $S(b) = M'^{-1}(j')$  and  $\#S(a) = \#S(b)$  for some  $j, j' \in J$ . By JDNM,  $\succsim_G^A = \succsim_{G'}^A$ . By Lemma 3, we know that  $j \succsim_{G'}^J j'$  iff  $\text{IF}(j, G') \geq \text{IF}(j', G')$ . By Consistency,  $a \succsim_G^A b$  iff  $\text{IF}(j, G') \geq \text{IF}(j', G')$ . Since  $\#M'^{-1}(j) = \#M'^{-1}(j')$ ,  $a \succsim_G^A b$  iff  $\#\{(p, q) \in C : qM'j\} \geq \#\{(p, q) \in C : qM'j'\}$ . This can also be written as  $a \succsim_G^A b$  iff  $\sum_{p \in P: aSp} \#C^{-1}(p) \geq \sum_{p \in P: bSp} \#C^{-1}(p)$ .

If  $a$  and  $b$  do not have the same number of papers (say  $a$  has less papers) and if they have no coauthor, then we can ‘add’ a dummy paper to  $a$ . The Dummy Paper condition implies that this paper has no effect on  $\succsim_G^A$ . We then add a second, third, ... dummy paper until  $a$  and  $b$  have the same number of papers. We then follow the same reasoning as in the first paragraph of this proof and we obtain

$$a \succsim_G^A b \iff \sum_{p \in P: aSp} \#C^{-1}(p) \geq \sum_{p \in P: bSp} \#C^{-1}(p).$$

Consider now a profile  $G = (P, S, C, M)$  and two authors  $a, b$  possibly with some coauthors. If they have no coauthors, we know how to compare them. Let  $G' = (P', S', C', M')$  be a profile as in the statement of Homogeneity of  $\succsim^A$ , where  $t = (\prod_{p \in P} \#S^{-1}(p))$ . By Homogeneity of  $\succsim^A$ ,  $\succsim_G^J = \succsim_{G'}^J$ .

We construct a new profile  $G'' = (P', S'', C', M')$  as follows. Consider the  $t$  clones in  $G'$  of a paper  $p$ . All of them are signed by the authors in  $S^{-1}(p)$ . The number  $t$  is a multiple of  $\#S^{-1}(p)$ . In  $G''$ , each author in  $S^{-1}(p)$  signs only a fraction of the  $t$  clones of  $p$ : this fraction is  $t/\#S^{-1}(p)$ . So, in the profile  $G''$ , each paper is signed by one and only one author. By NRA,  $\succsim_G^A = \succsim_{G'}^A = \succsim_{G''}^A$ . Replicating the same reasoning as in the beginning of this proof, we find  $a \succsim_G^A b$  iff  $a \succsim_{G''}^A b$  iff  $\sum_{p \in P': aS''p} \#C'^{-1}(p) \geq \sum_{p \in P': bS''p} \#C'^{-1}(p)$ . Or, equivalently,  $a \succsim_G^A b$  iff

$$\sum_{p \in P: aSp} \frac{t}{\#S^{-1}(p)} \#C^{-1}(p) \geq \sum_{p \in P: bSp} \frac{t}{\#S^{-1}(p)} \#C^{-1}(p).$$

Noticing that  $t$  on both sides of the inequality cancels out completes the proof.  $\square$

#### Proof of Theorem 2.

The proof is the same as in Theorem 1 up to the point where coauthors are admitted.

Consider now a profile  $G = (P, S, C, M)$  and two authors  $a, b$  possibly with some coauthors. If they have no coauthors, we know how to compare them. Let  $G' = (P', S', C', M')$  be a profile as in the statement of CDN. By CDN,  $\succsim_G^A = \succsim_{G'}^A$ . In  $G'$ , each paper is signed by one and only one author. So,  $a \succsim_G^A b$



iff  $a \succ_{G'}^A b$  iff  $\sum_{p \in P': aS'p} \#C'^{-1}(p) \geq \sum_{p \in P': bS'p} \#C'^{-1}(p)$ . Or, equivalently,  
 $a \succ_G^A b$  iff

$$\sum_{p \in P: aSp} \#C^{-1}(p) \geq \sum_{p \in P: bSp} \#C^{-1}(p).$$

□

## 5 Independence of the conditions

In this section, we prove that the conditions of Theorem 1 (and Theorem 2) are logically independent, that is, the theorem does not hold if any of these conditions is dropped.

### 5.1 Conditions of Theorem 1

Let us first prove the independence of the conditions used in Theorem 1. To this end, we provide, for each condition, an example satisfying all conditions but one. This shows that, in spite of the large number of conditions that we used in Theorem 1, none of them is redundant

1. ADNM. Let  $f$  be a real-valued mapping defined on  $2^A$  such that  $f(\{a\}) = 1$  for all  $a \in A$  and  $f(B) \neq f(B')$  for some  $B, B' \subset A$  with  $\#B = \#B'$ . Define  $j \succ_G^J j'$  iff  $g(j) \geq g(j')$  where

$$g(j) = \frac{\sum_{p: pMj} [\#C^{-1}(p) + f(S^{-1}(p))]}{\#M^{-1}(j)}$$

and  $a \succ_G^A b$  by (2).

2. IJS. Define  $j \succ_G^J j'$  iff  $g(j) \geq g(j')$  where

$$g(j) = \frac{\sum_{p: pMj} \left[ \#C^{-1}(p) + \frac{1}{2} \sum_{q: qCp} \#C^{-1}(q) \right]}{\#M^{-1}(j)}$$

and  $a \succ_G^A b$  iff  $h(a) \geq h(b)$  where

$$h(a) = \sum_{p: aSp} \frac{\#C^{-1}(p) + \frac{1}{2} \sum_{q: qCp} \#C^{-1}(q)}{\#S^{-1}(p)}.$$

3. Homogeneity of  $\succ^J$ . Define  $j \succ_G^J j'$  iff  $g(j) \geq g(j')$  where

$$g(j) = \sum_{p: pMj} \#C^{-1}(p)$$

and define  $\succ_G^A$  by (2).

4. Transfer. Define  $j \succsim_G^J j'$  iff  $g(j) \geq g(j')$  where

$$g(j) = \frac{\sum_{p:pMj} (\#C^{-1}(p))^2}{\#M^{-1}(j)}$$

and  $a \succsim_G^A b$  iff  $h(a) \geq h(b)$  where

$$h(a) = \sum_{p:aSp} \frac{(\#C^{-1}(p))^2}{\#S^{-1}(p)}.$$

5. Monotonicity.  $\succsim_G^J = J^2$  and  $\succsim_G^A = A^2$ .

6. JDNM. Define  $j \succsim_G^J j'$  by (1) and  $a \succsim_G^A b$  iff  $h(a) \geq h(b)$  where

$$h(a) = \sum_{p:aSp} \frac{\#C^{-1}(p) - (\#M(S(a)) - 1)\#C(p)}{\#S^{-1}(p)}.$$

7. Dummy Paper. Define  $\succsim_G^J$  by (1) and  $a \succsim_G^A b$  iff  $h(a) \geq h(b)$  where

$$h(a) = \sum_{p:aSp} \frac{[1 + \#C^{-1}(p)]}{\#S^{-1}(p)}.$$

8. Homogeneity of  $\succsim^A$ . Define  $\succsim_G^J$  by (1) and  $a \succsim_G^A b$  iff  $h(a) \geq h(b)$  where

$$h(a) = \sum_{p:aSp} \left( \frac{\#C^{-1}(p)}{\#S^{-1}(p)} + \frac{(\#S^{-1}(p) - 1)\#C^{-1}(p)}{\#S(a)\#S^{-1}(p)} \right).$$

9. NRA. Define  $\succsim_G^J$  by (1) and  $\succsim_G^A$  by (3).

10. Consistency. Define  $\succsim_G^J$  by (1) and  $a \succsim_G^A b$  iff  $h(b) \geq h(a)$  where

$$h(a) = \sum_{p:aSp} \frac{\#C^{-1}(p)}{\#S^{-1}(p)}.$$

## 5.2 Conditions of Theorem 2

Let us now prove the independence of the conditions used in Theorem 2.

1. ADN. Let  $f$  be a real-valued mapping defined on  $2^A$  such that  $f(\{a\}) = 1$  for all  $a \in A$  and  $f(B) \neq f(B')$  for some  $B, B' \subset A$  with  $\#B = \#B'$ . Define  $j \succsim_G^J j'$  iff  $g(j) \geq g(j')$  where

$$g(j) = \frac{\sum_{p:pMj} [\#C^{-1}(p) + f(S^{-1}(p))]}{\#M^{-1}(j)}$$

and  $a \succsim_G^A b$  by (3).

2. IJS. Define  $j \succsim_G^J j'$  iff  $g(j) \geq g(j')$  where

$$g(j) = \frac{\sum_{p:pMj} \left[ \#C^{-1}(p) + \frac{1}{2} \sum_{q:qCp} \#C^{-1}(q) \right]}{\#M^{-1}(j)}$$

and  $a \succsim_G^A b$  iff  $h(a) \geq h(b)$  where

$$h(a) = \sum_{p:aSp} \left[ \#C^{-1}(p) + \frac{1}{2} \sum_{q:qCp} \#C^{-1}(q) \right].$$

3. Homogeneity of  $\succsim^J$ . Define  $j \succsim_G^J j'$  iff  $g(j) \geq g(j')$  where

$$g(j) = \sum_{p:pMj} \#C^{-1}(p)$$

and define  $\succsim_G^A$  by (3).

4. Transfer. Define  $j \succsim_G^J j'$  iff  $g(j) \geq g(j')$  where

$$g(j) = \frac{\sum_{p:pMj} (\#C^{-1}(p))^2}{\#M^{-1}(j)}$$

and  $a \succsim_G^A b$  iff  $h(a) \geq h(b)$  where

$$h(a) = \sum_{p:aSp} (\#C^{-1}(p))^2.$$

5. Monotonicity.  $\succsim_G^J = J^2$  and  $\succsim_G^A = A^2$ .

6. JDNM. Define  $j \succsim_G^J j'$  by (1) and  $a \succsim_G^A b$  iff  $h(a) \geq h(b)$  where

$$h(a) = \sum_{p:aSp} [\#C^{-1}(p) - (\#M(S(a)) - 1)\#C(p)].$$

7. Dummy Paper. Define  $\succsim_G^J$  by (1) and  $a \succsim_G^A b$  iff  $h(a) \geq h(b)$  where

$$h(a) = \sum_{p:aSp} [1 + \#C^{-1}(p)].$$

8. CDN. Define  $\succsim_G^J$  by (1) and  $\succsim_G^A$  by (2).

9. Homogeneity of  $\succsim^A$ . Define  $\succsim_G^J$  by (1) and  $a \succsim_G^A b$  iff  $h(a) \geq h(b)$  where

$$h(a) = \sum_{p:aSp} \left( \#C^{-1}(p) + \frac{(\#S^{-1}(p) - 1)\#C^{-1}(p)}{\#S(a)} \right).$$

10. Consistency. Define  $\succsim_G^J$  by (1) and  $a \succsim_G^A b$  iff  $h(b) \geq h(a)$  where

$$h(a) = \sum_{p:aSp} \#C^{-1}(p).$$

## 6 Conclusion

We have defined a condition (Consistency) linking rankings of journals and rankings of papers that, we think, is very compelling. We have presented two pairs of rankings satisfying this condition: the ranking according to the impact factor on the one side and the ranking based on the total number of citations (weighted or not by the number of coauthors) on the other side. We have also shown that this condition can be used to characterize these pairs of rankings (in conjunction with some other conditions).

Much work remains to be done in the domain of consistent rankings. It would for instance be interesting to characterize the set of all pairs of rankings satisfying Consistency, Homogeneity of  $\succsim_G^J$ , Homogeneity of  $\succsim_G^A$  and Monotonicity (because these conditions are less subject to criticism than the other ones). Extending the concept of consistency to other rankings (departments, universities, papers, etc.) seems also important. Another potential path of research is the strengthening of Consistency. The present condition links rankings of journals and scientists when it is the case that journals (at least two of them) can be identified with a single scientist. A reinforcement of Consistency would consist in identifying a journal with several scientists: for instance, if journal  $j$  is identified with authors  $B \subset A$  and  $j'$  with  $B' \subset A$  and if there is a bijection  $\sigma$  between  $B$  and  $B'$  such that  $a \succsim_G^A \sigma(a)$  for all  $a \in B$ , then  $j \succsim_G^J j'$ . The stronger consistency condition could help us isolate a small set of interesting ranking systems.

Throughout this paper, we assumed that the only relevant information for ranking scientists and journals is represented in a profile. The scientific content of the publications, the age of a scientist, his/her country, the reason why a paper is cited or not are all aspects that are not taken into account. Although there are perhaps some contexts where the profile is the only relevant or available information, there are also many circumstances where more information than the profile is available and pertinent. Our work is by no means a plea in favor of using only profiles.

## Acknowledgements

During the preparation of this paper, Thierry Marchant benefited from a visiting position at the Université Paris Dauphine and at the Indian Statistical Institute, New Delhi. This support is gratefully acknowledged.

## References

Adler, R., Ewing, J., and Taylor, P. (2008). Citation statistics. Technical report, IMU–ICIAM–IMS. Joint Committee on Quantitative Assessment of Research. A report from the International Mathematical Union (IMU) in cooperation with the International Council of Industrial and Applied

- Mathematics (ICIAM) and the International Council of Institute of Mathematical Statistics (IMS), corrected version dated 6/12/08, available from <http://www.mathunion.org/fileadmin/IMU/Report/CitationStatistics.pdf>, last accessed 1 December 2009.
- Billaut, J.-C., Bouyssou, D., and Vincke, P. (to appear). Should you believe in the Shanghai ranking? An MCDM view. *Scientometrics*.
- Bornmann, L. and Daniel, H.-D. (2008). What do citation counts measure? A review of studies on citing behavior. *Journal of Documentation*, 64(1):45–80.
- Bouyssou, D. and Marchant, T. (2009). Bibliometric rankings of journals based on impact factors: An axiomatic approach. Technical report.
- Brown, L. D. (1996). Influential accounting articles, individuals, Ph. D. granting institutions and faculties: A citational analysis. *Accounting, Organizations and Society*, 21:723–754.
- Egghe, L. (2006). Theory and practise of the  $g$ -index. *Scientometrics*, 69:131–152.
- Garfield, E. (1972). Citation analysis as a tool in journal evaluation. *Science*, 178:471–479.
- Greiner, W., Neise, L., and Stöcker, H. (1995). *Thermodynamics and statistical mechanics*. Springer.
- Hirsch, J. E. (2005). An index to quantify an individual’s scientific research output. *Proceedings of the National Academy of Sciences*, 102:16569–16572.
- Kokko, H. and Sutherland, J. (1999). What do impact factors tell us? *Trends in Ecology and Evolution*, 14:382–384.
- Marchant, T. (2009a). An axiomatic characterization of the ranking based on the  $h$ -index and some other bibliometric rankings of authors. *Scientometrics*, 80:325–342.
- Marchant, T. (2009b). Score-based bibliometric rankings of authors. *Journal of the American Society for Information Science and Technology*, 60:1132–1137.
- Osterloh, M. and Frey, B. S. (2009). Research governance in academia: Are there alternatives to academic rankings? Working paper no. 2797, CESIFO.
- Palacios-Huerta, I. and Volij, O. (2004). The measurement of intellectual influence. *Econometrica*, 72:963–977.
- Pijpers, F. P. (2006). Performance metrics. *Astronomy and Geophysics*, 47:6.17–6.18.
- Pinski, G. and Narin, F. (1976). Citation influence for journal aggregates of scientific publications: theory, with application to the literature of physics. *Information processing and management*, 12(5):297–312.

- Quesada, A. (2009). Monotonicity and the Hirsch index. *Journal of Informetrics*, 3(2):158–160.
- Sidiropoulos, A., Katsaros, D., and Manolopoulos, Y. (2007). Generalized Hirsch  $h$ -index for disclosing latent facts in citation networks. *Scientometrics*, 72(2):253–280.
- Taber, D. F. (2005). Quantifying publication impact. *Science*, 309:2166.
- van Raan, A. F. J. (2006). Comparison of the Hirsch-index with standard bibliometric indicators and with peer judgment for 147 chemistry research groups. *Scientometrics*, 67:491–502.
- Woeginger, G. (2008a). An axiomatic analysis of Egghe’s  $g$ -index. *Journal of Informetrics*, 2(4):364–368.
- Woeginger, G. J. (2008b). An axiomatic characterization of the Hirsch-index. *Mathematical Social Sciences*, 56:224–232.