Ethnomathematics and cultural representations: Teaching in highly diverse contexts

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ABSTRACT

A great deal of the history of research in ethnomathematics has been dominated by the study of fundamental differences in ways of doing mathematics among various cultures. Understanding these differences is critical for the comprehension of human nature, and learning in relation to problem resolution. Using ethnomathematics as a program for the understanding of this fact, this article discusses how the study of different algorithms can contribute toward the incorporation, learning and celebration of the differences between diverse cultural groups. Ethnomathematics provides a basis for acknowledging the structures in diverse and often highly dynamic societies that are part of the dominant or majority community power structure. It also teaches learners to connect culture and mathematics and enriches this subject matter by understanding and bridging the often perceived dichotomy between academic mathematics and daily life. An ethnomathematics perspective provides a transformational space for students and teachers, and allows them to think of diversity as good, valuable, and necessary to living in a globalized interconnected world. Through a study of the algorithms immigrant students bring to the community, both teachers and students learn that culture influences the development of these methods, which are used to solve mathematical problems.

Keywords: Ethnomathematics. Algorithms. Representations. Semiotics. Universal Language. Immigrant Students.

Etnomatemática e representações culturais: ensinando em contextos altamente diversos

RESUMO

Uma grande parcela da história da pesquisa em etnomatemática tem sido dominada pelo estudo das diferenças fundamentais entre os modos de fazer matemática em várias culturas. O

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entendimento destas diferenças é um fator crítico para a compreensão da natureza humana e para o aprendizado em relação à resolução de problemas. Utilizando a etnomatemática como um programa voltado para o entendimento deste fato, este artigo discute como o estudo de diferentes algoritmos pode contribuir para a incorporação, a aprendizagem e a celebração das diferenças entre diversos grupos culturais. A etnomatemática providencia uma base para o reconhecimento das estruturas sociais que são diversificadas e altamente dinâmicas, e que estão submetidas à estrutura do poder majoritário e dominante das comunidades. A etnomatemática também ensina como os alunos devem conectar a cultura com a matemática para enriquecer esta disciplina através do entendimento da dicotomia percebida entre a matemática presente na vida cotidiana e na academia. A perspectiva etnomatemática providencia um espaço transformacional para que os alunos e professores pensem na diversidade como algo bom, valoroso e que é necessário para que possamos viver num mundo globalmente interconectado. Através do estudo de algoritmos que os alunos imigrantes trazem para a comunidade escolar, ambos professores e alunos aprendem que a cultura influencia no desenvolvimento destes métodos, que são utilizados na resolução de problemas matemáticos.

Palavras-chave: Etnomatemática. Algoritmos. Representações. Semiótica Linguagem Universal. Alunos Imigrantes.

INTRODUCTION

Learning is very much a process of acculturation by which individuals can learn to construct mathematical knowledge in a cooperative way. That may be all very well and good when a community is monocultural, but what about highly diverse communities, where the dominate group often "teaches" the minority population to ignore their own traditions, talents, and needs? In communities with diverse immigrant populations[1], the traditional school curriculum does not seem to have a rational way of embracing, including, understanding, studying, or assessing the importance of other methods for calculation or problem solving brought by newly arrived immigrants.

Good mathematical learning occurs with social and cultural interaction through dialogue, language, and through the negotiation of meaning of the symbolic representations between teacher and student. To include and exclude differences and traditions brought to the new community by immigrants and non-represented minorities is a moral decision made by governments and curriculum developers. To understand diverse cognitive strategies, it is necessary to envisage students in the context of this new cultural context which often includes a variety of traditions, behaviors, religions, and languages. The following discussion may act as mediator for actions that currently take place in classrooms in California in relation to the mathematics curriculum.

[1] In 2006, the population of immigrant students in the United States was 5.2 million, which represented 10.6% of the 49 million students attending school. There were more than 350 languages spoken among immigrant students and of those different languages 77% speak Spanish. In California, 25.4% of the students are immigrant. This represents 1.6 million of 6.3 million of students attending public schools and 85.3% of them speak Spanish.

MATHEMATICS CLASSROOMS IN CALIFORNIA

Classrooms with students from a variety of different linguistic and cultural traditions use a diversity of mathematical contexts and practices. In a true cultural integration, a relationship develops between formal and informal mathematics and language. Symbolic representations take on extremely important roles as well. However, many mathematics classrooms operate on the assumption that all students should receive the same mathematical content at the same time in the same way. Some mathematicians believe that there is only one right way to solve a mathematical problem.

This often happens, because in the context of highly diverse communities, teachers often come from mainstream and middle class contexts which are markedly different than those of their students. More often than not, schooling including the learning of mathematics has become a vehicle for the loss of self esteem, or of power of the student. How diverse people, despite their formal schooling experiences, actually come to learn, measure, classify, order and organize, infer, and model are all important aspects of diverse modes of teaching and learning of mathematics.

In so doing, many public school populations mirror the highly diverse and multicultural communities that they serve. Mathematics classrooms are most certainly part of this environment and students must be encouraged to solve problems by applying multiple strategies to assist other students to learn new ideas and processes, and to understand mathematical concepts through the use of real-life problem solving scenarios (ROSA, 2005). This kind of classroom has an eye toward equity because the teaching and learning emphasizes the treatment of all students respectfully and fairly by making use of their own different languages, strengths, learning styles, and cultural backgrounds in a deep not superficial manner. In this environment, effective teachers must learn to appreciate the diverse methods of learning and thinking, modify instruction, and plan accordingly.

For example, in many SDAIE[2] classrooms, by making use of the diversity and building an inclusive classroom environment where everyone's culture and background is respected, teachers learn to push students to come up with as many different algorithms, strategies, and thinking processes as possible for discussing or solving mathematical problems. Making use of how a variety of perspectives and different ways are used to understanding mathematical content, is as important as the actual answer. Diverse learning environments allow students to practice communicating and working with other people that are different from those that they are accustomed to learning and working with.

[2] Specially designed academic instruction in English (SDAIE) is a teaching approach intended for teaching various academic content (such as social studies, science, mathematics, and English) using the English language to students who are still learning English. SDAIE requires the student possess intermediate fluency in English as well as mastery of their native language. The instruction is carefully prepared so the student can access the English language content supported by material in their primary language and carefully planned instruction that strives for comprehensible input. SDAIE is a method of teaching students in English in such a manner that they gain skills in both the subject material and in using English.

The kind of mathematics classroom we strive for is a place that embodies an inclusive and democratic setting in which all students are active and engaged. As well, students learn to work and live with others that are vastly different than who they live with, or grew up with before immigrating. While learning to work with different people, they learn to work and navigate the dominant culture. All students need to learn about their own cultures and assess personal assumptions that affect their interaction with individuals that are different from themselves. In this case, critical thinking skills and the ability to work cooperatively are necessary approaches to develop an inclusive mathematics environment rooted on culturally relevant activities, which are based on a curriculum that empowers all learners. Methods of curricular reform in mathematics may be described as a progression from adding single pieces of cultural information to transforming the curriculum so that it equally values, indeed incorporates all cultural perspectives.

MATHEMATICS AND LANGUAGE

Mathematics is a language system that has its own history, symbols, syntax, grammar, and comes with an enormous variety of representations. It relies on an intensive use of different variables, signs for numbers, diagrams, formulas, and algorithms. Some of these signs and algorithms vary slightly between countries and present us with interesting and important historical reasons for doing so. Sometimes, the learners themselves see that some algorithms allow users to gain cognitive advantages and often provide better ways to facilitate calculation[3] . Why we use one method to the exclusion of another is related to history, politics, access, and equity.

Mathematics represents the foundation to humanity's scientific and cultural heritage. It is a construction of knowledge that deals with vital qualitative and quantitative relationships of space and time. It is well documented, and long understood that concepts of space and time are differentiated across cultures (HALL, 1992). Space and time as a human cultural constructs, as well interpret variations in patterns, problem-solving, and logical thinking processes that people use to understand and make meaning of their world. This is expressed, developed and contested through complex processes involving language, symbols, and social interaction. Mathematical literacy with its accompanying conceptual tools provide powerful numeric, spatial, temporal, symbolic, communicative, skills, knowledge, attitudes, and values that allow students to analyze; make and justify critical decisions; and take transformative action.

Working with relationships between language skills and mathematics, Aiken (1972); Cuevas (1984); Dawe (1983); Kessler, et al. (1985) pointed out that a

[3] For example, the left to right addition and subtraction methods allow many learners to do more mental math than the standard right-left USA pattern, with all borrowing and carrying complexities and headaches.

limited ability to speak English has considerable effects on the learning of mathematics. According to Mather and Chiodo (1994), the justification for the myth of the ability to speak English having a minimal effect on learning mathematics is that mathematics is often thought of as a universal language, and, therefore, an individual's knowledge is not tied to a particular cultural language. Orey (2008), the coordinator of The Algorithm Collection Project[4] -ACP argues that mathematics possesses important dialectical differences across cultures and curriculum. For example, one of the problems for immigrant students to California is related to syntax, the sentence structure and semantic components of language in the mathematics classes. In this perspective, Kessler et al (1985) stated that this is one area that relates to the lack of a one-to-one correspondence between mathematical symbols and the word they represent.

Frequently, mathematics is associated with the study of universals, and as mentioned above, is referred to as a universal language (PERKINS; FLORES, 2002). When people speak of universals, however, it is important to recognize that often something thought of as universal is merely universal to those who share the same cultural and historical background. The use of the English language presents us with an interesting parallel. Often South Asian, Australian, and USAan[5] English can be indecipherable between speakers from these regions. This means that language differences pose challenges for an effective leaning of mathematics. In this perspective, Perkins and Flores (2002, p. 346) stated that:

> Compared with the differences in language and culture faced by students who are recent immigrants, the differences in mathematical notation and procedures seem to be minor. Nevertheless, immigrant students confront noticeable differences between the way that mathematical ideas are represented in their countries of origin and the manner that they are represented in the United States.

Thus, Secada (1983) and Norman (1988) demonstrated that algorithms and fundamental operations are not universally the same because there are different mathematical ideas and procedures that are practiced in diverse cultures. The same point of view is shared by the ACP, which argues that immigrant students often come to school having learned different algorithms for the basic number operations, and that teachers and students can be trained to use them effectively.

[4] Visit http://www.csus.edu/indiv/o/oreyd/ACP.htm_files/Alg.html

[5] The authors choose to use the term USAan for citizens of the United States. Citizens of North, Central, and South America are Americans, and the continued use of American for USAan causes confusion and disrespect for those that live from Canada to Tierra del Fuego. Unfortunately Estadunidenses (Unitedstatesan) as used on Spanish and Portuguese does not translate as well. So we choose the term we heard us called in Costa Rica once, USAan.

THE CONTEXT OF ETHNOMATHEMATICS

Many educators operate under the assumption that mathematics is acultural, that it is a discipline without cultural significance, fail to see the connection between mathematics and culture (D'AMBROSIO, 2001). Because mathematics in any culture has been based upon certain values and needs, an immigrant's former cultural reference can interfere in the learning of mathematical concepts in the classrooms. For example, in his work with Native American populations, Davidson (1990) found that the interaction of culture and mathematical ideas are mutually reinforced when the application of culturally sensitive mathematical activities helps students to see the relevance of mathematics in their own culture and at the same time helps teachers to use this connection to teach even more sophisticated mathematics.

The use of mathematics in everyday life varies according to each different culture and its needs. Ethnomathematics, which is the study of mathematics within its diverse cultural contexts, is used to express relationships between culture and mathematics (D'AMBROSIO, 2001). Ethnomathematics is primarily concerned with connections that exist between the symbols, representations, and the imagery used to solve problems (VERGANI, 1998). The development of ethnomathematics as a program has challenged traditional concepts of Euro-Western centered mono-representational systems of mathematics into that of a world-centered multi-representational system of mathematics that represents mathematics as human endeavor (OREY; ROSA, 2007).

An ethnomathematical perspective encourages the study of diverse number systems, including the symbols and the representational systems of different cultural groups. The representational system of a culture often depends upon the unique types of mathematical knowledge that each culture develops over time. Cultural artifacts such as language, myths, and literature influence the representational system of different cultures and civilizations, and are more often then not related to the use of mathematics. According to D'Ambrosio (2001), an ethnomathematics-based curriculum is student-centered, anti-racist, anti-sexist, and grounded in the incorporation of mathematical ideas and activity that echo a diversity of cultures, particularly, those that experienced oppression or exclusion from mainstream society.

Ethnomathematics presents us with an alternative view towards the teaching and learning of mathematics. Allowing students to appreciate and understand more than one representation helps us to appreciate and understand mathematics as a potential for resolving conflict, and solving problems. Acknowledging how culture affects how we think and learn mathematics, builds one concrete example for understanding between humans. Ethnomathematics teaches us that both students and teachers can learn to model and value diversity in the classroom. To understand both the influence that culture has on mathematics and how this influence results in diverse ways in which mathematics is used and communicated (D'AMBROSIO, 2001) increases understanding between diverse groups.

Another way to think about an ethnomathematics program is to look at it from two vantage points, both based on a development of flexible thinking:

1) It can be viewed as the study of the mathematical practices of "others". That is, it can include multicultural mathematics, where teachers draw from mathematical activities from many cultures (ZASLAVSKY, 1996). A revision of the mathematics curriculum is necessary to include diverse cultural groups such as women, working people, ethnic and racial groups whose contributions and place in history have been distorted, marginalized, or ignored completely (D'AMBROSIO, 1990). What is needed is to devise some sort of equitable assessment tool that includes the understanding of, and appreciation for diverse modes of thought.
2) It can be viewed as people's own mathematical practice. That is, when we think about ethnomathematics as our own mathematics practice, a pedagogical approach starts with teachers and students who learn to think flexibility about how we use mathematics in everyday as well as academic contexts (ROSA; OREY, 2006). One way to do this is to have students keep a log for a day, a week, or some other period of time of how they or others use mathematics. Teachers then take this information and use it as a base from which to create contexts for problem solving and culturally relevant activities.

An ethnomathematics perspective allows educators to rethink how and what is taught. This perspective encourages teachers to recognize that there is mathematics in everything, not just the mathematics found in required school curriculum. How diverse people, despite their formal schooling experiences, actually come to learn, measure, classify, order and organize, infer, and model are all important aspects of diverse modes of teaching and learning of mathematics.

ETHNOMATHEMATICS AND REPRESENTATIONS

The notion of representation in ethnomathematics is profound. Especially in regard to natural, non-standard, informal, subjective, context-based representations of mathematical concepts. Goldin and Steingold (2001) demonstrated that representational systems are classified into three distinct categories: external, shared or negotiated, and internal. The external representations are the mathematical symbols, signs, characters, and signals. Moreover, these representations deal with such representations shared between teachers and learners. In so doing, "[…] the use of external, socially shared representations seems to be an important move in making mathematical knowledge more explicit" (NUNES; BRYANT, 1997, p.4). We believe that explicitation is paramount to the construction of the mathematical knowledge and must be one of the goals for teaching mathematics in schools. On the other hand, various forms of internal representation include verbal and syntactic, imaginistic and affective, and formal notational. Broadly speaking, there are most likely two important types of representation that affect understanding and solution of mathematics problems:

1) Instructional representations (definitions, examples and models) used by teachers to share knowledge to students, and

2) Cognitive representations that are constructed by the students themselves as they try to make sense of a mathematical concept or attempt to find a solution to a problem (MIURA, 2001). There are many representations in school mathematics which may help students to communicate, solve mathematical problems, and identify and change their attitudes towards mathematics.

Representations can be necessary to students' understanding of mathematical concepts because they can allow them to communicate mathematical approaches, arguments, and concepts to themselves and to others. As well, representations also are necessary in recognizing connections among related concepts and mathematical applications to realistic problems. For example, to become knowledgeable about fractions, students need a variety of representations that allow them to construct their understanding. They also need time to understand the differences and use of fractional notation such as ratio, indicated division, or fraction of a number. These experiences allow them to understand other common representations for fractions, such as points on a number line and how fractional representation is connected to the division algorithm. Unfortunately, these representations and others have often been taught and learned as if they were ends in themselves. This approach greatly limits the power and utility of fractional representations as cultural tools for the learning and doing of mathematics.

It is important that students are encouraged to represent their mathematical ideas in various ways that make sense and is culturally relevant to them, even if those representations are not conventional. At the same time, students need be given experience that allow them to learn conventional representational forms that facilitate their learning of mathematics, their communication with others, and further increases the need for them to be comfortable with different kinds of mathematical representations.

ETHNOMATHEMATICS AND SEMIOTICS

Semiotics is associated with representations. It is the study of the signs. However, signs are not isolated items; they occur within systems. The structure of a sign is to a great extent inherited from the system to which it belongs. The meaning of the sign is relational because they are always interpreted in a particular context. Signs are used by people in a social context as part of their participation in social groups. Semiotics is used as an analytic tool for the didactics of mathematics which is applicable in the cognitive, the social or the cultural level of investigation (WINSLOW, 2004).

According to this context, semiotic perspective helps teachers understand how the use of natural language, mathematics, and visual representations form a single unified meaning-making system. Since there are different semiotical approaches, it is important to discuss different points in which mathematical reflections can be enlightened by applying a certain type of semiotics.

Peirce's theory of signs and his classification from the point of view of the object of the sign (representant) is helpful in understanding different ways to represent, for example, the long division algorithm. Houser (1987) stated that Pierce defined a sign as anything which an individual so determined by something else, called its object, and so determines an effect upon a person, which effects the individual call its "representant". In this view, educators use signs all of the time, to interact with students. Peirce believed that signs are the matter or the substance of the thought and said that "life itself is a train of thought", that is, life and signs are fundamentally related and unseparable for all humans. Teachers present their students with signs (representants) in hopes of helping them to understand information (HOUSER, 1987).

Sometimes mathematical lessons revolve around coming to consensus or an understanding of the meaning of a sign, such as the symbol for division. Often, mathematical lessons simply use representations to help relate other ideas or signs. Sometimes students do not see the sign or symbol or algorithm as teachers assumed they would. In so doing, the example we use here, to initiate our discussion, is the long division algorithm as used in the United States.

Peirce's classification of signs from the point of view of the subject is helpful in understanding these representations. This context allowed Presmeg (1998) to develop a theoretical framework, drawing on literature from semiotics and ethnomathematics, to address the ways in which real experiences and cultural practices of students may be connected with mathematics classroom pedagogy. Her project investigated how mathematics educators could prepare prospective and practicing teachers to cope with cultural diversity. The first component of her project was the investigation of the ways that students could use their cultural identities and practices in constructing mathematical ideas that belong uniquely to them. The second was the investigation of ways teachers could facilitate students' construction of such uniquely personal cultural mathematics ideas in a high school classroom. The third component was the development of a grounded theoretical framework in which to situate the two previous components by using semiotic chaining. The high school project involved seven students from differing ethnic backgrounds. Evidence from these students made a strong case that traditional mathematics teaching does not facilitate a view of mathematics that encourages students to see the potential of mathematics outside the classroom. Although their own reports indicated that students were involved in many life activities with mathematical aspects, they continued to see mathematics as an isolated subject without much relevance to their lives.

Semiotic process may be used to illustrate connections as symbol systems constructed in a bridge between cultures. In order to do so, Peirce classified the relation of a sign to its object in one of three ways: as an icon, index, or symbol (HOUSER, 1987). An *icon* has some quality that is shared with the object. An *index* has a cause and effect link and a *symbol* denotes its object by virtue of a habit, law, or convention. A symbol then becomes an abstract representation of the object, much like written letters are related to verbal language. The symbol for division as used in North America can be interpreted as an icon. A drawn division symbol (representant) looks like the real division symbol used in public schools in the United States. In so doing, symbolism provides possible connections between mathematical ideas frozen in academic mathematics and practices, and different symbolism would facilitate the construction of different mathematics structures and concepts with increased relevance to students from different cultures.

It is paramount that educators understand Peirce's classification, because they must be aware that representation are perceived in different ways by students from different cultural background (HOUSER, 1987). What is an icon to teachers may be perceived as a symbol by students. In so doing, educators must try to learn symbols, icons, and signs that students interpret differently, and use this knowledge as a path and method for their instructional pedagogical practices. Teachers (interpretants) use the division symbol to represent a division algorithm. The interpretant related to this representant of the division symbol was different for students than for the teachers. So it is that, some students coming from the countries from the former Soviet Union and South America first viewed the iconic division symbol as used in the United States as representing the operation for a square root.

ALGORITHMS AND IMMIGRANT STUDENTS

Algorithmic strategies are the mathematical or more commonly the arithmetic actions themselves. These form part of the use of conventional and personal collective symbolic tools used in the solution of a problem that can be solved through mathematical operations and representations. These actions require tools and instruments that permit and grant adequate execution. Symbolic tools are the power and often guarantee the resolving of a problem.

There are different symbolic tools specific to diverse cultural groups. Algorithms are closely linked to mathematical, historical-cultural practices, languages and symbolical representations. The basic algorithms we learn and use are real objects created through an interaction of time, diverse peoples, language, and culture. Algorithms emerged from collective and individual operations whose purpose is to achieve specific goals. Algorithms are mutually agreed upon forms of solving problems guided by a set of concrete actions. The strategies that members of diverse cultural groups have created for everyday calculation developed from are in relation to their environment. These strategies have involved certain operations using symbolic tools and representations.

Algorithms frequently embody significant ideas, and an understanding of these ideas is a source of mathematical power. The notion of an algorithm as a method used to solve a problem, involves the following five aspects:

1. Presentation of the idea of an algorithm as a procedure guaranteed to solve a type of problem according to its cultural origin.
2. Experience with some culturally diverse algorithms.
3. The algorithms do not simply mean that they are rules for doing arithmetic.
4. Cultural mathematics practices and procedures can be “algorithmatized.”
5. There are different algorithms that can accomplish the same task.

However, there are teachers who believe that algorithms are embedded exclusively in mathematics as a body of pure knowledge. To contradict this point of view, Philipp (1996) stated that mathematics teachers must be encouraged to consider the use of alternative algorithms because they should recognize that several methods can be used to solve a problem other than the standardized methods. In so doing, invented and culturally-based algorithms can even be more effective than traditional ones if their logic is sound and simple. This perspective allows teachers with students of different racial and ethnic backgrounds to be encouraged the use invented algorithms their students might create as well as culturally-based algorithms they bring to the mathematics classroom.

Conventional logic has given rise to the modern western worldview, which has become a strong and dominate form of many conflicting dualisms, such as West versus East, North versus South, “pure” mathematics versus mathematics education, concrete versus abstract, and ethnomathematics versus academic mathematics. This perceived conflict in what is perceived as academic knowledge is legitimated through current models of curriculum design and assessment strategies. And often prevents students with diverse cultural backgrounds and practices from achieving success in traditional school mathematics curriculum, which often unintentionally promotes a disembodied-dualism, which continues to cultivate an essentialist view of mathematical intelligence as an ability to manipulate mathematical symbols so as to fit in a prescribed algorithm. This dismembers a culturally situated student thinking, limits their creative process, and most certainly contributes to low self esteem and expectations.

Kamii (1991) supports the idea that extensive practice with mental computation helps develop strong number sense. Since the standard algorithms tend to be optimized for pencil and paper computation and not for mental computation, practice in mental arithmetic will probably lead to alternative algorithms. Future studies might encourage the “best” practices or algorithms for mental calculation (BENJAMIN, 2006). The study of different culturally-based algorithms such as the division algorithm is a beneficial research activity that allows learners to investigate, classify and incorporate the results into lessons and learning activities. Algorithms can be influenced by history, culture and natural language. Since natural language expressions of algorithms tend to be verbose and ambiguous, they may influence the ways algorithms are expressed. This is one of the challenges faced by immigrant students in mathematics classrooms in California.

Often immigrant or English Language Learners (ELL) in SDAIE classes are considered "at-risk". The term "at-risk" describes students who are struggling in school because of linguistic, social, economic, and psychological factors. The label is generally a signal to educators that extra academic attention is needed. Richardson (1989) described how educators look at the personal and background factors students bring to the classroom, how these factors interact with classroom culture and practices, and how classroom practices impact the outside factors.

Immigrant students experience difficulties in learning mathematics that may have little to do with difficulties in the actual processing of mathematical ideas. Since many immigrant students often speak a language other than English, a typical approach to organized mathematics instruction currently observed in schools in the United States is not appropriate for their needs (DAVIDSON, 1990). A problem that comes frequently to highly diverse mathematics classrooms is related to students who may have difficulty in understanding the exact meaning of various symbols and representations.

Since a student's background knowledge and their entire repertoire of life experience determines the meaning derived from a mathematical problem, linguistic-cultural differences pose interesting challenges for students and teachers alike. Similarly, the classroom culture as set by the teacher influences how well students come to understand problems and how they conceptualize, interpret and solve problems.

In so doing, this aspect of curriculum provides an explicit and natural context for consideration of issues either relating to values, beliefs and preferences to mathematics, or involving mathematics as a cultural form of rational inquiry applied to contexts where values, beliefs, and preferences are keys aspects of a problems, task or situation.

Below, we would like to present a model for including alternative algorithms by sharing the history and some of the differences in the long division algorithm as found in a Sacramento public high school.

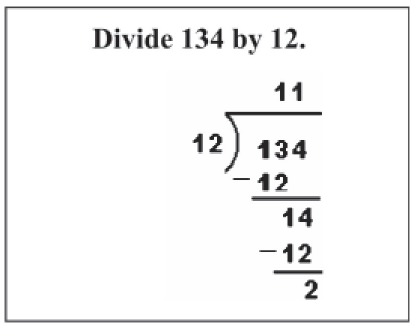
THE LONG DIVISION ALGORITHM

As alluded to earlier, mathematical symbols and representations can be seen as tools for conveying mathematical information. Understanding how immigrant students come to understand the meaning of new symbols and representations that may not be part of their former reality is paramount in their integration process into a new culture and value system, and developing flexible and creative problem-solving skills.

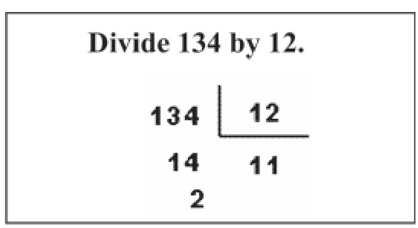
One of these experiences is related to the long division algorithm. The procedures as taught in USAan schools sometimes look different from what is used by the family, or that was learned in their country of origin. Perkins and Flores (2002, p.347) state that:

> As teachers encounter algorithms taught in other countries, they realize that the algorithms that they have learned are just some of the possible ways to compute answers. This realization can help teachers become more accepting when students deviate from the procedures or algorithms taught in class and use their procedures.

Schools in the United States, teach the following procedure for long division:



Brazilians, for instance, learned the following procedure for the same long division problem:



This method of doing long division, with some minor differences, was also shared by students who participated in the Algorithm Collection Project from Armenia, Iran, Pakistan, Russia, Ukraine, and Vietnam. In the case of the Brazilian division algorithm, the divisor is inside an L-shaped symbol. In the United States algorithm, the dividend is inside the symbol that looks like a denatured square root symbol. As well the Brazilian method encourages the use of mental calculation, where as the USAan method asks us to “show your work”. Thus, when some immigrant students see the North American form, they initially assume they are computing a square root.

One curiosity arising from the ACP is linguistically in nature. In Portuguese, division is always read as "134 divided by 12[6]". In English, it is said the same, but "dividing 12 into 134", or simply "12 into 134", or 12 goes into 134" are more common ways of saying it, monolingual speakers of English learning long division are often confounded at first by writing and thinking the opposite of what is written.

A CONCISE HISTORY OF DIVISION SYMBOLS

It is impossible to determine an exact date for the origin of the present arrangement of figure in the long division algorithm, partly because it developed and changed gradually over time. According to Cajori (1993) it appears that the Babylonians had ideograms translated from *Igi-Gal* to express denominator or division. They also possessed ideograms translated, as *Igi-Dua* for division.

Despite the Greek's fractional notation, they did not have a symbol for division. It seems that the notation for division had the same head as the notation for fractions. For example, Cajori (1993) stated that Diophantus (200-284), Greek mathematician, separated the dividend from the divisor by the words $\mu o \rho \iota o \nu$ to represent division. In the Hindu $Bakhsh\bar{a}l\bar{i}$ rithmetic division is marked by the abbreviation *bhâ* from *bhâga* which means part.

The Hindus designated fractions by writing the denominator beneath the numerator. The horizontal fraction bar was introduced by the Arabs, who first copied the Hindu notation, but later improved on it by inserting a horizontal bar between the two numbers. Several sources attribute the horizontal fraction bar to the Arabic writer al-Hassâr around 1200, who made use of the fractional line by writing the denominator below a horizontal line and over each of them the parts belonging to it.

Italian mathematician Fibonacci wrote in 1202 in his book *Il Liber Abbaci*, used the fractional line. He stated that when above any number a line is drawn, and above that is written any other number, the superior number stands for the part or parts of the inferior number. He called the superior number the *numerator* and the inferior number the *denominator*. Fibonacci symbolized division in fraction form with the use of a horizontal bar, but it is thought likely that Fibonacci adopted al-Hassar's introduction of this symbolization. Fibonacci was the first European mathematician to use the fraction bar as it is used today. He followed the Arab practice of placing the fraction to the left of the integer.

Cajori (1928) stated that the earliest mathematician to suggest a special symbol for division other than a fractional line was Michael Stifel (1487-1567), a German mathematician and algebraist, who wrote *Arithmetica Integra,* in 1545, often employed one or two lunar signs to perform short and long division. For example, the meaning of

[6] Cento e trinta e quatro dividido por doze.

the arrangement 8)24(or 8)24 is 24 divided by 8 (NCTM, 1969). Stifel also used the German capital 𝔇 to signify division, but he did not use Stifel's suggestion in arithmetic or algebraic computations. Simon Stevin (1548-1620) Flemish mathematician, in 1634, wrote *Euvres* in which he used the letter D to compute division problems. William Oughtred (1575-1660), English mathematician, in his book *Clavis Mathematicae*, written in 1631, emphasized the use of mathematical symbols such as division symbols. Oughtred wrote, in 1677, *Opuscula posthuma*, where he uses the symbols $\frac{4}{3}\Big)\frac{3}{2}\Big(\frac{9}{8}$ to mean $\frac{3}{2} \div \frac{4}{3} = \frac{9}{8}$. Joseph Moxon (1627-1691), wrote in 1679, the *Mathematical Dictionary* to explain mathematical terms and to show that D)$A + B - C$ means 𝔇. The book *Treatise of Algebra* written in 1685 by John Wallis (1616-1703), English mathematician, he factored 5940 as11)5)3)3)3)2)2)5940(2970(1485(495(165(55(11(1. German philosopher and mathematician, Gottfried Leibniz (1646-1716), stated in *Miscellanea Berolinensia,* in 1710, that division was commonly marked by writing the divisor beneath its dividend, with a line of separation between them (CAJORI, 1993).

In 1753, Gallimard used the inverted letter ᗡ for division, in his book *Methode d'arithmetique, d'a algebre et de geomètrie.* Portuguese mathematician, José Anastácio da Cunha (1744-1787), wrote *Princípios Matemáticos* in which he used the horizontal letter ⊐ as a symbol for division. In 1827, Peruvian author, Juan de Dios Salazar, in *Lecciones de Aritmetica*, division should be indicated by writing the dividend and the divisor on the same line, but inclosing the former in a parenthesis such as (20)5 meaning 20÷5. In relation to relative position of divisor and dividend, Cajori (1928, p. 273) stated:

> In performing the operation of division, the divisor and the quotient have been assigned various positions relative to the dividend. When the "scratch method" of division was practiced, the divisor was placed beneath the dividend and moved one step to the right every time a new figure of the quotient was to be obtained. In such cases, the quotient was usually placed immediately to the right of the dividend, but sometimes, in early writers, it was placed above the dividend. In short division, the divisor was often placed to the left of the dividend, so that *a)b(c* came to signify division.

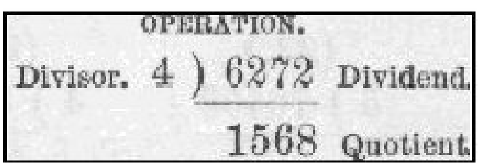
The long division method was found in 1827 in A *Course of Mathematics* written by Charles Hutton where he stated that to divide a number by the whole divisor at once, after the manner of long division. In 1833, F. Gerard described, in his book *Arithmétique de Bézout*, the following algorithm to divide 14464 by 8:

14464	**8**
	1808

In 1837, James Thomson, stated in his book *Treatise on Arithmetic,* that the French placed the divisor to the right of the dividend and the quotient below it. He also stated that they set up the division algorithm in a more compact and neat appearance. In his opinion, the French algorithm for division possessed the advantage of having the quotient near the divisor which allowed the fastest way to multiply the divisor by the dividend.

The short division method is found in 1844 in *Introduction to the National Arithmetic on the Inductive System* written by Benjamin Greenleaf (1786-1864) that described a method of operation by using short division; however the divisor could not exceed 12. In 1857, in his book *Higher Arithmetic, or the Science and Application of Numbers, Combining the Analytic and Synthetic Modes of Instruction;* James B. Thomson stated that "[…] the divisor is placed on the left of the dividend, and the quotient under it, merely for sake of convenience (p. 69)". In this perspective, it is interesting to observe that Thomson stated that the French place the divisor to the right of the dividend, and the quotient below it. This example shows that he valued different ways in which to solve problems.

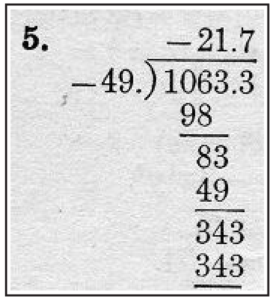
In the middle of the nineteenth century, some United States mathematics textbooks commonly showed long division with the divisor, dividend, and quotient on the same line, separated by parentheses, as 36)116(3.[7] According to Miller (2008), this notation is used, for example, in 1866 in the book *Primary Elements of Algebra for Common Schools and Academies* written by Joseph Ray. This same notation for long division is used in 1882, in the book *Complete Graded Arithmetic* written by James B. Thomson. In the examples used for short division, a vinculum almost attached to the bottom of the close parenthesis is placed under the dividend and the quotient is written under the vinculum.



From ***Complete Graded Arithmetic,* 1882**
(source: http://members.aol.com/jeff570/operation.html)

Miller (2004) stated that in the teacher's edition of the book *Elements of Algebra* written by Joseph Ray in 1888, the algorithm for division is shown below.

[7] This method was common at least until mid-century in the United States as noted by the famous Ma and Pa Kettle dialogue. See: http://www.csus.edu/indiv/o/oreyd/ACP.htm_files/Alg.html



From Elements of Algebra, 1888
(source: http://members.aol.com/jeff570/operation.html)

In accordance to Miller (2004), the book *Robinson's Complete Arithmetic* written by Daniel W. Fish in 1901, the Thomson's notation was used for short and long division, except that a vinculum was attached under the dividend to the close parenthesis.

FINAL CONSIDERATIONS

A comprehensive view of mathematics curriculum is implicit in an ethnomathematical perspective. All individuals possess potential for understanding and communication through a variety of mathematical signs and systems within cultural context. This allows them to gain new perspectives on human potential and on the organization of the mathematics curriculum. To our mind, mathematics can only be truly learned and taught if it includes culture, natural language, and visual representations that are culturally relevant to learners and teachers alike.

An ethnomathematical perspective helps all participants to come to understand and appreciate alternative viewpoints, cultural diversity, natural language, mathematics, and visual representations which form a unique system for meaning-making. In this context, reorienting teaching and learning to include ethnomathematics can engage and excite students about learning and encourages them to see themselves as being able to do mathematics. When we begin with validating a student's own cultural experiences we are, in fact serving as an essential components of understanding and celebrating the differences between us. A few specific suggestions are:

- Emphasize multicultural referents and relevancy in lessons (KRAUSE, 1983).
- Use basic mathematics vocabulary in the second language for individualized instruction whenever possible. Note that the vocabulary should not be used to focus on key words but should be used in context to develop understanding (FREEMAN; FREEMAN, 1988).

- Be aware of how other countries and cultures teach basic mathematical concepts (SECADA, 1983).
- Concerted efforts to be aware of and to explain any culturally-based terms (FREEMAN; FREEMAN, 1988).
- Most importantly is a unique connection observed by participants in the ACP, that of multilingualism and the long division used by students from Vietnam, Brazilian, and the former Soviet Union. Students who speak more than two languages and use long division algorithm, possess more confidence mathematically, and are better problem solvers, than monolingual North American long division algorithm users. There is an apparent link between multilingualism, and mathematics that need be further researched.

All students and teachers should understand why and how algorithms they use actually work. This understanding should be achieved as soon as possible, at the time of introduction of algorithms. Developing a theoretical framework by drawing on the literature of mathematics history, semiotics and ethnomathematics to address the ways in which real experiences and cultural practices of students are connected to the mathematics found in classroom pedagogy. There is hope that students who use different algorithms understand the relation to alternative forms of problem solving. In this context an ethnomathematics perspective helps reinforce available comparisons through the study of culturally diverse algorithms. Participants in the ACP develop an awareness of the qualities of culturally-based algorithms, and develop an appreciation of algorithms as natural creations, indeed extensions and/or artifacts of diverse cultural groups used to produce mathematical knowledge.

A sufficiently deep appreciation of the beauty, power, and sophistication of culturally diverse algorithms bridges the gap between standardized algorithms and the historical-cultural perspective of students. Learning that there are diverse ways of solving problems, as sued by one's peers, serves as a gateway to learn how other people solve problems, even as a basic as a simple division problem. In learning an algorithm one learns to confront the essence of the phenomena by which an algorithm comes to guarantee and accomplish its goal and objective. What we have seen in our work is that if we see mathematics as a universal language, then there are various accents and dialects worthy of study.

An ethnomathematics perspective provides a transformational space for students and teachers, and allows them to think of diversity as good, valuable, and necessary to living in a globalized interconnected world. The study of the algorithms immigrants bring to the community, encourages both educators and students learn that there is a difference between different and wrong. To see the value of a diverse community from a concrete, basic mathematics activity common to all peoples serves as an excellent gateway to understanding.

REFERENCES

AIKEN JR., L. S. Language factors in learning mathematics. *Review of Educational Research*, v.42, n.3, p.359-85, 1972.

BENJAMIN, A. *Secrets of mental math*. New York: Three Rivers Press, 2006.

CAJORI, F. *A history of mathematical notations*. Chicago: Open Court Publishing, 1928.

CAJORI, F. *A history of mathematical notations: two volumes bound as one*. New York: Dover Publications, Inc., 1993.

CUEVAS, G. J. Mathematics learning in English as a second language. *Journal of Research in Mathematics Education*, v.15, n.2, p.135-144, 1984.

D'AMBROSIO, U. *Etnomatemática*. São Paulo: Editora Ática, 1990.

D'AMBROSIO, U. What is ethnomathematics and how can it help children in schools? *Teaching Children Mathematics,* v.7, n.6, p.308–10, 2001.

DAVIDSON, D. M. An ethnomathematics approach to teaching language minority students. In: REYHNER, J. (Ed.). *Effective language education practices and native language survival*. Choctaw, OK: Native American Language Issues, 1990. p.143–148.

DAWE, L. Bilingualism and mathematical reasoning in English as a second language. *Educational Studies in Mathematics*, v.14, n.4, p.325-53, 1983.

FREEMAN, D.; Y. FREEMAN. *Sheltered English instruction*. Washington, DC: ERIC Document Reproduction Service no. ED 301070, 1988.

GOLDIN, G.; SHEINGOLD, N. System of Mathematical Representation and Development of Mathematical Concepts. In: F. R. Curcio (Ed.). *The Roles of Representation in School Mathematics*. Reston: National Council of teachers of Mathematics, 2001.

HALL, E. T. *Beyond culture.* Gloucester, MA.: Peter Smith Publishers, 1992.

HOUSER, N. Toward a Peircean semiotic theory of learning semiotics. *The American Journal of Semiotics*, v.5, n.2, p.251–274, 1987.

KAMII, C.; LEWIS, B. A. Achievement tests in primary mathematics: Perpetuating lower-order thinking. *Arithmetic Teacher*, v.38, n.9, p.4-9, 1991.

KESSLER, C.; QUINN, M. E.; HAYES, C. W. *Processing mathematics in a second language*: Problems for LEP children. In: Delaware Symposium VII on Language Studies. Newark, DC, USA. ERIC Document Reproduction Service No. ED 268821, 1985.

KRAUSE, M. C. *Multicultural mathematics materials.* Reston, VA: NCTM, 1983.

MATHER, J. R. C.; CHIODO, J. J. A mathematical problem: How do we teach mathematics to LEP elementary students? *The Journal of Educational Issues of Language Minority Students*, v.13, n.1, p.1-12, 1994.

MILLER, J. *Earliest uses of symbols of various mathematical symbols*. Available at http://members.aol.com/Jeff570/mathsym.html, 2008.

MIURA, I. T. The influence of language on mathematical representations. In: F. R. Curcio, (Ed.)*, The Roles of Representation in School Mathematics.* Reston: NCTM, 2001.

NCTM (National Council of Teachers of Mathematics). *Historical Topics for the Mathematics Classroom,* Reston: NCTM, 1969.

NCTM (National Council of Teachers of Mathematics). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: NCTM, 1989.
NORMAN, C. *Americans all.* Grand Rapids, MI: Instruction Fair, 1988.
NUNES, T.; BRYANT, P. *Learning and teaching mathematics:* An international perspective. East Sussex, UK: Psychology Press Ltd., 1997.
OREY, D. C. The *Algorithm Collection Project*. Available at http://www.csus.edu/indiv/o/oreyd/ACP.htm_files/Alg.html, 2008.
OREY, D.; ROSA, M. Cultural assertions and challenges towards pedagogical action of an ethnomathematics program. *For the Learning of Mathematics,* Canada, v.27, n.1, p.10-16, 2007.
PERKINS, I.; FLORES, A. *Mathematical notations and procedures of recent immigrant students*. Mathematics Teaching in the Middle School**,** v.7, n.6, p.346-351, 2002.
PHILIPP, R. Multicultural mathematics and alternative algorithms. *Teaching Children Mathematics*, v.3, n.3, p.128-133, 1996.
PRESMEG, N. C. A semiotic analysis of students' own cultural mathematics. In: OLIVIER, A.; NEWSTEAD, K. (Ed.). *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education.* Research Forum Report. Stellenbosch, South Africa, v.1, p.136-151, 1998.
RICHARDSON, V.; CASANOVA, U.; PLACIER P.; GUILFOYLE, K. *School Children At-Risk.* London: Falmer, 1989.
ROSA, M. Currículo e matemática: algumas considerações na perspectiva etnomatemática. *Plures Humanidades*, Ribeirão Preto SP, v.6, n.6, p.81-96, 2005.
ROSA, M.; OREY, D. C. Abordagens atuais do programa etnomatemática: delineando-se um caminho para a ação pedagógica. *BOLEMA,* v.19, n.26, p.19-48, 2006.
SECADA, W. G. *The educational background of limited English proficient students*: Implications for the arithmetic classroom. Washington, DC: Office of Bilingual Education and Minority Education. ERIC Document Reproduction Service No. ED 322284, 1983.
VERGANI, T. *Ethnomathematics and symbolic thought*. Paper presented at the First international conference in ethnomathematics, University of Granada, Spain, 1998.
WINSLOW, C. Semiotics as an analytic tool for the didactics of mathematics. *Nordisk Mathematikkdidaktikk,* v.9, n.2, p.81-99, 2004.
ZASLAVSKY, C. *The multicultural math classroom:* Bringing in the world. Portsmouth, NH: Heinemann, 1996.

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