Generalized Poisson–Neumann Polygonal Basis Functions for the Electromagnetic Simulation of Complex Planar Structures

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Abstract—Rooftop functions are commonly used for the discretization of planar currents in electromagnetic (EM) simulators. We describe the generalization of the rectangular and triangular rooftop functions to arbitrary polygonally shaped subdomains. It is shown that these generalized basis functions are solutions to a pertinent Neumann–Poisson problem, and we derive the integral equations satisfied by these basis functions. The new generalized polygonal functions allow for a more efficient meshing of complex geometrical structures in terms of polygonally shaped cells. They naturally model the current flow in the polygonal cells, satisfy the current continuity relation, and significantly enhance the EM simulation performance for complex geometrical structures. The increased simulation performance is demonstrated for a complex radio-frequency board interconnection layout and a spiral inductor on a silicon substrate.

Index Terms—Basis functions, complexity reduction, electromagnetic (EM) simulation, planar structures, Poisson–Neumann problem.

I. INTRODUCTION

Over the past decade, planar electromagnetic (EM) simulators1 have been extensively used for the time-harmonic characterization of planar structures in radio-frequency (RF) board microwave circuit and antenna applications. The EM behavior of the planar structure is governed by an integral equation in the unknown surface currents flowing on the planar metallization patterns. This integral equation is solved numerically by applying the method of moments (MoM). The planar structure is therefore typically subdivided or discretized into a mesh of rectangular and/or triangular cells. The fundamental reason for the triangular and/or rectangular subdivision scheme is that the currents can then be expanded in a basis of simple linear divergence and curl-conforming vector functions [1], also called rooftop functions or surface doublets [2]–[4], which satisfy the pertinent continuity equations and exhibit a locally constant charge distribution. The most important current continuity condition, in order for the currents to locally satisfy Kirchoff’s law, is for the normal component of the current to be continuous across the boundaries of adjacent cells in the mesh.

The above-mentioned rooftop functions associated with rectangular or triangular cells [5], [6] can be briefly described as follows. One vector function, which models the normal component of the current flowing across the cell side, is associated with each side of the cell. This vector function is constant along the corresponding side and varies linearly to zero along the adjacent sides of the cell. Rooftop functions with a rectangular subdomain have only one component in the direction normal to the corresponding cell side. Rooftop functions with triangular support, however, also have a component tangential to the cell side. This component is necessary to obtain the continuity of the normal current across the adjacent cell sides in the triangular cell. The next step, after triangles and rectangles, would seem to be quadrilaterals. Unfortunately enough, for general quadrilaterals, there do not exist linear rooftop functions with locally constant charge distribution that satisfy all of the relevant continuity requirements [7], [8].

In this paper, we describe the generalization of the rectangular and triangular rooftop functions to arbitrary polygonally shaped subdomains. In mathematical terms, it is shown that these generalized basis functions are solutions to a pertinent Neumann–Poisson problem. These generalized polygonal vector functions naturally model the current flow in a polygonal cell and by definition satisfy the normal current continuity condition across the edges of the cell. When used as current basis functions in a MoM numerical solution algorithm, they significantly enhance the simulation performance for complex geometrical structures, as will be demonstrated in Sections III and IV. Parts of the results of this paper were earlier very succinctly presented in [9], [10].

II. MATHEMATICAL FRAMEWORK

The commonly used triangular and rectangular rooftop functions are locally curl-free with a locally constant charge density [1]–[4]. In order to generalize these functions to cells with a more general shape, the pertinent question is: can we find a curl-free current density $\mathbf{J}$ with constant divergence $A$ over a general simply connected domain $D$ such that its flux $\phi$ has pre-assigned values $\phi(s)$ on its piecewise smooth boundary $\partial D$? In other words, we need

$$\text{div}_x \mathbf{J} = \frac{\partial}{\partial x} J_x + \frac{\partial}{\partial y} J_y = A$$

(1)

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1 Examples include EM (Sonnet Software, Liverpool, NY), Momentum (Agilent Technologies, Palo Alto, CA), IE3D (Zeland Software, San Francisco, CA), and Ensemble (Ansoft Corporation, Pittsburgh, PA).
\[ \text{curl}_z \mathbf{J} = \frac{\partial}{\partial y} J_x - \frac{\partial}{\partial x} J_y = 0 \] (2)

in \( D \) and flux
\[ \mathbf{J} \cdot \mathbf{n} = \phi(s) \] (3)
on \( \partial D \). Integrating (1) over \( D \) shows that the constant \( A \) and \( \phi(s) \) are related by
\[ A = \frac{1}{S} \int_{\partial D} \phi(s)ds \] (4)
where \( S \) is the surface of \( D \). Applied to a general polygonal domain, when we take \( \phi(s) = 1 \) on one side of the polygon and \( \phi(s) = 0 \) on the other sides, we obtain a generalization of the ubiquitous triangular and rectangular vector functions. Of course, these vector functions must be combined into doublets (rooftops) in order to guarantee the normal continuity of the current density.

A particular solution of (1) and (2) is given by \( J_x = 1/2Ax, \quad J_y = 1/2Ay \). Hence, by the Cauchy–Riemann equations [11], the real solution of (1) and (2) can be written in complex form as
\[ J_x - iJ_y = \frac{1}{2} Az^* + f(z) = \gamma z^* + f(z) \] (5)
where \( z = x + iy, \quad \gamma = A/2, \) and \( f(z) \) is analytic in \( D \). Note that the reader may indeed verify easily that \( P = J_x - \gamma x \) and \( Q = -J_y + \gamma y \) satisfy the Cauchy–Riemann equations, i.e.,
\[ \frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0. \] (6)

Defining the complex unit normal \( \eta \) and complex unit tangent \( \tau \) as
\[ \eta = n_x + in_y \quad \tau = t_x + it_y = i\eta \] (7)
we can write
\[ (J_x - iJ_y)\eta = \mathbf{J} \cdot \mathbf{n} - i\mathbf{J} \cdot \mathbf{t} = \phi - i\psi. \] (8)
Hence,
\[ i[\gamma z^* + f(z)] = \phi(s) - i\psi(s), \] (9)
on \( \partial D \).

It is important to note that the real and imaginary parts of an analytic function in \( D \) cannot be arbitrarily chosen on \( \partial D \). Note that (1)–(3) are a special case of the general inhomogeneous Poisson–Neumann problem, which will be introduced and studied in detail in Section II-B. We will now discuss a complete integral equation formalism in order to solve the Poisson–Neumann problem for the homogeneous and inhomogeneous cases in a rigorous manner.

A. Integral Equations for the Homogeneous Case
Let us first consider the homogeneous case
\[ \frac{\partial}{\partial x} J_x + \frac{\partial}{\partial y} J_y = 0 \] (10)
\[ \frac{\partial}{\partial y} J_x - \frac{\partial}{\partial x} J_y = 0 \] (11)
in \( D \) with boundary condition
\[ \mathbf{J} \cdot \mathbf{n} = \phi(s) \] (12)
on \( \partial D \). We wish to obtain an integral equation formulation yielding \( \mathbf{J} \cdot \mathbf{t} = \psi \) as a function of \( \phi \). It is seen that \( \mathbf{J} \) can be written as a gradient, i.e.,
\[ \mathbf{J} = \nabla K \] (13)
where \( K \) is a harmonic function, i.e., a solution of the Laplace equation
\[ \nabla^2 K = 0, \quad r \in D \] (14)
together with the Neumann boundary condition
\[ \frac{\partial}{\partial \mathbf{n}} K = \phi(s), \quad r \in \partial D. \] (15)
This problem admits a solution provided
\[ \int_{\partial D} \phi(s)ds = 0. \] (16)

From electrostatic theory [13], \( K(r) \) can be written as a single-layer potential
\[ K(r) = \int_{\partial D} \ln |r - r'(s')| \sigma(s')ds' \] (17)
with sources \( \sigma(s) \). The gradient of \( K(r) \) inside \( D \) is
\[ \nabla K = \int_{\partial D} \{ \nabla \ln |r - r'(s')| \} \sigma(s')ds'. \] (18)

On the interior boundary \( \partial D_- \), the gradient of \( K(r) \) is [14]
\[ \nabla K = \text{PV} \int_{\partial D} \{ \nabla \ln |r - r'(s')| \} \sigma(s')ds' - \pi \mathbf{n}(s) \sigma(s). \] (19)

This leads to the principal value integral equation
\[ \text{PV} \int_{\partial D} \left\{ \frac{\partial}{\partial \mathbf{n}} \ln |r(s) - r'(s')| \right\} \sigma(s')ds' - \pi \sigma(s) = \phi(s). \] (20)

Obtaining \( \sigma(s) \) from (20) formally solves the problem. From (19), we also find
\[ \text{PV} \int_{\partial D} \left\{ \frac{\partial}{\partial \mathbf{n}} \ln |r(s) - r'(s')| \right\} \sigma(s')ds' = \psi(s) \] (21)
relating the source term and the tangential component of \( \mathbf{J} \). Since
\[ \frac{\partial}{\partial \mathbf{n}} \ln |r(s) - r'(s')| - i \frac{\partial}{\partial t} \ln |r(s) - r'(s')| = \frac{\eta(s)}{z(s) - z(s')} \] (22)
we can combine (20) and (21) to yield
\[ \phi(s) - i\psi(s) = \eta(s) \text{PV} \int_{\partial D} \frac{\sigma(s')}{z(s) - z(s')}ds' - \pi \sigma(s). \] (23)
This can be written as
\[ \frac{\phi(s) - i\psi(s)}{\eta(s)} = \frac{\sigma(s)}{\eta(s)} - \frac{1}{\pi i} \text{PV} \int_{\partial D} \frac{\sigma(s')}{z(s') - z} ds'. \] (24)
\[
\mathcal{K}(h) = \frac{1}{\pi i} \int_{\partial D} \frac{h(z')}{z' - z} \, dz', \quad z \in \partial D.
\]

(26)

The Poincaré–Bertrand formula \cite{11} states that

\[
\frac{1}{\pi i} \int_{\partial D} \frac{1}{z' - z} \left[ \frac{1}{\pi i} \int_{\partial D} \frac{h(z'')}{z'' - z'} \, dz'' \right] \, dz' = h(z), \quad z \in \partial D.
\]

(27)

under quite general circumstances. Speaking in operator terms, this means that we simply have \(\mathcal{K}^2(h) = h\). Hence the source term \(\sigma\) can be eliminated from (25) by multiplication with \(\mathcal{I} - \mathcal{K}\), yielding

\[
\frac{\phi(s) - i \psi(s)}{\eta(s)} = \frac{1}{\pi i} \int_{\partial D} \frac{\phi(s') - i \psi(s')}{\eta(s')} \, ds', \quad z \in \partial D.
\]

(28)

From (8), we know that

\[
\frac{\phi - i \psi}{\eta} = J_x - i J_y = f(z), \quad z \in \partial D
\]

(29)

and hence the analytic function \(f(z)\) satisfies the principal value Cauchy equation

\[
f(z) = \frac{1}{\pi i} \int_{\partial D} \frac{f(z')}{z' - z} \, dz', \quad z \in \partial D.
\]

(30)

Equation (28) can be written as

\[
\phi(s) - i \psi(s) = \frac{1}{\pi} \int_{\partial D} \frac{\eta(s')}{z(s') - z(s)} \left[ \phi(s') - i \psi(s') \right] \, ds'
\]

(31)

which, in virtue of (22), can be decoupled in two real principal value integral equations as follows:

\[
\begin{aligned}
PV \int_{\partial D} \left\{ \frac{\partial}{\partial n} \ln |r(s) - r(s')| \right\} \psi(s') \, ds' + \pi \psi(s) \\
= -PV \int_{\partial D} \left\{ \frac{\partial}{\partial t} \ln |r(s) - r(s')| \right\} \phi(s') \, ds'
\end{aligned}
\]

(32)

\[
\begin{aligned}
PV \int_{\partial D} \left\{ \frac{\partial}{\partial n} \ln |r(s) - r(s')| \right\} \phi(s') \, ds' + \pi \psi(s) \\
= PV \int_{\partial D} \left\{ \frac{\partial}{\partial t} \ln |r(s) - r(s')| \right\} \psi(s') \, ds'.
\end{aligned}
\]

(33)

If \(\phi\) is known, we can determine \(\psi\) from (32), and if \(\psi\) is known, we can determine \(\phi\) from (33). Of course, given \(\phi\), we could also determine \(\psi\) from (33), but from a numerical stability point of view and Fredholm theory, it is always better to deal with integral equations of the second kind.

**B. Inhomogeneous Case: Pompeiu’s Formula**

Let us now introduce the general inhomogeneous case of the Poisson–Neumann problem. It consists of finding a vector field \(\mathbf{J}\) satisfying

\[
\frac{\partial}{\partial x} J_x + \frac{\partial}{\partial y} J_y = \rho
\]

(34)

\[
\frac{\partial}{\partial x} J_x - \frac{\partial}{\partial y} J_y = \zeta
\]

(35)

in \(D\) with the boundary condition

\[
\mathbf{J} \cdot \mathbf{n} = \phi
\]

(36)

on \(\partial D\), for a given \(\rho\), \(\zeta\), and \(\phi\).

For a general complex-valued, real-differentiable function \(F(z) = F(x, y)\), the complex derivatives with respect to \(z\) and \(z^*\) are defined \cite{11} as

\[
\frac{\partial F}{\partial z} = \frac{1}{2} \left( \frac{\partial F}{\partial x} - i \frac{\partial F}{\partial y} \right)
\]

(37)

\[
\frac{\partial F}{\partial z^*} = \frac{1}{2} \left( \frac{\partial F}{\partial x} + i \frac{\partial F}{\partial y} \right).
\]

(38)

For example, if \(F = u + i \psi\), we have

\[
\frac{\partial F}{\partial z^*} = \frac{1}{2} \left\{ \left( \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \right) + i \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) \right\}.
\]

(39)

We infer from (38) that \(\partial F/\partial z^* = 0\) when \(F(z)\) is analytic. Pompeiu’s generalization \cite{11} of Cauchy’s formula states that

\[
F(z) = \frac{1}{2\pi i} \int_{\partial D} \frac{F(z')}{z' - z} \, dz' - \frac{1}{\pi} \int_{D} \frac{\partial F}{\partial z^*} (z') \, dS',
\]

(40)

When \(F(z)\) is analytic, this reduces to Cauchy’s formula. We will now show that Pompeiu’s formula is essential for the solution of the Poisson–Neumann problem. Defining

\[
F(z) = J_x(z) - i J_y(z)
\]

(41)

we have by the definition of the complex derivatives

\[
\frac{\partial F}{\partial z^*} = \frac{1}{2} (\rho + i \zeta).
\]

(42)

Hence, from (8) and (39), we obtain the following representation:

\[
J_x(z) - i J_y(z)
\]

\[
= \frac{1}{2\pi i} \int_{\partial D} \frac{\phi(s') - i \psi(s')}{z(s') - z} \, ds' - \frac{1}{\pi} \int_{D} \frac{\rho(z') + i \zeta(z')}{z' - z} \, dS',
\]

(43)

Suppose further that we can describe \(\rho\) and \(\zeta\) by two-dimensional (2-D) polynomials. Then it is possible to write

\[
\frac{1}{2} (\rho + i \zeta) = \sum_{0 \leq k, l \leq m} a_{k,l} z^k \bar{z}^l.
\]

(44)
Defining

$$H(z) = \sum_{0 \leq k, l \leq m} \frac{1}{l+1} a_{kl} z^k \bar{z}^{l+1}$$  \hspace{1cm} (45)$$

we have $\partial H / \partial \bar{z}^* = 1 / (2 \rho + i \zeta)$. Hence,

$$H(z) = \frac{1}{2 \pi i} \oint_{\partial D} \frac{H(z')}{z' - z} dz' - \frac{1}{\pi} \iint_D \frac{\partial F}{\partial z^*} \left( \frac{z'}{z' - z} \right) dS',$$ \hspace{1cm} (46)

Subtracting (46) from (39), we obtain

$$F(z) - H(z) = \frac{1}{2 \pi i} \oint_{\partial D} \frac{F(z') - H(z')}{z' - z} dz', \hspace{1cm} z \in D$$ \hspace{1cm} (47)

implying that $F(z') - H(z')$ is analytic in $D$. Taking into account (30), (32), and (33) remain valid, if we apply them to the transformed variables $\hat{\phi}$ and $\hat{\psi}$ defined as

$$\hat{\phi} = \phi - \Re \left[ \eta H(z) \right], \hspace{1cm} \hat{\psi} = \psi + 3 \Im \left[ \eta H(z) \right].$$ \hspace{1cm} (48)

For example, if $\rho = A$ and $\zeta = 0$, as in (1) and (2), we have

$\hat{H}(z) = \gamma z^* \bar{z}$ as in (5) and $\hat{\phi} = \phi - \Re \left[ \eta H(z) \right]$. A comprehensive algorithm for the solution of the Poisson–Neumann problem is, therefore, as follows.

- Find $H(z)$ by formal integration with respect to $z^*$, i.e.,

$$H(z) = \int_0^* \frac{1}{2} \left( \rho + i \zeta \right) dz^*.$$ \hspace{1cm} (49)

- Transform $\phi$ to $\hat{\phi}$ as follows:

$$\hat{\phi} = \phi - \Re \left[ \eta H(z) \right], \hspace{1cm} z \in \partial D.$$ \hspace{1cm} (50)

- Solve

$$\text{PV} \oint_{\partial D} \left\{ \frac{\partial}{\partial \bar{n}} \ln |r(s) - r(s')| \right\} \hat{\psi}(s') ds' + \pi \hat{\psi}(s)$$

$$= -\text{PV} \oint_{\partial D} \left\{ \frac{\partial}{\partial \bar{n}} \ln |r(s) - r(s')| \right\} \hat{\phi}(s') ds'.$$ \hspace{1cm} (51)

- Result

$$J_x(z) - i J_y(z) = H(z) + \frac{1}{2 \pi} \oint_{\partial D} \frac{\hat{\phi}(s') - i \hat{\psi}(s')}{z(s) - z} ds', \hspace{1cm} z \in D.$$ \hspace{1cm} (52)

As an example, we take one of the well-known basis functions over the square, i.e.,

$$F(z) = J_x(z) - i J_y(z) \equiv x - 1.$$ \hspace{1cm} (53)

From $H(z) = z^* / 2$, we can write (52) as

$$\frac{z}{2} - 1 = \frac{1}{2 \pi} \oint_{\partial D} \frac{\hat{\phi}(s') - i \hat{\psi}(s')}{z(s') - z} ds', \hspace{1cm} z \in D.$$ \hspace{1cm} (54)

The left-hand side of (54) is clearly analytic in $D$ and the right-hand side of the same equation can be expressed as a Cauchy integral. Hence, we need

$$\frac{\hat{\phi}}{\eta} - i \frac{\hat{\psi}}{\eta} = \frac{z}{2} - 1$$ \hspace{1cm} (55)

and, hence, by (48)

$$\frac{\hat{\phi}}{\eta} - i \frac{\hat{\psi}}{\eta} = \frac{z}{2} - 1 + \frac{z^*}{2} = x - 1$$ \hspace{1cm} (56)

as required by (8).

C. Some Remarks

Using (41) and (42), we can interpret $J_0$ defined by

$$J_x - i J_y = H(z) = \oint_0^* \frac{1}{2} (\rho + i \zeta) dz^*$$ \hspace{1cm} (57)

as a particular solution of the Poisson–Neumann problem, without considering the boundary values. The solution incorporating the boundary conditions is then given by

$$\mathbf{J} = \mathbf{J}_0 + \nabla \mathbf{K}$$ \hspace{1cm} (58)

where $K$ can be written as the single-layer potential

$$K(r) = \oint_{\partial D} \ln |r - r(s')| \hat{\delta}(s') ds'$$ \hspace{1cm} (59)

with the source term $\hat{\delta}(s)$ satisfying

$$\text{PV} \oint_{\partial D} \left\{ \frac{\partial}{\partial \bar{n}} \ln |r(s) - r(s')| \right\} \hat{\delta}(s') ds' - \pi \hat{\delta}(s) = \hat{\phi}(s)$$ \hspace{1cm} (60)

where

$$\hat{\phi}(s) = \phi(s) - \mathbf{J}_0 \cdot \mathbf{n}.$$ \hspace{1cm} (61)

The key numerical point is the solution of integral (51). If we define the operators

$$\hat{\mathcal{N}} \hat{\phi} = \text{PV} \oint_{\partial D} \left\{ \frac{\partial}{\partial \bar{n}} \ln |r(s) - r(s')| \right\} \phi(s') ds'$$ \hspace{1cm} (62)

$$\hat{T} \phi = \text{PV} \oint_{\partial D} \left\{ \frac{\partial}{\partial \bar{n}} \ln |r(s) - r(s')| \right\} \phi(s') ds'$$ \hspace{1cm} (63)

then the pertinent equation can be written in a concise fashion as

$$\hat{\mathcal{N}} \hat{\psi} + \pi \hat{\psi} = -\hat{T} \hat{\phi}.$$ \hspace{1cm} (64)

Alternatively, we may use the integral equation formulations (32), (20), and (21) to yield

$$\hat{\mathcal{N}} \hat{\phi} + \pi \hat{\phi} = \hat{T} \hat{\psi}$$ \hspace{1cm} (65)

$$\hat{\mathcal{N}} \hat{\sigma} - \pi \hat{\sigma} = \hat{\phi}$$ \hspace{1cm} (66)

$$\hat{T} \sigma = \hat{\psi}.$$ \hspace{1cm} (67)

Note that (65) and (66) are Fredholm integral equations of the second kind and are therefore to be preferred above integral equation (67), which is Fredholm of the first kind. Fig. 1 shows vector plots for a traditional rectangular and triangular cell together with the corresponding result for a general polygonal cell.
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KNOCKAERT et al.: GENERALIZED POISSON–NEUMANN BASIS FUNCTIONS FOR EM SIMULATION OF COMPLEX PLANAR STRUCTURES

5

Fig. 1. Vector plots of the current distribution for rectangular, triangular, and general polygonal cells.

Fig. 2. Vector plots of the current distribution for T- and L-shaped cells.

Fig. 2 shows vector plots for more specific T- and L-shaped cells. It appears that these vector functions quite naturally model the current flow in the polygonal cells. Our results were obtained for \( \text{div}_s \mathbf{J} = -1/S \) with \( S \) the surface area of the cell and for \( \mathbf{J} \cdot \mathbf{n} = 1/L \) along one edge, where \( L \) is the length of that edge and \( \mathbf{J} \cdot \mathbf{n} = 0 \) along the other edges. The pertinent boundary integral equations are solved numerically by applying the MoM discretization along the edges of the cell using the classical one-dimensional (1-D) pulse basis functions with mid-point collocation.

III. PLANAR APPLICATIONS: CONVENTIONAL MESH VERSUS GENERALIZED POLYGONAL MESH

The MoM discretization for planar structures is succinctly illustrated in Fig. 3. The metallization patterns are conventionally meshed using rectangular and triangular cells. The linear cell sizes are of the order \( \lambda/20 \) to \( \lambda/10 \), where \( \lambda \) is the wavelength associated with the maximal frequency in the frequency band of interest. Maxwell’s equations are translated into integral equations by applying the boundary conditions on the planar structures. The surface currents are modeled with rooftop basis functions defined over the rectangular and triangular cells. The boundary conditions are imposed by applying the Galerkin testing procedure. This results in a discrete interaction matrix equation. The solution of this matrix equation yields the expansion coefficients for the unknown surface current.

The efficiency of the EM solution process depends strongly on the density and quality of the mesh. When simulating complex geometrical structures at RF frequencies, the meshing with rectangles and triangles leads to a much higher number of cells than needed by the wavelength criterion. This makes the EM simulators less attractive for simulating complex interconnect structures as the computer memory and time requirements are prohibitively high. The efficiency of the discretization is strongly improved when the restriction imposed by the use of rectangular and triangular cells is removed. This is realized by applying the concept of mesh reduction. Starting from an initial mesh of rectangular and triangular cells, a reduced mesh is constructed by merging two or more adjacent cells. This results in a mesh with a lower number of polygonally shaped cells. The mesh reduction step can be repeated up to the level in which each disconnected metallization pattern is represented by only one “generalized” cell. With each reduced mesh, an EM interaction matrix system can be built and solved. This mesh reduction process is illustrated in Fig. 4. The calculation of the interactions in the generalized mesh relies on the definition of generalized basis functions for polygonally shaped cells as defined in Section II. We have applied the generalized polygonal vector functions in the MoM simulation to model the surface current distribution of complex geometrical structures. For this we used the commercial code Momentum from Agilent Technologies in its RF mode.

IV. NUMERICAL SIMULATIONS

As a first example, we take the configuration of Fig. 5. It consists of the interconnection layout for a 35.6 mm × 43.67 mm
Fig. 5. RF board interconnection layout.

RF board circuit. The substrate is FR4 with a thickness of 30 ml. The lumped components are removed from the board and replaced by port connections, resulting in a total number of 60 ports. The interconnection structure is meshed at 1 GHz with an imposed mesh density of 20 cells per wavelength. For comparison purposes, we first used the rectangular/triangular meshing algorithm of Momentum in the microwave mode. The resulting mesh is shown in the top of Fig. 6. The corresponding interaction matrix has a size of 3428. Due to the geometrical complexity, this mesh contains a lot of redundant elements. The polygonal mesh (as obtained using the Momentum RF mode) corresponding to the imposed mesh density (bottom of Fig. 6) gives a much smaller interaction matrix size of 733. The simulation statistics for the two simulations are compared in Table I.

The polygonal mesh yields a threefold memory reduction and a fourteenfold speed improvement in the EM simulation. Some of the simulated $S$-parameters up to 1 GHz for the 60-port interconnection layout are displayed in Fig. 7 for both the rectangular/triangular mesh and the polygonal mesh. It is the selected results (transmission, reflection, crosstalk, and ground bounce) in which a designer is typically interested. The port numbers are indicated in the bottom of Fig. 6. Fig. 8 shows that the obtained results are almost identical, with the maximum difference just above $-60$ dB. The available amount of RAM is large enough to store the entire matrix.

As a second example, we take the configuration of Fig. 9. Fig. 9 shows the layout of a four-turn octagonal spiral inductor on a silicon substrate with a line width of 15 $\mu$m and a separation between the windings of 5 $\mu$m. Surrounding the inductor is a metallization ring which acts as the patch for the return current in the structure. This metallization ring is also connected to
the silicon substrate using a number of square-shaped vias. The radius of the inner winding is 65 \( \mu \text{m} \). The silicon substrate has a thickness of 500 \( \mu \text{m} \) and a resistivity of 0.15 \( \Omega \text{m} \). The \( \text{SiO}_2 \) layer on top of the silicon substrate has a thickness of 8 \( \mu \text{m} \). The spiral structure is meshed with 20 cells per wavelength at 20 GHz, using a classical rectangular/triangular subdivision (Fig. 10) and the new polygonal subdivision obtained with mesh reduction (Fig. 11). An edge mesh consisting of a band of narrow cells near the edges of the metallization is created in order to accurately model the higher current distribution near the edges of the metal. Note that this edge mesh is retained in the mesh reduction process, as can be seen in Fig. 11, in order to retain highly accurate simulation results. For length/width ratios that become very large, as in the edge mesh, the integral equations of Section II can be approximated by neglecting the width and hence by using a line approximation. The mesh with the rectangular/triangular cells (Momentum in the microwave mode) results in a discretized matrix equation with 1266 unknowns, while the reduced polygonal mesh (Momentum in RF mode) has only 506 unknowns. The simulation statistics are given in Table II.

For large values of \( N \), the time savings will be much more substantial. EM simulations or measurements yield \( S \)-parameter data, which can be used directly as a model for the spiral inductor in subsequent design steps. However, it is more convenient and useful for design purposes to use a number of derived quantities. For the spiral inductor, the most important ones are the inductance value and the quality factor. In order to extract these quantities, the reflection coefficient \( S_{11} \) seen at the first port is simulated with the second port shorted. This reflection coefficient is transformed to an input impedance from which the inductance value \( L \) and the quality factor \( Q \) of the spiral inductor are easily calculated by means of the following formulas:

\[
Z_{in} = R + j \omega L = \frac{1 + S_{11}}{1 - S_{11}} Q = \frac{\omega L}{R},
\]

The resulting plots for the simulated and measured values for \( L \) and \( Q \) as a function of frequency are shown in Fig. 12.
V. CONCLUSION

We have introduced new polygonal vector functions that are the generalization of the rectangular and triangular rooftop functions. These functions allow for a more flexible meshing of complex geometrical structures. The generalized polygonal functions satisfy the current continuity relation and are therefore very well suited to model current flow in planar structures. When applied in a planar EM simulator, and combined with a new mesh reduction technology to eliminate the redundancy in the EM equations, they result in a significant performance enhancement for complex geometrical structures.

REFERENCES


Daniël De Zutter (M’92–SM’96–F’00) was born in 1953. He received the M.Sc. degree in electrical engineering, Ph.D. degree, and the degree equivalent to the French Aggrégation or the German Habilitation from Ghent University, Gent, Belgium, in 1976, 1981, and 1984, respectively.

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