The analytic function $G$, is defined for $\text{Im } t < 0$, and $G = \text{Re } G$, is recovered by letting $\text{Im } t \to 0$. Furthermore, $\xi(t)$ denotes roots of the equation $\tau(\xi(t)) = t - t'$, with $\text{Im } \xi \geq 0$; in the integral, which arises from the two sides of the branch cut due to $(1 - \xi^2)^{1/2}$. Im $\tau(\xi) > 0$ while $\text{Im } \tau(\xi^*) < 0$, with $\xi^*$ denoting the complex conjugate of $\xi$. One may show that the contribution involving $\xi(t)$ on the right side of (10) generates the first term inside the braces in (5), while the integral can be reduced to a closed form that generates the second term. The choice of the complex source initiation time in (4b) corresponding to the complex source location in (4a) ensures that the manipulations leading to (10) are legitimate. In fact, any $t'$ with $\text{Im } t' > b/u$ is acceptable and will generate pulses with smoother profiles. For real source location and initiation time ($b = 0$), (10) may be shown to reduce to (2). Details of the derivation, with emphasis on the spectral implications of the solution, may be found in [8].

REFERENCES

Integral Equation for the Fields Inside a Dielectric Cylinder Immersed in an Incident E-Wave

LUC F. KNOCKAERT, MEMBER, IEEE, AND DANIEL DE ZUTTER

Abstract—The scattering of a time-harmonic E-wave by a dielectric cylinder is solved by a single integral equation. Two alternatives are investigated to derive such a single integral equation. Interest is focused on the fields inside the dielectric. In this case the integral equation has only the incident electric field as its source term.

I. INTRODUCTION

In a recent paper [1] it was shown by Marx that the determination of the scattered and transmitted transient electromagnetic waves due to a homogeneous dielectric body can be reduced to the solution of a single integral equation. In the present communication we restrict ourselves to the scattering of a time-harmonic E-wave by a dielectric cylinder. For this case we show that two alternative formulations exist which solve the problem by a single integral equation. A first formulation is particularly suited for the determination of the fields inside the dielectric. The second integral equation yields the result which is already given in [1] and is better suited for the calculation of the scattered fields.

The formulation uses the heuristic approach to delta functions and Green’s functions rather than a rigorous treatment in the theory of distributions.

II. INTEGRAL EQUATION RELEVANT TO THE FIELDS INSIDE THE DIELECTRIC

The geometry of the problem is shown in Fig. 1. $S$ is the cross section of the cylinder bounded by the closed curve $c$. The outside of the cylinder is denoted by $S_{out}$ and $\mathbf{n}$ is the outward pointing unit normal vector to the boundary $c$. The cylinder consists of a dielectric material characterized by its constitutive parameters $\epsilon, \mu, \sigma$. The medium outside $S$ is characterized by $\epsilon_{out}, \mu_{out}$, and $\sigma_{out}$. If we suppose that $c$ is a continuously differentiable curve, one can easily prove that the following equations are satisfied [2]:

\[
\int_c E'(t) \frac{\partial G}{\partial n} (t'|r') - \frac{\partial E}{\partial n} (t) G(t|r') \, dc(t)
\]

and

\[
\int_c E_{sc}(t) \frac{\partial G_0}{\partial n} (t'|r') - \frac{\partial E_{sc}}{\partial n} (t) G(t|r') \, dc(t)
\]

The superindex sc stands for the scattered electric field. The total electric field, i.e., the sum of the incident and reflected field, is denoted by $E$. Introducing the incident electric field $E_i$, (2) can also be written as:

\[
\int_c E(t) \frac{\partial G_0}{\partial n} (t'|r') - \frac{\partial E}{\partial n} (t) G(t|r') \, dc(t)
\]

The integration in (1), (2), and (3) extends over the boundary $c$ and $\mathbf{r}$ is the position vector of a variable integration point on $c$. If the observation point $\mathbf{r}'$ lies on the boundary, the integrals in (1), (2), and (3) are principal value integrals. In that case an infinitesimal small neighbourhood of $\mathbf{r} = \mathbf{r}'$ is excluded from the integration range. The notation $\partial \mathbf{n}$ stands for the normal derivative. $G$ is the Green’s functions for the scalar wave equation in two dimensions:

\[
\nabla^2 G (r)|r' = k^2 G (r)|r' = \delta (r - r')
\]

\[
G = \frac{i}{4} H_0^{(2)} (k |r - r'|)
\]
where
\[ k = [\omega^2 \mu (\varepsilon + \sigma / j \omega)]^{1/2}. \]  
(5)

Similar equations hold for the Green's function \( G_0 \) for the medium outside the cylinder.

As a first step we introduce a new unknown \( \xi \) defined on \( c \) by
\[ \int_c G(\hat{r'} \hat{r}) \xi(\hat{r}) \, d\hat{r} = E(\hat{r}). \]  
(6)

The observation point \( \hat{r} \) is either located inside the cylinder or lies on the boundary \( c \). The unknown \( \xi \) divided by \( j \omega \mu \) can be interpreted as a surface current density at the surface of the cylinder and flowing in the axial direction. This current density radiates into an unbounded medium with constitutive parameters \( \varepsilon, \mu, \) and \( \sigma \) and yields the correct electric field of the original problem for points located inside the cylinder or on its boundary \( c \). For \( \hat{r} \) on \( c \), the normal derivative of \( E \) becomes \( [3] \):
\[ \frac{\partial E}{\partial n}(\hat{r}) = -\frac{\xi(\hat{r})}{2} + \int_c \frac{\partial G}{\partial n}(\hat{r'} \hat{r}) \xi(\hat{r'}) \, d\hat{r'}. \]  
(7)

Substitution of (6) and (7) into the left hand member of (1) yields:
\[ I(\hat{r}) = \int_c G(\hat{r} \hat{r'}) \frac{\xi(\hat{r})}{2} \, d\hat{r} + \int_c f(\hat{r'}, \hat{r}) \xi(\hat{r'}) \, d\hat{r'}, \]  
(8)

where
\[ f(\hat{r'}, \hat{r}) = \int_c \left[ \frac{\partial G}{\partial n}(\hat{r'} \hat{r}) G(\hat{r} \hat{r'}) - G(\hat{r} \hat{r'}) \frac{\partial G}{\partial n}(\hat{r'} \hat{r}) \right] \, d\hat{r}. \]  
(9)

We now prove that \( I(\hat{r'}) \) is equal to the right hand member of (1) provided that \( \hat{r'} \) is not located outside \( c \). Starting from
\[ \nabla^2 G(\hat{r} \hat{r'}) + k^2 G(\hat{r} \hat{r'}) = \delta(\hat{r} - \hat{r'}) \]
\[ \nabla^2 G(\hat{r} \hat{r'}) + k^2 G(\hat{r} \hat{r'}) = \delta(\hat{r} - \hat{r'}) \]  
(10)
we multiply the first equation in (10) with \( G(\hat{r} / \hat{r'}) \) and the second one with \( G(\hat{r'}/\hat{r}) \). Subtracting the results, integration over \( S \) and application of Green's theorem shows that
\[ f(\hat{r'}, \hat{r'}) = \begin{cases} G(\hat{r} / \hat{r'}) / 2, & \hat{r'} \text{ in } S, \\ 0, & \hat{r'} \text{ on } c. \end{cases} \]  
(11)
To obtain (11), \( \hat{r'} \) must be on \( c \) and this must be regarded as a limiting case of \( \hat{r'} \) approaching \( c \) from the inside. To obtain zero for \( \hat{r'} \) on \( c \), \( \hat{r'} \) must also approach \( c \) from the inside. For those reasons the validity of (6) becomes restricted to \( \hat{r} \) inside \( S \) or on \( c \), as we postulated above. Consequently from (8) and (6) it follows that (1) is automatically satisfied by the introduction of \( \xi \). As a last step we introduce (6) and (7) into (3). The final result is the single integral equation satisfied by \( \xi \):
\[ \int_c \xi(\hat{r}) K(\hat{r'}, \hat{r}) \, d\hat{r} = E'(\hat{r'}). \]  
(12)

The kernel \( K(\hat{r'}, \hat{r}) \) is given by
\[ K(\hat{r'}, \hat{r}) = G_0(\hat{r'} \hat{r}) / 2 + G(\hat{r'} \hat{r}) / 2 + \int_c \left[ \frac{G(\hat{r} \hat{r'})}{2} \frac{\partial G_0}{\partial n}(\hat{r} \hat{r'}) \right. \]
\[ - \left. \frac{\partial G}{\partial n}(\hat{r} \hat{r'}) G_0(\hat{r} \hat{r'}) \right] \, d\hat{r}. \]  
(13)

The source term is simply the incident electric field.

The integrals in (12) and (13) are principal value integrals. This means that \( \hat{r'} = \hat{r} \) is excluded from the integration range in (12) and that \( \hat{r} = \hat{r'} \) are excluded from the integration range in (13). The contributions from these points have been taken into account explicitly. Consequently the integral in (13) is well defined and the interchange of the order of integration that leads to (12) is allowed.

III. INTEGRAL EQUATION RELEVANT TO THE FIELDS OUTSIDE THE DIELECTRIC

As an alternative to the substitution introduced in (6), we can define a new variable \( \xi \) as follows:
\[ \int_c G_0(\hat{r'} \hat{r}) \xi(\hat{r'}) \, d\hat{r'} = E_0(\hat{r}). \]  
(14)

In this case the observation point \( \hat{r} \) is either located in \( S_{out} \) or on the boundary \( c \). The unknown \( \xi \) divided by \( j \omega \mu \) can again be interpreted as a surface current density at the surface of the cylinder. In case the current density radiates into an unbounded medium with constitutive parameters \( \varepsilon_{out}, \mu_{out} \) and \( \sigma_{out} \) and only yields the correct electric field of the original problem for points located outside the cylinder or on its boundary \( c \). For \( \hat{r} \) on \( c \), the normal derivative of \( E_0 \) now becomes \( [3] \):
\[ \frac{\partial E_0}{\partial n}(\hat{r}) = \frac{\xi(\hat{r})}{2} + \int_c \frac{\partial G_0}{\partial n}(\hat{r} \hat{r'}) \xi(\hat{r'}) \, d\hat{r'}. \]  
(15)

Substituting (14) and (15) into the left hand member (2) and going through analogous calculations as above, shows that (2) is satisfied.
From (1) we derive the following integral equation for $\varphi^* (r')$:

$$\varphi^* (r') = \frac{E(r')}{2} + \int_c \left[ E(r) \frac{\partial G}{\partial n} (r|r') - \frac{\partial E^i}{\partial n} (r) G(r|r') \right] dc(r). \quad (16)$$

The kernel $K(r', r)$ is

$$K(r', r) = -G_0 (r' | r) - 2 - G(r | r) + \int_c \left[ G_0 (r | r') \frac{\partial G}{\partial n} (r | r') - \frac{\partial G^i}{\partial n} (r | r') G(r | r') \right] dc(r'). \quad (17)$$

This integral equation is the one found by Marx [1]. The source term in (16) is considerably more complicated than the source term in (12). As the electric field is continuous across $c$, (6) and (14) show that

$$\int_c G(r|r') \psi_{in}(r') \, dc(r') = \int_c G_0 (r' | r) \psi_{out} (r') \, dc(r') + E^i(r). \quad (18)$$

We have introduced the subscripts "in" and "out" to emphasize that the unknowns $\varphi^*$ in (6) and (14) are different. The subscript "in" indicates that definition (6) yields the correct electric field inside the cylinder, whereas (14) yields the correct electric field outside the cylinder. Both definitions, however, are valid on the boundary $c$.

**ACKNOWLEDGMENT**

The authors like to thank the reviewer for the interesting comments leading to a more correct presentation of the mathematical derivation of the results. Dr. De Zutter is a Research Associate of the National Fund for Scientific Research of Belgium.

**REFERENCES**


**Addendum to "A Simple Expression for Estimating Attenuation by Fog at Millimeter Wavelengths"**

EDWARD E. ALTSHULER, FELLOW, IEEE

The standard error of the estimate for (1) in the above paper is 0.146 dB. Since this is comparable in magnitude to some of the attenuations at the longer wavelengths, (1) should not be used for wavelengths greater than 1 cm.

Manuscript received March 2, 1986.
The author is with the Rome Air Development Center, Electromagnetic Sciences Division, Hanscom AFB, MA 01731.
IEEE Log Number 8609144.


**Correction to "Wide-Angle Microwave Lens for Line Source Applications"**

G. LARRY LEONAKIS, MEMBER, IEEE

In the above paper, three equations were printed with errors in them. The left sides of (4) and (5) should be squared. This is easily observable by dimensional inspection. Also the expression for $b$ in (12) is missing the $-2g$ term on the right side. This error has been reprinted in the work done in [2]. The equations are correct in the original work in [1].

**REFERENCES**


Manuscript received February 27, 1986.
The author is with Hughes Aircraft Company, 2260 E. Imperial Highway, Building R11, mail stop 10046, El Segundo, CA 90245.
IEEE Log Number 8609016.