Comment on “An Upper Bound on Run-Length Coding Entropy”

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In the above letter,\(^1\) the authors prove that their equation (11) has one and only one solution in the interval \(0 < x < 1\) when \(1 < R < (M + 1)/2\) and, in the interval \(x > 1\) when \(R > (M + 1)/2\).

It is my aim to provide an alternative proof to Theorem 1 and to show that a stronger result can be obtained with the help of Descartes’ rule of signs \([1]\) and symmetry considerations.

The equation under consideration is the following:

\[
f(M, R, x) = (M - R)x^{M+1} - (M - R + 1)x^M + Rx + 1 - R = 0 \quad (1)
\]

It is clear that \(f(M, R, x)\) admits a double zero \(x = 1\).

Descartes’ rule of signs states that the number \(Z\) of positive zeros of a polynomial and the number \(C\) of sign changes of the sequence of its coefficients are related in the following way: \(C - Z \geq 0\) and \(C - Z\) is an even number. For \(1 < R < M\), the number of sign changes of \(f(M, R, x)\) is \(C = 3\) by inspection. Since there is already a positive double root, the conclusion that there is an additional positive root \(x(M, R)\) follows straightforwardly from Descartes’ rule of signs.

Secondly it is not difficult to show that

\[ x^{M+1} f(M, R, x^{-1}) = -f(M, M + 1 - R, x) \] (2)

which implies that \( x(M, M + 1 - R) = 1/x(M, R) \) and this proves Theorem 1 implicitly. Moreover, if one calculates the corresponding entropies by formulas (8)-(10) of the above letter\(^1\) we have the following elegant result

\[ H(M, R) = H(M, M + 1 - R) \] (3)

For instance, in Table 1 of the above letter\(^1\) we have \( H(6, 2) = H(6, 5) = 1.9729 \).

References