Space mapping method for the design of passive shields

Peter Sergeant,a) Luc Dupré, and Jan Melkebeek
Department of Electrical Energy, Systems and Automation, Ghent University, Sint-Pietersnieuwstraat 41, B-9000 Gent, Belgium

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The aim of the paper is to find the optimal geometry of a passive shield for the reduction of the magnetic stray field of an axisymmetric induction heater. For the optimization, a space mapping algorithm is used that requires two models. The first is an accurate model with a high computational effort as it contains finite element models. The second is less accurate, but it has a low computational effort as it uses an analytical model: the shield is replaced by a number of mutually coupled coils. The currents in the shield are found by solving an electrical circuit. Space mapping combines both models to obtain the optimal passive shield fast and accurately. The presented optimization technique is compared with gradient, simplex, and genetic algorithms.

I. INTRODUCTION

In the classical approach to design a passive shield, an objective function is minimized by iteratively evaluating it. The objective function contains a finite element model in order to determine an objective value. This approach has proven to work well. However, for complicated (nonlinear) finite element models, the computational effort is huge.

If the finite element model is replaced by a computationally fast analytical model, the computation time of the optimization becomes low. As, however, no analytical solutions exist for complicated models, the unavoidable simplification causes this coarse model—and the optimal solution it yields—to be inaccurate.

The presented space mapping algorithm combines both models to find the optimal solution fast and accurately. The shielding application is an induction heater whose magnetic stray field should be reduced in a given target area by a passive shield with given material properties.

II. THE FINE MODEL: FINITE ELEMENTS

The objective function $F$ of the fine model has to associate an objective function value with the three geometrical parameters to be optimized, i.e., the radial position, the thickness, and the height of the shield. The function $F$ uses a time-harmonic, quasistatic finite element model of the axisymmetric induction heater with geometry of Fig. 1. The domain is defined by the workpiece, the excitation coil, and the air surrounding the induction heater. The chosen shield material is copper ($\sigma=5.8\times10^7\text{ S/m}$). Details about the model as well as an experimental verification can be found in Ref. 1.

With the calculated electromagnetic quantities, the objective function value $K$ is determined,

$$K = w_1 B_{\text{avg}} + w_2 P_P + w_3 (P_{w0} - P_w) + w_4 C_p,$$

where $w_1 B_{\text{avg}}$ is the average flux density in the target area with surface $S_T$: $B_{\text{avg}} = \int_{S_T} B|ds|/S_T$, where $|B| = \sqrt{|B_x|^2 + |B_y|^2}$. $w_2 P_P$ is the power dissipated in the passive shield, $w_3 (P_{w0} - P_w)$ is the power in the workpiece without shield $P_{w0}$ minus the power with shield present $P_w$ indicating the disturbing influence on the heating process, and $w_4 C_p$ is the investment cost to build the shield. The weighting coefficients $w_i$ determine the impact of each term on the solution.

III. THE COARSE MODEL: CIRCUIT MODEL

The aim of the objective function $C$ of the coarse model is to obtain an objective value for a passive shield in a fast but not necessarily accurate way. The objective function $C$ has the same structure as the objective function $F$ of the fine model and also the objective value is given by the same expression (1).

In the coarse model, the axisymmetric shield of Fig. 1 is replaced by a set of equivalent conducting coils, Fig. 2(a). Each coil has its own resistance, self-inductance, and mutual inductances between the coil and all other coils. Although the workpiece and the excitation coil can be modeled by coils as well [Fig. 2(a)], they are represented by a voltage source in the resulting electrical network of Fig. 2(b). The sources $U_k$ with $k=1\cdots M$ represent the voltages induced in the open coils $k$ of the replaced passive shield. Consequently, as all equivalent coils are open, the $U_k$ result from the non-distorted magnetic field, generated by the excitation coil in presence of the workpiece. The induced voltages are ob-

![FIG. 1. Geometry of the finite element domain (only half of the domain is shown), with the workpiece to be heated and the circular excitation coil of the axisymmetric induction heater, the cylindrical passive shield, and the target area in which the field should be reduced.](image)
tained by (a priori) finite element calculation of the unshied induction heater. This voltage source representation of the induction heater stray field has the advantages that the stray field is modeled more accurately and that also ferromagnetic workpieces can be modeled, while the analytical expressions used in the coarse model (see below) can only take into account nonferromagnetic objects. The disadvantage is that the third term in the objective value—the change of $P_w$—cannot be calculated in the coarse model.

The components in Fig. 2(b) are calculated by well-known analytical formulas for nonferromagnetic passive shields (shielding by eddy currents).

The resistance of coil $k$ is $R_k = \rho l_k / S$, with $\rho$ the resistivity of the passive shield material, $c_k = 2 \pi R_k$ the circumference of coil $k$ with radius $R_k$ and cross section $S$. $S$ is chosen such that the section of all coils together is equal to the cross section of the passive shield. The shield is "remelted" into a number of coils.

The self-inductance of coil $k$ with coil radius $R_k$ and wire radius $r_k$ is found by Wien's formula: $L_k = \mu_0 R_k \left(1 + \frac{r_k^2}{8 R_k^2} \ln \left(\frac{8 R_k}{r_k}\right) - 0.0083 \frac{r_k^2}{R_k^2} - 1.75\right)$. The mutual inductance between two coupled coils with radii $R_k$ and $R_l$ at distance $d$ (Maxwell’s formula): $M_{kl} = \mu_0 \sqrt{R_k R_l} \left[\frac{2}{\sqrt{m}} - \frac{1}{\sqrt{m}}K(m) - \frac{2}{\sqrt{m}}E(m)\right]$.

The flux density in the point $(r, z)$ of coil $k$ with radius $R_k$ and current $I_k$ is given by Biot-Savart’s law,

$$B_r = \frac{\mu_0 I_k}{2 \pi R_k} \gamma \left[\frac{1 + \alpha^2 + \beta^2}{Q - 4 \alpha} E(m) - K(m)\right],$$

$$B_z = \frac{\mu_0 I_k}{2 \pi R_k} \left[\frac{1 - \alpha^2 - \beta^2}{Q - 4 \alpha} E(m) + K(m)\right],$$

with $\alpha = r / R_k$, $\beta = z / R_k$, $\gamma = z / r$, $m = 4 \alpha / Q$, and $Q = (1 + \alpha)^2 + \beta^2$.

We summarize the approximations in the coarse model that make it different from the fine one. (1) The replacement of the continuous thin shield by a number of coils with discrete positions is accurate if the number of coils is sufficiently high. (2) As the current distribution in the equivalent coils is assumed to be uniform, no skin effect is taken into account. Consequently, the method is valid only for shields that are thinner than the skin depth. Thick shields, however, can be modeled as well if more than one row of coils is considered. (3) The choice to model the induction heater by a voltage source instead of mutually coupled coils makes it impossible to find the third term in the objective value. This term becomes zero in the coarse model.

### IV. THE SPACE MAPPING TECHNIQUE

The aim of the space mapping technique is the optimization of the passive shield, using the accuracy of the fine model, and taking advantage of the fast evaluation of the coarse model.

Let us denote the fine model by $f(x_f): \Omega \rightarrow \mathbb{R}^m$. Here, $x_f$ contains the fine model variables (radial position, thickness, and height of the shield) in the model parameter space $\Omega$. $f'(x_f)$ is the corresponding magnetic-field pattern. $F[f(x_f)]: \Omega \rightarrow \mathbb{R}$ is the objective function associated with $f(x_f)$. This function $F$—explained in Sec. II—uses the field pattern $f(x_f)$, the latter being accurate but expensive to evaluate, to find the objective value (1). The solution of the optimization problem is

$$x_f^* = \arg \min_{x_f \in \Omega} F[f(x_f)],$$

where $x_f^*$ represents the optimal set of fine variables.

### TABLE I. Comparison of the number of fine model evaluations and the total CPU time for optimization algorithms.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>One variable</th>
<th>Three variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gradient</td>
<td>12</td>
<td>32</td>
</tr>
<tr>
<td>Simplex</td>
<td>19</td>
<td>37</td>
</tr>
<tr>
<td>Genetic</td>
<td>60</td>
<td>145</td>
</tr>
<tr>
<td>Space mapping</td>
<td>4a</td>
<td>12b</td>
</tr>
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The coarse model is represented by \( c(x_i) : \Omega \to \mathbb{R}^m \). The variable \( x_i \) (radial position, thickness, and height) in the model parameter space \( \Omega \) can be optimized by minimizing the coarse objective function \( C[c(x_i)] \). This function \( C \)—explained in Sec. III—uses the analytical model to find the coarse objective value \( C \) using the coarse model, thus by solving \( f \) for the coarse function \( C \) is found at 180 mm.

A mapping function \( p \) has to be established between the two variables \( x_i \) and \( c_i \): \( x_i = p(c_i) \) so that the field pattern in the fine model can be approximated by the coarse field pattern: \( f(x_i) = c[p(x_i)] \). The mapping function \( p \) is found by solving (\( \epsilon \) small positive value),

\[
\left| f(x_i) - c(x_i) \right| < \varepsilon.
\]

In this crucial step called parameter extraction, the best coarse parameter \( x_i = p(c_i) \) is searched that yields a magnetic-field pattern \( c(x_i) \) similar to the magnetic-field pattern \( f(x_i) \). For the mapping, the agressive space mapping (ASM) technique is used. If the correct mapping function \( p \) is found, the optimum \( x_i^* \) of the fine model is obtained by using the coarse model, thus by solving

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V. SIMULATION RESULTS

For the gradient method, the MATLAB function \( \text{fmincon} \) was used. Table I shows that 13 min of CPU time were needed to minimize the not perfectly smooth (due to finite element round-off) fine objective function of one variable. The optimization of the coarse function by this algorithm requires only 9 s.

The simplex method is known as robust but rather slow; it is slightly slower than the gradient algorithm. As it does not use gradients, the nonsmooth character of the fine function is not a problem.

Genetic algorithms are usually very robust and very slow. Although more than 1 h is needed for the optimization of one variable (Table I), good “individuals” are found rather quickly. For the optimization, two subpopulations were chosen: the first consisting of ten individuals, and the second consisting of five individuals (one variable) or ten individuals (three variables).

The space mapping algorithm needs only four evaluations of the fine model for the optimization. Both \( x_f \) and \( x_c \) represent the shield height. In addition to four evaluations of the fine model \( f(x_i) \), three optimizations of the coarse model \( c(x_i) \) are needed: one to optimize \( x_i \) by (3) and two for the mapping (4).

The optimization of the one variable (the shield height) works. Firstly, the objective function \( C \) of the coarse model is optimized by executing (3). The coarse optimum \( x_c^* \) is found at 0.18 m and becomes the starting argument for the fine objective function: \( x_f = x_c^* \). Secondly, in this argument \( x_f \), the fine objective value \( F_f = F(x_f) \) is calculated as well as its derivative. The latter improves the efficiency of the parameter extraction (see Sec. 5.F in Ref. 4), but requires two fine model evaluations. Thirdly, the parameter extraction searches the coarse argument \( x_{c1} \) that yields the same response as the fine argument \( x_{f1} \)—see (4). The aim is to find the mapping function \( p \) from \( x_{f1} = p(x_{c1}) \). The fourth step is to determine an updated fine argument \( x_{f2} \). If the found argument \( x_{f1} \) is smaller than the coarse optimum \( x_c^* \), the ASM chooses a fine argument \( x_{f2} \) that is larger than \( x_c^* \). The function value \( F_f = F(x_{f2}) \) is calculated as well as the derivative, which is about zero in this point. In the second parameter extraction, (4) is solved in order to find the coarse argument \( x_c^* = p(x_{f2}) \) with approximately the same function value and gradient as \( x_{f2} \). The argument \( x_c^* \) seems to be equal to the coarse optimum \( x_c^* \). If the mapping function \( p \) is correct, the corresponding fine argument \( x_{f2} \) is equal to the fine optimum \( x_{f1} \) so that the algorithm is terminated.

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