Optimization of multilayered nonlinear crystalline alloys for shielding

Peter Sergeant, a Luc Dupré, Lode Vandebossche, and Jan Melkebeek
Department of Electrical Energy, Systems and Automation, Ghent University, Sint-Pietersnieuwstraat 41, 9000 Gent, Belgium

(Submitted on 10 November 2004; published online 10 May 2005)

A multilayered material is presented for the shielding of sinusoidal magnetic fields. The shield consists of alternating layers of aluminum and (nonlinear hysteretic) steel. The optimization variables are the total shield thickness and the fraction of steel. The optimization goals are to obtain the desired shielding factor, to minimize the shield thickness, and to minimize the eddy currents and hysteresis losses. In the objective function, analytical expressions are used for the shielding factor and the electromagnetic losses. Simulation results of a planar shield show that both the optimal shield thickness and the fraction of steel decrease with the frequency. © 2005 American Institute of Physics. [DOI: 10.1063/1.1852322]

I. INTRODUCTION

Shielding techniques aim at a predefined field reduction with minimal electromagnetic losses. An optimization of multilayered crystalline material is presented to find an optimal shield for the shielding of a sinusoidal magnetic source field. The shield consists of several alternating layers of aluminum and nonlinear steel, characterized by a Preisach distribution function1 in the Rayleigh area.2 The paper describes how the shielding factor and the electromagnetic losses from a multilayered nonlinear planar shield are calculated by analytical expressions.3,4 Only the fundamental component of a multilayered nonlinear planar shield are calculated by analytical expressions.3,4 The shielding factor is defined in a point with the same coordinate as the edge of the shielded region and has thickness as the source field.

II. SHIELDING FACTOR AND LOSSES

We consider the shielding problem of Fig. 1 (geometry of Hobung5): a multilayered planar shield is placed in an imposed magnetic field \( H_0 \) with a sinusoidal distribution in space (wavelength \( \lambda \)). Notice that an underlined symbol symbolizes a time phasor, a complex representation of a quantity that varies sinusoidally in time: \( \Xi = X \sin(\omega t) \) with \( \omega \) as the angular velocity. A bar on top of a symbol means a space vector \( \bar{X} \). The shield, consisting of \( n \) layers (alternating nonlinear steel and aluminum) has to mitigate \( H_0 \) in the shielded area by a given shielding factor. The latter is defined as \( |H_s|/|H_0| \) with \( H_s \) as the magnetic field in the shielded area in a point with the same \( x \) coordinate as \( |H_0| \). The surface \( A \) is the edge of the shielded region and \( B \) borders the source region. Layer \( l \) has thickness \( d_l \), conductivity \( \sigma_l \), and a field-dependent permeability \( \mu_l(H) \). Its surface closest to the source is labeled \( \beta \) and its other surface is \( \alpha \). This geometry is an “unwrapped” version of a cylindrical shield with radius \( r = \lambda / 2\pi \) in a uniform, transverse flux field.4

We use Hobung’s analytical expressions of Ref. 4 to calculate the shielding factor of the planar shield consisting of linear materials. The nonlinearity is taken into account by dividing every nonlinear layer \( l = 1 \cdots n \) with \( \mu = \mu(|H|) \) into \( p = 1 \cdots m \) fictitious linear sublayers with constant \( \mu_{l,p} \) (see Fig. 1). Moreover, \( \mu_{l,p} \) is complex in order to model hysteresis.

The permeability \( \mu \) must be determined in every sublayer. Notice that in this paper only working conditions in the Rayleigh region 2 are considered, which is acceptable for shielding problems. For \( h(t) > 0 \), the induction \( b(t) \) equals

\[
b(t) = c_1 h(t) + c_2 h(t)^2 \quad \text{if} \quad h(t) > 0,
\]

when starting from the demagnetized state and applying a monotonously increasing or decreasing time-varying magnetic field \( h(t) \). However, for materials exhibiting hysteresis, \( b(t) \) is not merely a function of \( h(t) \), but of its history as well. Usually, \( b(t) \) is split into a reversible part \( b_{rev}[h(t)] \) and an irreversible part \( b_{irr}[h(t), h_{past}(t)] \). To present the latter, the classical Preisach model is used.1 In order to have in the Preisach model a virgin curve as described by (1), one must choose \( b_{rev}(t) = c_1 h(t) \) and the Preisach distribution function \( P(\alpha, \beta) = c_2 \).

![Fig. 1. A multilayered planar shield (n layers). Each nonlinear layer is divided into m linear sublayers.](image)
According to this Preisach model, a periodic variation of $h(t)$ between the extremal values $h_s$ and $-h_m$ corresponds with an ascending and a descending branch:

$$b_{\text{ascend}}[h(t)] = c_1 h(t) + c_2 h^2(t) - \frac{c_2}{2} [h(t) - h_m]^2,$$

$$b_{\text{descend}}[h(t)] = c_1 h(t) - c_2 h^2(t) + \frac{c_2}{2} [h(t) + h_m]^2.$$

The corresponding extremal values for $b(t)$ are $b_m = c_1 h_m + c_2 h_m^2$ and $-b_m$, respectively. Concerning $\mu$, only its fundamental component $\mu'$ is considered. $\mu'$ is a function of the sinusoidal magnetic field $H$. (3) and (4) give rise to a nonsinusoidal time variation of the induction $b(t)$. The fundamental harmonic of $b(t)$ and $\mu$ is given by

$$\mu' = (c_1 + c_2) |H| - \frac{4}{3\pi} c_2 |H| = \frac{B}{H}.$$

As (5) needs a magnetic field $H$ as input value, some approximations are unavoidable because the analytical algorithm accepts only one $\mu'_l$ per sublayer $p$ in layer $l$ while the real $\mu'$ differs from point to point:

- The $x$ dependence of $H_x = \cos(kx)$ and $B_x = -\sin(kx)$ (with expressions in Ref. 4) is taken into account by taking the rms value (divide by $\sqrt{2}$).
- The $y$ dependence is “discretized” by the sublayers of every layer $l$, each having its own $\mu'_l$, $p = 1 \cdots m$.
- The space vector $\vec{H}$ is taken into account by calculating both $\mu'(H_x / \sqrt{2})$ and $\mu'(H_y / \sqrt{2})$ and averaging them,

$$\mu' = \frac{\frac{H_x}{\sqrt{2}} + \frac{B_x}{\mu_0 \sqrt{2}}}{\frac{H_y}{\sqrt{2}} + \frac{B_y}{\mu_0 \sqrt{2}}}.$$

As neither $\mu'_l$ nor $H$ are known in the sublayers, the problem has to be solved iteratively. Firstly, $b(t)$ is calculated for every sublayer $p = 1 \cdots m$ in all layers $l = 1 \cdots n$ by evaluating (5) with $H_0$ as input argument. After calculating the shielding factor using the calculated $\mu'_l$, the distribution of $H$ in the shield is found from the expressions in Ref. 4, and the $\mu'_l$ are updated. Iteratively, the $H$ and the $\mu'_l$ are updated until the procedure converges. With the $\mu'_l$, known in all sublayers, the shielding factor, $H_x$, and $H_y$, are calculated as explained in Ref. 4.

Once the shielding factor and $\vec{H} = H_x \vec{i} + H_y \vec{j}$ are known, the current density is calculated by $J = \nabla \times \vec{H}$. Per meter length in the $z$ direction, the eddy current losses $P_{\text{ec}}$ and the hysteresis losses $P_{\text{hy}}$ are calculated from proper expressions for a sublayer with thickness $d$ (see paragraph 1.3.1 in Ref. 5):

$$P_{\text{ec}} = \frac{1}{2} \int_0^H \int_0^d \frac{J(x,y) \cdot J'(x,y)}{\sigma} dy,$$

$$P_{\text{hy}} = \frac{1}{2} \int_0^H \int_0^d \Re(H(x,y) \cdot j \omega B'(x,y)) dy.$$
IV. OPTIMIZATION OF THE SHIELD

The optimization tries to find a shield that achieves the desired field reduction with minimal thickness and electromagnetic losses. For several frequencies, an optimization of the thickness and the fraction of steel is carried out for a shield with alternating layers of aluminum and nonlinear steel (properties given in Sec. III). The number of layers is a design parameter and the wavelength was chosen as \( \lambda = 0.6 \pi \, \text{m} \). The optimization goals are to (1) obtain the desired shielding factor, (2) minimize the losses due to eddy currents and hysteresis, and (3) minimize the thickness.

The minimization algorithm is the MATLAB function `fgoalattain`. It was used to find the optimal thickness and fraction such that the shield has exactly the desired shielding factor and that the losses and thickness are minimal. An objective function executes the procedure explained in Sec. II to find for a given thickness, fraction, frequency, wavelength, and number of layers the shielding factor and the losses. It produces two objective values: the shielding factor that the optimization routine tries to make the desired 0.01, and the penalization term with a weighted contribution of the losses and the thickness.

Optimizations were carried out for several frequencies and for both two and ten layers. This resulted in Fig. 3 where the optimal thickness, fraction, and the minimized losses are shown. All shields in this figure have a shielding factor of 0.01 (100 times field reduction). The layers were divided into ten sublayers. For all curves shown, it can be seen that for lower frequencies, a higher fraction of steel is used. Here, a ferromagnetic material provides shielding in a frequency range where eddy current shielding is poor. More steel requires a thinner shield because of the better shielding due to the higher product \( \mu \sigma \) but it also causes more losses because of the lower conductivity compared to aluminum.

In accordance with Sec. III, the losses for the shield with order A-Al-…-Fe-B are much lower than for the shield with the opposite order, A-Fe-…-Al-B, because there is no reciprocity concerning the source and the shielded area. To obtain low losses, the first layer near the source must be steel. The conclusion is that, in general, a higher fraction of steel reduces the required shield thickness but causes more losses.

Finally, it is interesting to notice that the amplitude of the imposed field \( H_0 \) also determines the shielding factor because \( \mu \) is a function of \( |H| \). The shielding factor is better for higher amplitudes, which is good because strong fields need the most field reduction.

---