Generalisation of the dynamic Preisach model toward grain oriented Fe–Si alloys

L. Dupré, G. Bertotti, V. Basso, F. Fiorillo, J. Melkebeek

*Department of Electrical Power Engineering, Laboratory for Electrical Machines and Power Electronics, St. Pietersnieuwstraat 41, B-9000 Gent, Belgium

Istituto Elettrotecnico Nazionale “Galileo Ferraris”, Corso Massimo d'Azeglio 42, I-10129, Torino, Italy

Abstract

A generalisation of the dynamic Preisach model is proposed, for which the magnetisation rate \( \frac{d\phi}{dt} \) for each Preisach dipole is proportional to the difference between the effective field and \( x + k_\phi \phi \) or \( \beta + k_\phi \phi \), where \( k_\phi \) is an extra material parameter. With this modification, the area enclosed by the macroscopic hysteresis loops for a sinusoidal flux becomes dependent on the magnetisation frequency \( f \) according to the law \( C_0 + C_1 f \). The model has been compared with experimental results on grain oriented Si–Fe alloys and the correlation with the statistical loss theory is shown. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Hysteresis; Dynamic Preisach model; Statistical loss theory

1. Introduction

It is a fact that the classical Preisach model (CPM) [1] has received considerable attention over many years, because it summarises many key features of hysteresis in a mathematically elegant way. An improvement was obtained through more sophisticated models, such as the product model and the moving model [2], in order to take into account the non-congruency experimentally observed. However, a drawback of these generalisations is that they are basically rate-independent, and that they do not allow the user to describe the excess losses as defined in the statistical loss theory [3]. In Ref. [4], a rate-dependent Preisach model, denoted by DPM, was introduced by assuming that the switching of each elementary Preisach dipole cannot occur instantaneously, but at a finite rate controlled by the difference between the external field and the loop threshold fields. In Ref. [3], it was shown that the loop area predicted by the model follows a law of the form \( C_0 + C_1 \sqrt{f} \). According to the statistical loss theory [4], this frequency dependence corresponds to a linear relation between the number \( n \) of active correlation regions in the material and the excess field \( H_{\text{ex}} \), for which \( n \) becomes zero in the limit of vanishing magnetisation frequency. In Ref. [4], it was shown that for grain oriented materials these two limitations are not satisfied. In this paper, a generalisation of DPM is proposed by introducing an extra material parameter, resulting first in a finite slope for the quasi-static elementary loop of the Preisach dipoles and second in a more general frequency dependence of the excess losses.

2. Statistical loss theory

A general approach to the calculation of iron losses in soft magnetic laminated materials under unidirectional flux \( \phi(t) \) is based on the separation of losses into three components: the hysteresis losses \( P_h \), the classical losses \( P_c \) and the excess losses \( P_e \). According to the statistical loss theory [4], the magnetisation process in a given cross section \( S \) of the magnetic lamination of thickness \( d \) can be described in terms of \( n \) simultaneously active
correlation regions. For several alloys \( n \) is a linear function of the excess field \( H_{\text{exc}} = n_0 + H_{\text{exc}}/V_0 \).

When Eq. (1) holds, the excess losses under a sinusoidal flux excitation with frequency \( f \) and maximum induction \( B_m \), can be written as

\[
P_e = 2B_m f\left(\sqrt{n_0^2 V_0^2 + 16\sigma GSV_0 B_m f} - n_0 V_0\right).
\]  

(2)

Here, \( \sigma \) is the electrical conductivity and \( G = 0.1357 \). For materials where \( n_0 \) is negligibly small, Eq. (2) reduces to the approximate formula

\[
P_e = 8\sqrt{\sigma GSV_0} r^{1.5} B_m^{1.5}.
\]  

(3)

Fig. 1 shows the loss curves \( (P_e + P_c)/f \) measured on the grain-oriented 3 wt% Si-Fe material considered in this paper. A good correspondence was found between the measured loss values and Eq. (2). The values for \( n_0 \) and \( V_0 \) are given in Table 1. \( n_0 \) and \( V_0 \) are parameters depending on the cross section \( S \) of the lamination. The data of Fig. 2 and Table 1 were obtained taking \( S = 8.62 \times 10^{-6} \text{ m}^2 \).

### 3. Generalisation of the dynamic Preisach model

The scalar rate-independent Preisach model [1] provides quite an accurate description of hysteresis effects in magnetic materials. In this model, each Preisach dipole has a rectangular nonsymmetric hysteresis loop defined by two characteristic parameters \( \alpha \) and \( \beta \) (\( \beta \leq \alpha \)), see Fig. 3(a) (dashed line). Depending on the history of the magnetic field, the magnetisation of the dipole, denoted by \( \phi \), takes the value +1 or −1 (see Fig. 4).

In the dynamic Preisach model (DPM) [3], the switching rate of each dipole is given by

\[
\frac{d\phi}{dt} = \begin{cases} 
  k_d(H_e(t) - z) & H_e(t) > z \text{ and } \phi < +1, \\
  k_d(H_e(t) - \beta) & H_e(t) < \beta \text{ and } \phi > -1, \\
  0 & \text{in the other cases.}
\end{cases} 
\]  

(4)

Then, the magnetisation \( M(t) \) is obtained from

\[
M(t) = M_{\text{exc}}(H_e(t)) + \frac{1}{2}\int_{-\infty}^\infty \phi(\alpha, \beta) P(\alpha, \beta) d\alpha 
\times \int_{-\infty}^\alpha d\beta \phi(\alpha, \beta) M(\alpha, \beta),
\]  

(5)

where \( P(\alpha, \beta) \) is the Preisach distribution function and the effective field \( H_e(t) \) is obtained from the applied field \( H_a(t) \) and the corresponding magnetisation \( M(t) \):

\[
H_e(t) = H_a(t) + k_m M(t).
\]
divided into three subregions, i.e., two regions in which \( \phi \) equals \( +1 \) and \( -1 \), respectively, and a third region, in which the dipoles are in an intermediate state \( (-1 < \phi < +1) \). The shape of the third region is defined by the time variation of the magnetic field and reduces, for quasi-static flux excitations \( (f \to 0) \), to the same line of the CPM, separating subregions 1 and 2 (see Fig. 4).

It is shown in Ref. [3] that the dynamic hysteresis losses obtained by this DPM follow the frequency dependence of Eq. (3).

For magnetic materials where \( n_0 \) may not be neglected in Eq. (2), like grain oriented materials, a generalised dynamic Preisach model (GDPM) is needed. Here, a GDPM is proposed where the characteristic behaviour of the Preisach dipoles is given by

\[
\frac{d\phi}{df} = \begin{cases} 
  k_d(H_s(t) - k_a \alpha - \phi) & H_s(t) > \alpha + k_a \phi \\
  k_d(H_s(t) - k_a \phi - \beta) & H_s(t) < \beta + k_a \phi \\
  0 & \text{in the other cases}.
\end{cases}
\]

(6)

This generalisation amounts to replacing the dipoles in the classical Preisach theory by dipoles having a characteristic with a finite slope defined by \( k_a \) (see Fig. 3(a) and (b) (full line)). In this way, a new switching rule for the Preisach dipole is defined. Note that Eqs. (6) and (4) have static limits which correspond to a different elementary hysteresis operator. The most direct consequence is that, according to Eq. (5), two different Preisach distribution functions should be used to obtain the same static output.

Again, the Preisach plane is divided into three subregions in which \( \phi = +1, -1 \) and \( -1 < \phi < +1 \), respectively. Now, the shape of the third region is defined by \( k_a \) and the time variation of the magnetic field. In particular, the third region takes a limit width of \( 2k_a \) in case of quasi-static excitation (see Fig. 4). Fig. 5 compares
Fig. 6. Number of active correlation regions obtained from GDPM.

Table 2

<table>
<thead>
<tr>
<th>$M$ (T)</th>
<th>$n_0$ (1/m$^2$)</th>
<th>$V_0$ (A/m)</th>
<th>$n_0$ from Eq. (11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>38</td>
<td>0.06</td>
<td>27</td>
</tr>
<tr>
<td>0.5</td>
<td>38</td>
<td>0.084</td>
<td>0.084</td>
</tr>
<tr>
<td>0.75</td>
<td>36</td>
<td>0.108</td>
<td>0.108</td>
</tr>
<tr>
<td>1</td>
<td>36</td>
<td>0.152</td>
<td>0.152</td>
</tr>
<tr>
<td>1.25</td>
<td>29</td>
<td>0.192</td>
<td>0.192</td>
</tr>
<tr>
<td>1.4</td>
<td>28</td>
<td>0.205</td>
<td>0.205</td>
</tr>
<tr>
<td>1.5</td>
<td>22</td>
<td>0.268</td>
<td>0.268</td>
</tr>
</tbody>
</table>

Fig. 7. Comparison between the measured losses and the losses from GDPM.

In case of a low-frequency $f$ and a complete switching of the dipole from $+1$ to $-1$ and from $-1$ to $+1$, the excess loss for one dipole per period, corresponding to the shaded area in Fig. 3(c) and (d), is given by

$$P_s = \frac{16}{k_n k_d} f H_m^2.$$  

Consequently, from GDPM, for low frequencies we obtain for the excess losses

$$\frac{P_s}{f} = \int_{-H_m}^{H_m} \int_{-\beta}^{\beta} P(x, \beta) d\beta.$$  

Taking into account that for low frequencies, Eq. (2) reduces to (see Ref. [5])

$$\frac{P_s}{f} = \frac{\sigma G S 16 B_m^2}{n_0} f,$$

we have

$$n_0 = \sigma G S B_m^2 k_n k_d.$$  

The $n_0$-values obtained from the slope of $P_s/f$ at low frequencies and the $n_0$-values obtained from Eq. (11), are compared in Table 2.

4. Experimental validation of GDPM

Fig. 7 compares the loss per cycle obtained from GDPM with the experimental losses. Here, the losses under sinusoidal $B$-excitation, calculated iteratively, include the hysteresis losses and the excess losses, whereas...
the experimental losses are obtained from the measured losses minus by the classical losses, i.e.,

\[ P_e = \frac{1}{6} \sigma \pi^2 d^2 B_m^2 f^2. \]  

(12)

The same values for \( k_n \), \( k_d \) and \( k_m \) are used as in the previous section. A good correspondence between the calculated and measured losses is observed for low magnetisation levels, while for higher levels, the measurements are overestimated by GDPM.

Fig. 8 shows different types of BH-loops under sinusoidal flux of 50 Hz and 1 T. The measured quasi-static BH-loop is given by the filled square line. Notice that the Preisach distribution function, used in all simulations, gives a very accurate description of the measured quasi-static BH-loops. The dynamic BH-loop, obtained from GDPM (hysteresis and excess contributions) is given by the open square line. To compare the calculated BH-loops with the measured one, the open square loop must be modified to take into account the classical losses \( P_e \). Therefore, for each point in the BH-loop obtained from GDPM, the magnetic field is increased by the classical field

\[ H_c = \frac{1}{12} \sigma d^2 \frac{dM}{df}. \]  

(13)

Finally, Fig. 8 shows the satisfactory correspondence between the measured dynamic BH-loop (filled circle line) and the numerically obtained BH-loop (open circle) from the GDPM combined with Eq. (13).

5. Conclusions

In this paper, a generalisation of the dynamic Preisach model has been discussed. An additional material parameter, denoted by \( k_n \), defines the finite slope of the elementary dipole characteristic in the quasi-static case. A new switching rule for the Preisach dipoles was defined. With this new formalism, the area enclosed by the macroscopic hysteresis loops under sinusoidal B-excitation, becomes dependent on the magnetisation frequency \( f \) according to the law \( \sqrt{1 + Cf} - 1 \). The significance of this generalisation has been clarified by relating the new model to the statistical loss theory. The model has been applied successfully to the interpretation of losses in grain oriented Si-Fe alloys.

Acknowledgements

The research was carried out during a scientific stay of the first author at IEN Galileo Ferraris. The first author is post-doctoral researcher of the Fund of scientific research-Flanders. (F.W.O.-Vlaanderen).

References