Relation between the microstructural properties of electrical steels and the Preisach modelling

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Abstract

In this paper, the relation of the microstructural properties of electrical steels and the material parameters in the dynamic Preisach model is established. Particular experimental work was performed in order to separate the influence of grain size from the crystallographic texture. First, the parameters defined in the dynamic Preisach model has been fitted using the hysteresis loss characteristics. Next, we identified and separated the effect of average grain size from the crystallographic texture on each material parameter defined in the Lorentzian–Preisach distribution function. Here, the same texture dependence was identified for all kinds of magnetisation processes, i.e. for the irreversible and reversible parts of the quasi-static magnetisation and for the AC excess loss behaviour. Finally, the Preisach distribution function is rewritten explicitly including the grain size and crystallographic texture dependence. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

The electromagnetic properties of electrical steels, like magnetisation curves and loss characteristics, are principally determined by the microstructural properties of the material, described by grain size and crystallographic texture. For this reason, to apply any model, it is indispensable to clarify and to identify the relation between the microstructure, controlled by the steel production techniques, and the macroscopic electromagnetic behaviour of the material. On the one hand, there is a huge empirical knowledge about how the grain size and the crystallographic texture influence the macroscopic magnetic properties. On the other hand, there is a lack of knowledge of how to quantify the effect of the microstructure on the electromagnetic properties, which may be described by models like the Preisach model \cite{1}, but also by the Stoner–Wohlfarth model \cite{2} or the Jiles–Atherton model \cite{3}.

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The main goal of this paper is to reveal the correlation between microstructural properties and the material parameters in the Preisach theory describing the macroscopic behaviour of electrical steels, using the microstructural and the macroscopic electromagnetic characterisation of structurally controlled alloys. Herefore, we use intensively the combined lamination-dynamic Preisach model. This model, solved by advanced numerical techniques, has proven to be very efficient to describe in detail the electromagnetic behaviour of electrical steels under arbitrary alternating excitation conditions, see e.g. Refs. [4,5]. In general, in the Preisach theory, the material is characterised by a distribution function of the two switching fields $a$ and $b$. The shape of this distribution function contains the information on the microstructure and hence on the material behaviour. In order to establish the relation between the Preisach distribution and the microstructure, the shape of this distribution is described by the limited number of fitting parameters in a Lorentzian function. Finally, it is shown that similar conclusions could be made from the results obtained with the statistical power loss model of Bertotti [6—8].

2. Dynamic Preisach modelling

2.1. Magnetic characterisation by the Preisach theory

The scalar Preisach hysteresis model initially introduced in Ref. [1] and extensively further developed in other papers, is one of the most accurate ways of describing the hysteresis effects encountered in magnetic materials. In this way, the hysteresis model provides the flux density $B$ as a function of the magnetic field $H$ and its history $H_{past}$. The material is described by a (infinite) set of dipoles. Each dipole has a rectangular non-symmetric hysteresis loop defined by two characteristic parameters, denoted by $a$ and $b$ ($b < a$). The density of these dipoles is represented by the Preisach distribution function $P(a, b)$ (PDF), characterising the material, cf. e.g. Ref. [9]. The resulting magnetisation of the entire material is obtained from the accumulated magnetisation of all the dipoles.

In the classical Preisach model (CPM), under the action of the magnetic field strength $H$, a given elementary dipole will by in the $+1$ state when $H > a$ and in the $-1$ state when $H < b$. In the case that $b < H < a$, the state of the Preisach dipole will depend on the last extremal value $H_{last}$ kept in the memory outside the interval $[b, a]$. Explicitly:

$$
M_d(a, b, t) = +1: \quad H(t) > a \text{ or } (b < H(t) < a \text{ and } H_{last} > a),
$$

$$
M_d(a, b, t) = -1: \quad H(t) < b \text{ or } (b < H(t) < a \text{ and } H_{last} < b). \quad (1)
$$

Notice that the magnetisation $M_d$ of the dipole does not depend on the rate of variation of the magnetic force $H(t)$. The induction $B(H(t), H_{past}(t))$ takes on the following form in the Preisach model:

$$
B(H(t), H_{past}(t)) = \mu_0 H(t) + \frac{1}{2} \int_{-H}^{H} \int_{-a}^{a} d\beta d\alpha M_d(a, \beta, t)P(a, \beta).
$$

Here $P(a, \beta)$ is assumed to be negligibly small when either $a > H_t$ or $\beta < -H_t$. This value $H_t$ is obtained experimentally.

In the literature, several measurement techniques are prescribed to construct the PDF, e.g. Refs. [10,11]. These techniques are mainly based on the construction of transition curves or equivalently the Everett theory [12]. In this paper we will proceed with a proper analytical expression for the distribution function. Several analytical expressions for the Preisach function can be found, see e.g. Ref. [13]. Here, the analytical
expression used is of the Lorentzian type, which has proven to be very valid for silicon iron alloys, i.e.

\[ P(x, \beta) = P_{\text{rev}}(x, \beta) + P_{\text{irr}}(x, \beta) = \delta_{x,\beta} \left( \frac{k_2}{1 + (x/c)^2} + d \right) + \frac{k_1}{(1 + ((x - a)/b)^2)(1 + ((\beta + a)/b)^2)} \]  

(3)

with \( \delta_{x,\beta} \) the usual Kronecker delta symbol.

Notice that the first term in Eq. (3) only represents Preisach dipoles for which \( x = \beta \), i.e. dipoles that contribute only to reversible processes. The second term gives rise to dipoles fully responsible for the hysteresis losses.

The parameter, often used to describe the mean field effects in the Preisach model, see e.g. Ref. [14] must not be considered as we only focus on the right description of the hysteresis losses as such.

Much more important is the presence of the dynamic hysteresis effects. Indeed, as the CPM is rate-independent, the non-negligible excess losses, see the statistical power loss model of Bertotti, [15], are not taken into account. Therefore, we pass to the dynamic Preisach model (DPM), introduced by Bertotti [16]. In the DPM, the dipoles are assumed to switch at a finite rate, proportional to the difference between the local magnetic field \( H(t) \) and the elementary loop switching fields \( x \) or \( \beta \). The factor of proportionality, denoted by \( k_d \), is an extra material parameter, describing the basic properties of domain wall dynamics in metallic systems. Explicitly, the evolution in time of the magnetisation \( M_d \) is given by

\[
\frac{dM_d}{dt} = k_d(H(t) - x), \quad H(t) > x \text{ and } M_d < +1,
\]

\[
\frac{dM_d}{dt} = k_d(H(t) - \beta), \quad H(t) < \beta \text{ and } M_d > -1,
\]

\[
\frac{dM_d}{dt} = 0, \quad \text{in the other cases}.
\]

(4)

Now, the magnetisation \( M_d \) may take each value between \(-1\) and \(+1\). Notice that the switching of the dipoles is rate dependent and hence the \((M_d,H)\)-characteristic depend on the rate of \( H(t) \). It is shown in Ref. [17] that the extra area enclosed by the \((M_d,H)\)-characteristic, compared with the CPM-case, corresponds to the excess losses.

2.2. AC magnetic characterisation using the combined lamination dynamic Preisach model

To build up a numerical model which includes the three types of losses, i.e. the hysteresis losses, the excess losses and the classical losses, a combined lamination-dynamic Preisach model (CL-DPM) is considered in Refs. [5] and [6]. An Euclidean coordinate system is defined with the \( x \)-axis perpendicular to the lamination. Throughout the lamination, the time dependent flux \( \phi(t) \) flows in the \( z \)-direction and the magnetic field has only one component, viz. \( H = H \cdot \hat{e}_z \). Eliminating the edge effects, we may assume \( H \) to vary in the \( x \)-direction only. One obtains the governing equation for \( H \) from the Maxwell equations:

\[
\frac{1}{\sigma} \frac{\partial^2 H}{\partial x^2} = \frac{\partial B}{\partial t},
\]

(5)

where \( B \) is the magnitude of the magnetic induction \( \vec{B} = B(x) \cdot \hat{e}_z \), while \( \sigma \) is the electrical conductivity of the material. Due to this conductivity, eddy currents \( J = -\hat{\partial} H / \hat{\partial} x \cdot \hat{1}_y \) are generated in the material. In Eq. (5),
the relation between \( B \) and \( H \) is given by the DPM:

\[
B(H(t), H_{\text{past}}(t)) = \mu_0 H(t) + \int_0^{H} \left( \frac{k_2}{1 + (H/c)^2} + d \right) \, dz + \frac{1}{2} \int_{-H}^{H} \, dz \int_{-H}^{z} \, d\beta \, M_d(z, \beta, t) \times \frac{k_1}{(1 + ((z - a)/b)^2)(1 + ((z + a)/b)^2)}. \tag{6}
\]

Now, combining Eq. (5) with Eqs. (6) and (4) one has

\[
\frac{1}{\sigma} \frac{\partial^2 H}{\partial x^2} = \mu_{\text{rev}} \frac{\partial H}{\partial t} + q_1(H(x, t), H_{\text{past}}(x, t))H - q_2(H(x, t), H_{\text{past}}(x, t)) \tag{7}
\]

with

\[
q_1(H(x, t), H_{\text{past}}(x, t)) = \frac{k_d}{2} \int_{D_1(x, t)} P_{H}(x, \beta) \, dz \, d\beta + \frac{k_d}{2} \int_{D_2(x, t)} P_{H}(x, \beta) \, dz \, d\beta \tag{8}
\]

and

\[
q_2(H(x, t), H_{\text{past}}(x, t)) = \frac{k_d}{2} \int_{D_1(x, t)} \beta P_{H}(x, \beta) \, dz \, d\beta + \frac{k_d}{2} \int_{D_2(x, t)} \beta P_{H}(x, \beta) \, dz \, d\beta. \tag{9}
\]

The reversible differential permeability equals:

\[
\mu_{\text{rev}}(H) = \mu_0 + \left( \frac{k_2}{1 + (H/c)^2} + d \right). \tag{10}
\]

The domains \( D_1 \) and \( D_2 \) in the \((x, \beta)\)-plane represent dipoles in an intermediate state, switching to positive ( + 1) and negative (−1) saturation, respectively. Of course, the time and space dependence of \( D_1 \) and \( D_2 \) is through the local magnetic field \( H(x, t) \) and its history \( H_{\text{past}}(x, t) \).

We refer to Ref. [5] for the numerical solution of the highly non-linear parabolic problem (7). It is well known that this CL-DPM allows us to predict the electromagnetic behaviour of electrical steels under arbitrary periodic alternating excitation conditions, see e.g. Refs. [4–6,11].

### 3. Application of the combined lamination-dynamic Preisach model

#### 3.1. Experiments

Six different 3.2 wt% SiFe non-oriented 0.35 mm thick electrical steel coils, denoted by \( C_1, C_2, C_3, \ldots, C_6 \), were produced from the same casting, to assure a wide variety of average grain sizes, but without significant changes in the crystallographic texture. The grain size distribution was determined by means of quantitative image analysis (Table 1) and the texture was evaluated by the orientation distribution function (ODF) from X-ray diffraction measurements.

For each material, Epstein samples and single strips (100 mm x 660 mm) were cut with angles from 0 to 90° with respect to the rolling direction, in steps of 15°. For each single strip, the power losses under sinusoidal flux excitation were measured for frequencies between 20 and 400 Hz, and for different polarisations, ranging from 0.3 to 1.8 T, in steps of 0.1 T.
Simultaneously, on the Epstein samples, almost 15 to 20 quasi-static hysteresis loss measurements were performed between 0.03 and 1.8 T by a hysteresograph. In this way, the quasi-static BH loops and the $W_{\text{hyst}}(B_m)$ hysteresis loss characteristics were determined experimentally.

The measurements performed, have been planned in such a way that, by means of a simple evaluation of the obtained results, the effect of average grain size could be easily separated from the effect of crystallographic texture. Indeed, from the microstructural analyses, we may conclude that, when considering the strips of the six materials for a fixed cutting direction, the influence of the grain size on the magnetic properties may be determined. When focusing on one material and studying the variation of the magnetic properties in terms of the cutting angle, the texture dependency may be determined.

The fitting of the seven parameters $k_1, k_2, a, b, c, d$ and $k_4$ defined in Eqs. (3) and (4) worked out in two steps. The first step is based on the quasi-static characterisation while the second one is based on the AC-properties.

### 3.2. Quasi-static characterisation: fitting of the PDF

For the quasi-static characterisation, the procedure starts from the $W_{\text{hyst}}$-characteristics. The six parameters in the Lorentzian PDF are fitted such that

$$W_{\text{hyst,meas}}(B_m) = W_{\text{hyst,calc}}(B_m), \quad 0.1 \text{T} < B_m < 1.5 \text{T}$$

(11)

where $B_m$ is the amplitude of the quasi-static BH-loop (obtained by varying the induction from $B_m$ to $-B_m$ and back to $B_m$, without any higher order loops). As the Preisach model uses the magnetic field strength $H$ as input, Eq. (11) must be rewritten as

$$W_{\text{hyst,meas}}(H_m) = W_{\text{hyst,calc}}(H_m)$$

(12)

together with

$$B_{m,\text{meas}}(H_m) = B_{m,\text{calc}}(H_m),$$

(13)

where $H_m$ is the extremal value of the magnetic field corresponding with $B_m$. Here, Eq. (11) is equivalent with Eqs. (12) and (13). The parameters $k_1, a$ and $b$, entering the PDF of Eq. (3) and defining the calculated hysteresis losses $W_{\text{hyst,calc}}(H_m)$, are fitted using the characteristic (12) while the remaining parameters $k_2, c$ and $d$ are obtained by considering Eq. (13). Indeed, the area enclosed by the quasi-static BH-loop is given by

$$W_{\text{hyst,calc}} = \int H \, dB = \int H \, d(B_{\text{rev}} + B_{\text{irr}}) = \int H \, dB_{\text{irr}},$$

(14)

where $B_{\text{rev}}$ is given by the first and the second term in the RHS of Eq. (6), while $B_{\text{irr}}$ is given by the last term in Eq. (6).

Once $k_1, a$ and $b$ are obtained, we can proceed by analysing $k_2, c$ and $d$. Herefore, we start by constructing the following single valued characteristic:

$$B_{m,\text{rev,meas}}(H_m) = B_{m,\text{meas}}(H_m) - B_{m,\text{irr,calc}}(H_m),$$

(15)
where \( B_{\text{m, irr, calc}}(H_m) \) may be calculated by the Preisach model, considering only the irreversible part of the PDF, i.e.

\[
B_{\text{m, irr, calc}}(H_m) = \frac{1}{2} \int_{-H_m}^{H_m} \int_{-H_m}^{z} \frac{k_1}{(1 + ((z - a)/b)^2)(1 + ((\beta + a)/b)^2)} \, dz \, \, d\beta.
\]  (16)

Then, the three parameters \( k_2, c \) and \( d \) are calculated such that

\[
B_{\text{m, rev, meas}}(H_m) = B_{\text{m, rev, calc}}(H_m)
\]  (17)

is fulfilled by

\[
B_{\text{m, rev, calc}}(H_m) = \mu_0 H_m + \int_{0}^{H_m} dz \left( \frac{k_2}{1 + (z/c)^2} + d \right)
\]  (18)

The results are summarised in Figs. 1–6. The parameters \( k_2, c \) and \( d \), describing \( P_{\text{rev}}(z, \beta) \) show a less smooth variation due to the non-perfectness of the measurements. However, these last three parameters are less important with respect to the iron losses. Using the Lorentzian distribution function the \( W_{\text{hyst, meas}}(B_m) \)-characteristic was approximated by \( W_{\text{hyst, calc}}(B_m) \) within 3%.

### 3.3. AC characterisation: fitting of the parameter \( k_d \)

The excess losses \( W_{\text{exec}} \) in the electrical steels are described by the parameter \( k_d \), defined in Eq. (4). The fitting of this parameter is done in such a way that the measured iron losses under sinusoidal flux conditions equal the iron losses calculated with the CL-DPM, for different induction levels and different frequencies, i.e.

\[
W_{\text{tot, meas}}(B_m, f) = W_{\text{tot, calc}}(B_m, f), \quad 0.1 \, \text{T} < B_m < 1.5 \, \text{T}, \ 20 \, \text{Hz} < f < 400 \, \text{Hz}.
\]  (19)
Taking into account Eq. (11), and considering the principle of loss separation, the total losses under sinusoidal alternating flux conditions given by Ref. [15]

\[
W_{\text{tot,meas}}(B_m, f) = W_{\text{byst}}(B_m) + W_{\text{clas}}(B_m, f) + W_{\text{exc}}(B_m, f)
\equiv W_{\text{byst}}(B_m) + \frac{1}{6}\sigma\pi^2 B_m^2 t^2 f + 8.67\sqrt{\sigma GSV_o B^3} \sqrt{f}.
\]
Here \( V_0 \) is a fitting parameter with the dimensions of a magnetic field, describing microstructural features like the average grain size \( \Phi \) and the crystallographic texture. In addition, \( t \) and \( S \) stand for the thickness and the cross-sectional area of the lamination.

From Eqs. (11) and (20), it is clear that to obtain the parameter \( k_d \), such that Eq. (19) holds, it is sufficient to fit the slope of the characteristic

\[
[W_{\text{tot,meas}}(B_{\text{mv}}, f) - W_{\text{elas}}(B_{\text{mv}}, f)](\sqrt{f})
\]

as this slope, under certain conditions, see Ref. [17], is proportional to \( k_d^{-0.5} \).
Fig. 6. Parameter $k_2$, defining the reversible part of the PDF.

The parameter $k_d$ is calculated iteratively by considering the slope of the function (21) which is calculated using the CL-DPM (7)–(10). The values for $k_d$ are depicted as a function of the cutting angle and as a function of the grain size in Figs. 7 and 8, respectively. Finally, we observed that the $k_d$ parameter is magnetisation independent.
Fig. 8. Parameter $k_d$, defining the dynamic hysteresis effects as a function of the grain size. A small deviation from a monotonously decreasing function is observed at low values for the grainsize, due to a measuring error of the grainsize.

4. Separation of the effect of grain size from the influence of the crystallographic texture

Each of the seven parameters could be written as a linear function of the reciprocal value of the average grain size $\Phi$. Indeed, after several tentatives, it turned out that in case of the materials under investigation, it is sufficient to apply the linear dependence of the parameters on the reciprocal values of average grain size $\Phi$, as follows:

$$ p = P_{(1)} \frac{1}{\Phi} + P_{(2)}, \quad (22) $$

where $p$ is one of the seven parameters under investigation.

In this way, $P_{(1)}$, as well as $P_{(2)}$, easily quantify the texture dependence – or independence. Now, our intention is to prove that the extracted texture dependence of the examined parameters can lead to a single normalised texture dependent function.

Therefore, for a better comparison of the angle (texture) dependence of the coefficients $P_{(1)}$ and $P_{(2)}$ in Eq. (22), the following normalisation was applied:

$$ |P_{\text{norm}}| = \frac{(P_{\text{act}} - P_{\text{min}})}{(P_{\text{max}} - P_{\text{min}})}, \quad (23) $$

where $P_{\text{act}}$ equals the actual value of the investigated coefficient in Eq. (22).

4.1. Quasi-static characterisation – use of the $W_{\text{hyst}}(B_m)$ characteristics

4.1.1. Irreversible part of the magnetisation

For parameter $a$, an evident linear dependence on the reciprocal values of the average grain size was obtained with almost constant slopes, see Fig. 9 and thus according to Eq. (22):

$$ a = A_{(1)} \frac{1}{\Phi} + A_{(2)} \quad (24) $$
Fig. 9. Fitting results based on data of the $W_{\text{hyd}}(B_m)$ relation for parameter $a$, as a function of the reciprocal value of the average grain size.

Fig. 10. Values of the coefficients $A_{(1)}$, $A_{(2)}$, $B$ and $1/K_1$ as a function of the cutting angle. $A_{(1)}$ and $A_{(2)}$ are coefficients of the linear fits of data in Fig. 1 for each cutting angle; $B$ and $1/K_1$ were calculated as averaged values of $b$ and $1/k_1$ for each cutting angle in Figs. 2 and 3.

For parameters $b$ and $k_1$ the same type of plot provided almost horizontal lines, which means that their grain size dependence is negligible, i.e. $b = B_{(2)}$ and $k_1 = K_{1,(2)}$.

In case of the parameter $a$, the coefficient $A_{(1)}$ proved to be texture independent, with an average value of $A_{(1)} = 1.296 \pm 0.1616$, while the texture dependence was quantified in the angle dependence of the $A_{(2)}$ coefficient.
The texture dependence of the parameter $b$ and $1/k_1$ was simply calculated by averaging out their values for each cutting direction over the six materials. The average values are denoted by $B_{(2)} \equiv B$ and $1/K_{1,(2)} \equiv 1/K_1$. The calculated values and the relative errors of the evaluation are presented in Fig. 10. In Fig. 11, the angle dependence is presented for the three normalised texture dependent coefficients, $A_{(2)}$, $B$ and $1/K_1$, applying Eq. (23). The shape of the three texture dependencies was found to be very similar, within an acceptable error. For further considerations, a common texture dependent curve can be proposed, denoted by $T_x(x)_{rev}$ which was derived as the average of the three curves of Fig. 11.

4.1.2. Reversible part of the magnetisation

The fitting of the model parameters of the reversible part of the Preisach function has revealed that there is no significant grain size dependence in any of the three parameters $c$, $d$ and $k_2$. According to Eq. (22), we obtain $c = C_{(2)}$, $k_2 = K_{2,(2)}$ and $d = D_{(2)}$.

Furthermore, in the case of the parameter $k_2$, neither the grain size nor the angle dependence was exceeding the dispersion of the values and $k_2$ can be simply considered as a constant during the fitting of the other model parameters ($k_2 = 5.5310 \times 10^{-3}$).

Furthermore, the angle dependence of the parameters $c$ and $d$ averaged out for the different grain sizes and denoted by $C_{(2)} \equiv C$ and $D_{(2)} \equiv D$ is shown in Fig. 12. Applying the usual normalisation process according to Eq. (23), very similar texture dependent characteristics were found with respect to the irreversible part of the magnetisation (compare Fig. 11 with Fig. 13). The average of the two normalised curves in Fig. 13 can unambiguously characterise the texture dependence of the reversible part of the magnetisation, denoted by $T_x(x)_{rev}$.

4.2. AC characterisation – evaluation of $k_d$-parameter

The AC characterisation, using the combined lamination-dynamic Preisach model described in Section 2, has provided the parameter $k_d$, which showed a very evident texture and grain size dependence when
Fig. 12. Values of the coefficients $C$ and $D$ as a function of the cutting angle – $C$ and $D$ were calculated as averaged values of $c$ and $d$ for each cutting angle in Figs. 4 and 5.

Fig. 13. Angle dependence of parameter $C$ and $1/D$, normalised according to Eq. (23).

Considering this parameter as a function of the cutting angle (Fig. 7) and as a function of the reciprocal value of average grain size (Fig. 8):

$$k_d = \frac{1}{K_{d,1}} \frac{1}{\Phi} + K_{d,2}$$

(25)
In Fig. 8, the $K_{d,(2)}$ coefficient of Eq. (22) was found zero, which could be expected for parameters describing the dynamic behaviour. The coefficient $1/K_{d,(1)}$ proved to show a similar texture dependence as was found for the other parameters $a, b, c, d, k_1, k_2$, see Fig. 14.

5. Discussion of the results

The analysis of the material parameters in the Preisach theory as a function of grain size and cutting angle has permitted the authors to extract the effect of grain size from the influence of crystallographic texture:

- In the irreversible part of the magnetisation under quasi-static conditions, only the parameter $a$ is grain size dependent, i.e.:

$$a = A_{(1)} \frac{1}{\Phi} + A_{(2)},$$

where $A_{(1)}$ is constant, and $A_{(2)}$ holds the texture dependence.

The grain size independent parameters $b$ and $1/k_1$, defining, respectively, the shape and the peak value of the PDF, were averaged out in each cutting direction. Together with $A_{(2)}$, their averaged values $B$ and $1/K_1$ were normalised according to Eq. (23), which permitted us to compare their cutting angle, and equivalently their texture dependence. Fig. 11 shows the same shape for the function describing the texture dependence.

- For the reversible part of the magnetisation under quasi-static conditions, the three parameters $c, d$ and $k_2$ were found to be grain size independent. This is a comprehensive conclusion, considering that in this phase of the magnetisation, the rotational process is dominant. The $k_2$ parameter was found to be grain size and texture independent, with an estimated value of $k_2 = 5.531 \pm 0.3606 \times 10^{-3}$. The parameters $c$ and $d$, averaged out for each cutting angle, were found to be strongly correlated, in the form of $c \propto 1/d$. This correlation resulted in a very similar texture dependence after normalisation, see Fig. 13, which permitted us to construct one texture dependent function for the reversible part of the magnetisation, denoted by $T_x(z)_{rev}$. 

![Graph](image-url)
In the case of the *AC characterisation*, the $k_d$ parameter, which describes the dynamic (excess) loss behaviour, was found to be both grain size and texture dependent according to the following form:

$$
k_d = \frac{1}{K_{d,(1)} \Phi},
$$

where $K_{d,(1)}$ holds the texture dependence. After normalisation, it is seen that $K_{d,(1)}$ has the same texture dependence, denoted by $T_a(x)_{\text{dyn}}$, as the parameters in the Lorentzian Preisach distribution function.

Indeed, to obtain a better comparison, we have plotted the texture dependence of different contributions of the magnetisation, after that the normalised values for the parameters $A_{(2)}, B$ and $1/K_1$ for the irreversible part and the normalised parameters $C$ and $1/D$ for the reversible part, were averaged out. Plotting these data together with the normalised form of the dynamic part a very good agreement was found between the texture dependence of the three magnetisation mechanisms, see Fig. 15.

### 6. Concluding remarks

In this paper, extended experiments together with fitting techniques and numerous numerical calculations using the combined lamination-dynamic Preisach model were performed to study the effect of the microstructure of the material on the material parameters in the Lorentzian distribution function of the Preisach model. Handling separately the irreversible, the reversible and the dynamic (anomalous) contributions to the electromagnetic behaviour, the fitting process was performed. It has been verified that it is possible to separate quantitatively the effect of the average grain size from the influence of the crystallographic texture.

As a result of the sample set up, the authors were able to observe separately the influence of texture and grain size. Indeed, the examination of the magnetic behaviour, in particular the magnetic parameters, as a function of the cutting angle for each material results in the texture dependence. Meanwhile, the grain size dependence has to result from the examination of samples cutted under the same angle, however for the different materials.
In general the authors may conclude that only the parameter \( a \), defining the position of the peak in the PDF, shows a grain size as well as a texture dependence. The other material parameters, i.e. \( b, k_1, c \) and \( d \), the former two defining the shape and the peak value of the PDF, do not show a grain size dependence.

The texture dependence was identified for \( A_{(2)}, b \) and \( k_1 \) parameters:

\[
A_{(2)}, b, \frac{1}{k_1} \sim T_x(\alpha)_{\text{irrev}}.
\]  
(28)

The parameters \( c, d \) and \( k_2 \), defining the reversible part of the magnetisation were found independent of the grain size, which can be expected for the part of the magnetisation dominated by the rotation process. The \( k_2 \) was found to be also texture independent. The value found for \( k_2 \) equals \( 5.531 \pm 0.3606 \times 10^{-3} \). The texture dependence of \( c \) and \( d \) was found strongly correlated:

\[
c \sim \frac{1}{d} \sim T_x(\alpha)_{\text{rev}}.
\]  
(29)

The dynamic (anomalous) part of the magnetisation was described by a single parameter \( k_d \), which proved to be grain size as well as texture dependent:

\[
k_d = \frac{1}{K_{d,(1)}}, \quad \text{with } K_{d,(1)} \sim T_x(\alpha)_{\text{dyn}}.
\]  
(30)

Moreover, the three kinds of texture dependence were clearly found identical:

\[
T_x(\alpha)_{\text{irrev}} \equiv T_x(\alpha)_{\text{rev}} \equiv T_x(\alpha)_{\text{dyn}} \equiv T_x(\alpha).
\]  
(31)

These conclusions permit us to rewrite the Lorentzian distribution function in the Preisach model, involving concrete texture and grain size dependencies:

\[
P(\alpha, \beta) = \delta(\alpha - \beta) \left( \frac{5.531 \times 10^{-3}}{1 + (\alpha/l_1(T_x))^2} + \frac{1}{l_2(T_x)} \right) + \frac{1}{l_3(T_x) \left( 1 + \left( \frac{\alpha - (1.3/\Phi + l_4(T_x))}{l_5(T_x)} \right)^2 \right) \left( 1 + \left( \frac{\beta + (1.3/\Phi + l_4(T_x))}{l_5(T_x)} \right)^2 \right)} \]

with \( l_i \) a linear function of the normalised texture function \( T_x(\alpha) \):

\[
l_i = A_i T_x(\alpha) + B_i, \quad i = 1, 2, 3, 4, 5.
\]  
(33)

Here, the constants \( A_i \) and \( B_i \) are properly chosen values. Eq. (30) may be applied for the parameter \( k_d \).

Finally we may point out that the conclusions mentioned above clearly confirm some previously published conclusions related to the microstructural dependence of the statistical power loss model of Bertotti [18], where the \( V_0 \) internal coercive field was found to depend linearly on the grain size:

\[
V_0 = V_1 \Phi, \quad V_1 \sim T_x(\alpha).
\]  
(34)

Moreover, in Ref. [17], the following relation between the material parameters in the dynamic Preisach model and the material parameters in the statistical power loss model was proven:

\[
k_d = \frac{H_{\text{max}}}{256 V_0 \sigma GSB^2} \left( \int d\alpha d\beta P(\alpha, \beta) \cdot C(h(x), \alpha, \beta, H_{\text{max}}) \right)^2
\]  
(35)
and by numerical experiments, it was shown that for a fixed value of $k_d$, the value for $V_0$ does not change with the variation of the parameter $a$, which is the only parameter in the Lorentzian distribution function showing a grain size dependency. Therefore, from Eqs. (34) and (35), it is clear that $1/k_d$ and $V_0$ must have the same grain size dependence. Furthermore, the $W_0$ coefficient of the power function representation of the hysteresis losses ($W_{\text{hyst}} = W_0 B^n$) provides the same texture and grain size dependence as the parameter $a$ in the Lorentzian PDF. Indeed:

$$W_0 = \frac{W_1}{\phi} + W_2, \quad \text{with} \quad W_1 = \text{const.} \quad \text{and} \quad W_2 \sim T(x).$$

Finally, it can be shown that the parameter $a$ as well as the parameter $W_0$ are directly related to the coercive force $H_c$ of the material.

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**References**