A computational model for the iron losses in rotating electrical machines

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Abstract

In this paper we deal with a mathematical model for the evaluation of the electromagnetic iron losses in rotating electrical machines under no load conditions. This model is based on a two level machine model, i.e. first level: tooth region model, second level: lamination model. The presented problems of electromagnetic field computations are coupled with refined material models based on the Preisach theory. The model is validated by the comparison of numerical results and experimental values from measurements. © 1998 Elsevier Science Ltd. All rights reserved.

1. Introduction

Iron losses can account for a significant part of the total losses of an electrical machine. At the other hand, nowadays the efficient use of electricity is strongly emphasized. Electrical drive systems offer considerable opportunity to obtain major improvements in this respect. Consequently, it is important to increase the accuracy and reliability of the modelling and simulation of the iron losses.

A numerical model based on a single valued material characteristic cannot describe adequately the phase difference between the magnetic flux density $B$ and the magnetic field strength $H$ in the case of a rotating magnetic flux excitation. This type of excitation in electrical machines results from the complexity of the magnetic circuit and of the magnetic motoric force distributions.

In this paper we present the inclusion of the vector Preisach model, as described in [1], in the magnetic field calculations for a 2D-domain $D$. This domain $D$ represents one tooth region of the stator of an asynchronous machine. The magnetic behaviour of the material can be described in terms of the macroscopic fields, taking into account the hysteresis phenomena.

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The boundary $\partial D$ is divided into six parts, namely 3 flux gates and 3 flux walls, as shown in Fig. 1. The three enforced flux patterns through $\partial D_1$, $\partial D_2$ and $\partial D_3$ are obtained by numerical field calculations or by local measurements in the electrical machine.

On the basis of the computed field patterns in the domain $D$, the local excitation conditions for the magnetic material will be derived. The models described in [2] and [3] will be used to investigate the local material response. This will lead to a detailed knowledge of the local iron losses.

Finally, the global machine losses, evaluated from this new method, are compared with the measured machine losses.

2. Material models

2.1. Scalar hysteresis model

If $H$ and $B$ are unidirectional, the $BH$-relation can be described by a scalar Preisach model in which the material is assumed to consist of small dipoles, each being characterized by a rectangular hysteresis loop as shown in Fig. 2 [4]. The magnetization of the dipole is given by

$$M_d = \begin{cases} 
+1 & : H(t) > \alpha \text{ or } (\beta < H < \alpha \text{ and } H_{\text{last}} > \alpha) \\
-1 & : H(t) < \beta \text{ or } (\beta < H < \alpha \text{ and } H_{\text{last}} < \beta)
\end{cases}$$

where $H_{\text{last}}$ is the last extreme value of $H$ kept in memory. The characteristic parameters $\alpha$ and $\beta$ are distributed statistically according to a Preisach function $P_s(\alpha, \beta)$ which is a material parameter. This distribution function can be identified directly when using a proper measurement technique [5].

The $BH$-relation reads:

$$B(H, H_{\text{past}}) = \int_{-H_m}^{H_m} \int_{H_m}^{H_m} d\beta \eta_s(\alpha, \beta, t) P_s(\alpha, \beta).$$

Here $\eta_s(\alpha, \beta, t)$ gives the value of the magnetization $M_d$ for the dipole with parameters $\alpha$ and $\beta$ at time $t$. Consequently, the induction $B$ depends upon the magnetic field $H(t)$ and its history, denoted by $H_{\text{past}}(t)$.
2.2. Vector hysteresis model

In this model, as described in [1], the magnetic field vector $\vec{H}$ and the magnetic induction vector $\vec{B}$ are no longer unidirectional. The vector $\vec{H}$ is projected on an axis $\vec{d}$, which encloses an angle $\theta$ with the fixed $x$-axis, $-\pi/2 < \theta < \pi/2$, see Fig. 3. The resulting component $H_\theta$ ($=H_x \cos \theta + H_y \sin \theta$) is used as the input of the scalar Preisach model on the axis $\vec{d}$.

The $BH$-relation is now given by, see [6],

$$B(H,H_{\text{past}}) = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} d\theta B_\theta(H_{\theta},H_{\text{past},\theta}) \tilde{1}_\theta,$$

(3)

Fig. 3. (Md,H)-characteristic of a Preisach dipole.

Fig. 2. ($M_d$,$H$)-characteristic of a Preisach dipole.

Fig. 3. Vector Preisach model.
with

\[ B_0(H_\theta,H_{\text{past}},0) = \int_{-H_m}^{H_m} \int_{-H_m}^{H_m} d\beta \eta_\tau(\theta,\alpha,\beta,t)P_t(\alpha,\beta), \]  

(4)

where \( \eta_\tau(\theta,\alpha,\beta,t) \) is obtained from the component \( H_\theta \), and thus depends on \( \bar{H}(t) \) and \( H_{\text{past}}(t) \). The Preisach function \( P_t \) in this rotational model can be evaluated from the distribution function \( P_\tau \), entering (2), see [3].

2.3. Extension to a rate dependent hysteresis model

The frequency dependence of the hysteresis effects may have a large influence on the magnetic behaviour of the material as pointed out in [2]. Therefore, in that paper a rate-dependent scalar Preisach model has been incorporated in the magnetodynamic field calculations under arbitrary alternating excitation conditions. Here, the switching of the dipoles is no longer instantaneous, but proceeds at a finite rate. The main consequence of this improvement is the enlargement of the hysteresis loops with increasing frequency. This effect allows the modelling of the extra losses, appearing at increasing frequency, together with the classical eddy current losses.

Unfortunately, at present no experimentally validated rate-dependent vector hysteresis model is available.

3. Two level machine model

3.1. First level: tooth region model

We consider the single tooth region of Fig. 1, where the electrical conductivity \( \sigma \) is assumed to be zero. The relevant Maxwell equations for the magnetic field \( \mathbf{H} = H_\alpha \mathbf{1}_x + H_\tau \mathbf{1}_y \) and the magnetic induction \( \mathbf{B} = B_x \mathbf{1}_x + B_y \mathbf{1}_y \), in the 2D domain \( D \) with boundary \( \partial D \) are, see e.g [7],

\[ \text{rot} \, \mathbf{H} = 0, \]  

(5)

\[ \text{div} \, \mathbf{B} = 0, \]  

(6)

where the relation between \( \mathbf{H} \) and \( \mathbf{B} \) is defined by the material characteristics obtained from the vector Preisach hysteresis model, described above.

Enforcing a total flux \( \phi_s(t) \) through the parts \( \partial D_s \), \( s = 1,2,3 \), of \( \partial D \), we arrive at the boundary conditions (BCs)

\[ \phi_s(t) = \int_{\partial D_s} \mathbf{B} \cdot \mathbf{n} \, dl, \quad t > 0, \quad s = 1,2,3 \]  

(7)

\[ \mathbf{H} \times \mathbf{n} = 0 \text{ on } \partial D_s, \quad t > 0, \quad s = 1,2,3, \]  

(8)

where \( \mathbf{n} \) is the unit outward normal vector to the boundary part \( \partial D_s \).
At the other hand an assumed zero flux leakage through \( \partial D_4, \partial D_5 \) and \( \partial D_6 \) results in the additional BCs:
\[
\hat{B} \cdot \hat{n} = 0 \text{ on } D_s, \ t > 0, \ s = 4,5,6.
\] (9)

The demagnetized state of the material at \( t = 0 \) is expressed by the initial condition (IC)
\[
H(x,y,t) = 0,
\begin{cases}
\eta_i(x,y,\theta,x,\beta,t = 0) = +1, & x + \beta < 0 \\
\eta_i(x,y,\theta,x,\beta,t = 0) = -1, & x + \beta > 0
\end{cases}
\quad \psi(x,y) \in D, \ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}.
\] (10)

Details of a numerical approximation method, viz a modified finite element–finite difference method, for the problem (5)–(10) can be found in [8]. Comprehensive accounts on finite element methods for electrical machines are e.g. [9] and, more recently, [10].

As the enforced fluxes \( \phi_s(t), \ s = 1,2,3 \), are periodic in time, we may use a complex Fourier decomposition for the local vector fields \( \vec{H}(x,y;t) \) and \( \vec{B}(x,y;t) \), viz
\[
\vec{H}(x,y;t) \equiv \sum_{k=-\infty}^{+\infty} H_k(x,y) \cdot e^{j(k\omega t + z_k)},
\] (11)
\[
\vec{B}(x,y;t) \equiv \sum_{k=-\infty}^{+\infty} B_k(x,y) \cdot e^{j(k\omega t + \beta_k)}.
\] (12)

Here, \( \omega \) is \( 2\pi \) times the basic frequency; \( z_k \) [resp. \( \beta_k \)] and \( H_k \) [resp. \( B_k \)] are the phase angle and the amplitude of the \( k \)th harmonic of \( \vec{H} \) [resp. \( \vec{B} \)], see also [8].

Using the local field patterns obtained from the tooth region model, see (12), we may investigate the local material behaviour from a lamination model.

3.2. Second level: lamination model

The magnetic behaviour of ferromagnetic laminations can be described in terms of the macroscopic fields, taking into account the interacting hysteresis and eddy current phenomena.

We consider a single lamination of length \( l \), width \( w \) and thickness \( 2d \), see Fig. 4, the cartesian coordinate system being chosen in a natural way as indicated. Throughout the sheet, which is assumed isotropic, the time dependent total flux vector \( \phi(t) \) flows parallel to the \((x,y)\)-plane. This flux vector is constructed out of (12). The magnetic field and the magnetic induction in the lamination model take the form \( \vec{H} = H_x \hat{x} + H_y \hat{y} \) and \( \vec{B} = B_x \hat{x} + B_y \hat{y} \) respectively. As \( d \ll w \) and \( d < l \), eliminating the edge effects, we may assume \( H_x, H_y \) and \( B_x, B_y \) to vary in the \( z \)-direction only.

Next, we take into account the constitutive relation, \( \vec{J} = \sigma \vec{E} \), between the electric field \( \vec{E} \) and the current density \( \vec{J} \) (both parallel to the \((x,y)\)-plane) in the relevant Maxwell equations, viz
\[
\text{rot} \, \vec{E} = -\partial_t \vec{B}/\partial t
\] (13)
Two different types of excitations can be considered.

3.2.1. (a) alternating excitation conditions
Here, we may assume $H_y$, $B_y$ and $\varphi_y(t)$ to be identically zero. The equations above simplify to the parabolic DE for $H_x$

$$\frac{1}{\sigma} \cdot \frac{\partial H_x^2}{\partial z} = \frac{\partial B_x}{\partial t}, \; 0 < z < d, \; t > 0,$$

along with the BCs

$$\frac{\partial H_x}{\partial z}(z = 0, t) = 0, \quad \frac{\partial H_x}{\partial z}(z = d, t) = \frac{\sigma}{2} \frac{d\varphi_x}{dt}, \; t > 0.$$ 

and ICs

$$H_x(z, t = 0) = 0, \begin{cases} \eta_s(z, x, \beta, t = 0) = +1 & : x + \beta < 0 \\ \eta_s(z, x, \beta, t = 0) = -1 & : x + \beta > 0 \end{cases} \; 0 > z > d.$$  

Here, the magnetic induction $B_x(z, t)$ can be related to the magnetic field $H_x(z, t)$ by either the scalar rate independent or rate dependent Preisach hysteresis model. A modified finite element–finite difference approximation method for the BVP (15)–(17) is described in detail in [2].

3.2.2. (b) rotational excitation conditions
Now, the governing differential equations for the magnetic field ($H_x$, $H_y$) are found to be

$$\frac{1}{\sigma} \cdot \frac{\partial H_x}{\partial z} = \frac{\partial B_y}{\partial t}, \; 0 < z < d, \; t > 0,$$
\[
\frac{1}{\sigma} \frac{\partial^2 H_y}{\partial z^2} = \frac{\partial B_y}{\partial t}, \quad 0 < z < d, \quad t > 0,
\]
while the BCs become
\[
\frac{\partial H_x}{\partial z}(z=0,t) = \frac{\partial H_x}{\partial z}(z=d,t) = 0, \quad \frac{\partial H_x}{\partial z}(z=d,t) = \frac{\partial H_y}{\partial z}(z=d,t) = \frac{\sigma}{2} \frac{\partial \varphi_z}{\partial t}, \quad t > 0
\]
(20)

The ICs, again describing the demagnetized state at \( t = 0 \), are now given by
\[
H_x(z,t=0) = 0, \quad H_y(z,t=0), \quad \eta(t,z,0,\alpha,\beta,t=0) = +1 \quad \alpha + \beta < 0 \quad \pi \leq \theta \leq \frac{\pi}{2}, \quad 0 < z < d.
\]
(21)

Here, the magnetic induction \( B \) is related to the magnetic field \( H \) by the vector rate independent Preisach hysteresis model, introduced in [1]. In [3] we dealt with a modified finite element–Crank Nicholson method to solve numerically the resulting BVP.

Notice that, due to the complexity of the material model used, (15), (18) and (19) are highly nonlinear partial differential equations with memory.

The total electromagnetic losses in the lamination per unit volume during a time interval \([T_1, T_2]\) (where \( T_2 - T_1 \) is an integer multiple of the excitation period) are calculated by summing up the hysteresis losses and the eddy current losses. These losses are given by, see e.g. [7]
\[
P_h = \frac{1}{2d} \int_{-d}^{d} \int_{T_1}^{T_2} \left( H_x \frac{\partial B_x}{\partial t} + H_y \frac{\partial B_y}{\partial t} \right) dt
\]
(22)
and
\[
P_e = \frac{1}{2d} \sigma \int_{-d}^{d} \int_{T_1}^{T_2} \left( \left( \frac{\partial H_x}{\partial z} \right)^2 + \left( \frac{\partial H_y}{\partial z} \right)^2 \right) dt.
\]
(23)

4. Numerical results and concluding remarks

We consider a three phase 3 kW 4-pole induction motor, described in detail in [11]. The “measured” iron losses are obtained by measuring or calculating the different components of the power balance. The results are listed in Table 1. Multiplication of the three phase currents and voltages, measured by means of a data-acquisition system, readily produces the electrical power input. The resistance of the stator windings were measured immediately after the tests, allowing an accurate calculation of the stator joule losses. The joule losses in the rotor cage are predicted using the 2D FE-motor model. The mechanical friction losses have been estimated from the instantaneous deceleration when interrupting the power supply.

The experimentally obtained machine iron losses are compared with the global machine losses that are predicted by the combined tooth region-lamination model. Due to periodicity,
only three neighbouring teeth in the stator must be considered. The enforced total fluxes \( \phi_s \) through the parts \( \partial D_s \), \( s = 1, 2 \) in Fig. 1 are obtained from local measurements in the electrical machine (notice that \( \phi_3 = -\phi_1 - \phi_2 \)). The flux through the gate \( \partial D_1 \) for each of the three neighbouring teeth is given in Table 2 by its Fourier decomposition

\[
\phi_1(t) = \sum_k A_k \cos(k\omega t + \gamma_k).
\]  

(24)

The local flux patterns in a tooth region were calculated when using first a single valued material characteristic and next a vector Preisach model.

Table 3 shows the symmetry for each pair of positive and negative harmonics for point 1 in Fig. 1 for tooth1. This corresponds to alternating field vectors. Notice that for point 2 this symmetry is lost, reflecting a rotational magnetic induction \( \mathbf{B} \).

Similar remarks could be made for each point in tooth1, tooth2 and tooth3.

The local flux patterns obtained from the tooth model are now used as input for the combined magneto dynamic-rate independent vector Preisach model. The evaluation of the dynamic \( B_xH_x \)- and \( B_yH_y \)-loops are performed using the flux patterns obtained above, first with a single valued material characteristic, next with a vector hysteresis model.

The resulting dynamic BH-loops are shown in Fig. 5 and Fig. 6 for the points 1 and 2 in tooth1, see Fig. 5. We see that for point 1 the dynamic BH-loops are identical, while for point 2 the loops show a discrepancy. However the enclosed area, giving the iron loss density, is nearly the same.

To evaluate the local losses, we add to the losses calculated from the model (18)–(21) extra dynamic electromagnetic losses to take into account the rate dependent hysteresis effects. These extra dynamic losses represent the difference between the losses evaluated from the model (15)–

<table>
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<th>k</th>
<th>( A_k ) (Wb)</th>
<th>( \gamma_k ) (°)</th>
<th>( A_k ) (Wb)</th>
<th>( \gamma_k ) (°)</th>
<th>( A_k ) (Wb)</th>
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that is coupled first with the rate independent and next with the rate dependent Preisach model. Here, the alternating excitation used is the excitation which is obtained when we project the rotating waveform on that axis that gives rise to the maximum amplitude.

### Table 3

<table>
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<tr>
<th>$k$</th>
<th>$B_k(T)/\text{point 1}$</th>
<th>$B_k(T)/\text{point 2}$</th>
<th>$B_k(T)/\text{point 1}$</th>
<th>$B_k(T)/\text{point 2}$</th>
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</table>

(17), that is coupled first with the rate independent and next with the rate dependent Preisach model. Here, the alternating excitation used is the excitation which is obtained when we project the rotating waveform on that axis that gives rise to the maximum amplitude.

Fig. 5. Dynamic BH-loops in point 1.
To evaluate the total iron losses in the electrical machine, we sum up the local iron losses obtained from the combined lamination–vector hysteresis model and being corrected in the way just mentioned. Here we consider 4 cases:

**case1:** use of all harmonics in the flux patterns of the tooth region, obtained with a hysteretic characteristic

**case2:** use of all harmonics in the flux patterns of the tooth region, obtained with single valued characteristic

**case3:** use of the basic harmonic in the flux patterns of the tooth region, obtained with hysteretic characteristic

**case4:** use of the basic harmonic in the flux patterns of the tooth region, obtained with single valued characteristic

The results are given in Table 4. This leads us to the following conclusions.

- The global machine losses predicted by the combined tooth region–lamination model, using the flux patterns of **case 1**, are equal to 64.96 W, which is in fairly good agreement with the measured machine losses of 72.4 W.
Case 2 makes clear that the calculated global iron losses are not influenced substantially by the difference in the flux patterns when using either a single valued material characteristic or a vector hysteresis characteristic in the tooth region model.

Case 3 and case 4 show that neglecting the higher harmonics, results in a significant underestimation of the global iron losses.

The discrepancy between the measured iron losses (72.4 W) and the calculated global iron losses according to case 1 may rest upon the following causes: accuracy of the measurement of the global iron losses, the discrepancy between the material characteristics before and after the construction of the machine, anisotropy effects, etc. The inclusion of anisotropy effects in the models is an actual research topic of the authors. Preliminary numerical results are very promising.

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References


<table>
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