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On a magnetodynamic model for the iron losses in non-oriented steel laminations

L R Dupré†, R Van Keer‡ and J A A Melkebeek†

† Department of Electrical Power Engineering, University of Gent, Sint-Pietersnieuwstraat 41, B-9000 Gent, Belgium
‡ Department of Mathematical Analysis, University of Gent, Galglaan 2, B-9000 Gent, Belgium

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Abstract. In this paper we deal with a numerical model for the evaluation of the magnetic iron losses in one steel lamination. The magnetodynamic model for the magnetic field strength \( H \) is coupled with the Preisach model. The resulting boundary value problem for \( H \) is solved numerically by a modified finite element–finite difference method. The model is validated by the comparison of numerical experiments and measurements. The dynamic behaviour of two materials is investigated.

1. Introduction

The evaluation of the electromagnetic losses in electric machines relies on the computation of the electromagnetic fields inside the ferromagnetic laminations. This requires an accurate description of the hysteresis and eddy current effects. In [1] a modified finite element (FE)–finite difference (FD) method is presented in which the classical Preisach hysteresis model (CPM) is embedded in magnetodynamic field calculations for one lamination. The numerical results for the magnetic losses, obtained for the materials considered in [1], are in good agreement with the experimental values. However, for other materials a systematic discrepancy has been observed. Actually, the frequency-dependence of the hysteresis effects may have a large influence on the magnetic behaviour of the material, as pointed out in [2]. Therefore a rate-dependent Preisach model (RPM), recently introduced by Bertotti [3], will be incorporated into magnetodynamic field calculations as already briefly reported on in [4]. Thus, in this recent model the magnetic behaviour of the Preisach dipoles is rate-dependent. We recall that, in the Preisach theory, a ferromagnetic material is composed of elementary dipoles [5, 6]. In the CPM, these dipoles switch instantaneously, whereas in the RPM they are assumed to switch at a finite rate.

The most important consequence of this improvement is the enlargement of the hysteresis loops with increasing frequency. This effect allows the modelling of the extra losses appearing at increasing frequency together with the classical eddy current losses.

The magnetic behaviour of the material, as described by the RPM, may be incorporated in the magnetodynamic model in a direct and correct manner. The coupling of this magnetodynamic model with other valuable hysteresis models which have been reported in the literature [7, 8] turns out to be more cumbersome for the presented study.

In the next section the RPM is described to some extent, starting from the fundamental equations proposed by Bertotti [3]. We emphasize the frequency-dependence of the magnetic behaviour of the dipole. Subsequently, we deal with the magnetodynamics of one lamination. In section 3 we develop a fully discrete approximation method, namely a modified Galerkin FE–FD technique, for the highly nonlinear parabolic boundary value problem.
for the magnetic field strength in the lamination. Here, the major difficulty is due to the fact that the governing parabolic PDE is an equation with memory, reflecting the hysteresis effects. In section 4 the numerical results for the magnetic losses obtained from the magnetodynamic model, when using either the CPM or the RPM, are discussed and compared with experimental values.

2. The mathematical model

2.1. The classical Preisach model versus the rate-dependent Preisach model

2.1.1. The behaviour of the elementary Preisach dipole.

The elementary dipoles, composing the ferromagnetic material, are characterized by the values of the switching fields \( \alpha \) and \( \beta \), see figure 1 (dotted line). In the CPM the magnetization \( M_d \) of the dipole only takes the value +1 or −1. Explicitly,

\[
M_d = \begin{cases} 
+1 & H(t) > \alpha \text{ or } (\beta < H < \alpha \text{ and } H_{last} > \alpha) \\
-1 & H(t) < \beta \text{ and } (\beta < H < \alpha \text{ and } H_{last} < \beta) 
\end{cases}
\]

(1)

Here \( H_{last} \) is the last extreme value kept in memory outside the interval \([\beta, \alpha]\). Thus the CPM is rate-independent.

In the RPM of [3] the dipoles are assumed to switch at a finite rate, proportional to the difference between the local magnetic field \( H(t) \) and the elementary loop switching fields \( \alpha \) and \( \beta \). The factor of proportionality, denoted by \( k \), is an extra material parameter. Explicitly, the evolution in time of the magnetization \( M_d \) is given by

\[
\frac{dM_d}{dt} = \begin{cases} 
k(H(t) - \alpha) & H(t) > \alpha \text{ and } M_d < +1 \\
k(H(t) - \beta) & H(t) < \beta \text{ and } M_d > -1 \\
0 & \text{in the other cases.} 
\end{cases}
\]

(2)

To give an idea of the \((M_d, H)\) characteristic of one dipole in the RPM, we consider two relevant examples.

2.1.1.1. Example 1 (the symmetric case). For the imposed magnetic field \( H(t) \) as in figure 2 (broken line), the corresponding \((M_d, H)\) loops are shown in figure 1 (broken and chain lines). As may be observed from figure 1, the dipole may switch completely for sufficiently low frequency, but this is no longer the case for higher frequencies.

In addition the area enclosed by the \((M_d, H)\) loop during one cycle is given in figure 3 as a function of the frequency, for the indicated set of data. Below a first critical frequency \( f_k,1 \), the extra enclosed area in comparison with the case of the CPM is proportional to \( \sqrt{f} \). Above a second critical frequency \( f_k,2 \), the total area is proportional to \( 1/f \) and may become smaller than the classical area \( 2(\alpha - \beta) \).

2.1.1.2. Example 2 (the asymmetric case). A more complex situation results for an imposed magnetic field such as in figure 2 (full line). Due to the asymmetry of field strength relative to the \( \alpha \) and \( \beta \) parameters, the corresponding \((M_d, H)\) characteristics are asymmetric as well, as shown in figure 1 (full line).

2.1.2. Material characterization. The relative density of the Preisach dipoles is represented by the distribution function \( P(\alpha, \beta) \) [5, 6]. Correspondingly, the induction \( B(H(t), H_{past}(t)) \) takes on the following form in the Preisach model:

\[
B(H(t), H_{past}(t)) = \frac{1}{2} \int_{-H_m}^{H_m} d\alpha \int_{-H_m}^{\alpha} d\beta \eta(\alpha, \beta, t) P(\alpha, \beta).
\]

(3)

Here \( P(\alpha, \beta) \) is assumed to be negligibly small when either \( \alpha > H_m \) or \( \beta < -H_m \). This value \( H_m \) of the magnetic field is directly obtained from the experimental evaluation of \( P \). Moreover \( \eta(\alpha, \beta, t) \) takes on the time-dependent value of the magnetization \( M_d \) for the dipole with the parameters \( \alpha \) and \( \beta \). Of course, this results in the induction \( B \) depending upon the magnetic field \( H(t) \) and its history \( H_{past}(t) \). As mentioned above, in the CPM \( \eta(\alpha, \beta, t) \) only takes the
values $+1$ or $-1$, whereas in the RPM $\eta(\alpha, \beta, t)$ varies within the whole range from $-1$ to $+1$, according to (2).

### 2.2. Magnetodynamics in one lamination

Throughout the lamination, shown in figure 4, which is assumed to be isotropic, the time-dependent flux $\phi$ flows in the $z$ direction and the magnetic field has only one component, namely $\vec{H} = H \hat{e}_z$. Because $d \ll w$, eliminating the edge effects, we may assume $H$ to vary in the $x$ direction only. From the Maxwell equations the governing equation for $H$ is

$$\frac{1}{\sigma} \frac{\partial^2 H}{\partial x^2} = \frac{\partial B}{\partial t}$$  \hspace{1cm} (4)$$

where $B$ is the magnitude of the magnetic induction $\vec{B} = B(x)\hat{e}_z$, and $\sigma$ is the electrical conductivity of the material. Due to this conductivity eddy currents $\vec{J}$ are generated in the material.

In terms of the magnetodynamic model, $\frac{\partial B}{\partial t}$ must be related to the magnetic field $H(t)$, both for the CPM and for the RPM. In the former case one simply has, compare with [1],

$$\frac{1}{\sigma} \frac{\partial^2 H}{\partial x^2} = \mu_d(H(x, t), H_{\text{past}}(x, t)) \frac{\partial H}{\partial t}$$  \hspace{1cm} (5)$$

where $\mu_d$ is the differential permeability of the magnetic material.

In the RPM however, (3) and (4) combined with (2), written out for $\eta(\alpha, \beta, t)$, leads to

$$\frac{1}{\sigma} \frac{\partial^2 H}{\partial x^2} = \mu_{\text{rev}} \frac{\partial H}{\partial t} + k_1(H(x, t), H_{\text{past}}(x, t)) H - k_2(H(x, t), H_{\text{past}}(x, t))$$  \hspace{1cm} (6)$$

with

$$k_1(H(x, t), H_{\text{past}}(x, t)) = \frac{k}{2} \int_{D_1(x,t)} P(\alpha, \beta) \, d\alpha \, d\beta$$

$$+ \frac{k}{2} \int_{D_2(x,t)} P(\alpha, \beta) \, d\alpha \, d\beta$$  \hspace{1cm} (7)$$

$$k_2(H(x, t), H_{\text{past}}(x, t)) = \frac{k}{2} \int_{D_1(x,t)} \alpha P(\alpha, \beta) \, d\alpha \, d\beta$$

$$+ \frac{k}{2} \int_{D_2(x,t)} \beta P(\alpha, \beta) \, d\alpha \, d\beta.$$  \hspace{1cm} (8)$$

Herein $\mu_{\text{rev}}$ is the reversible differential permeability. $D_1$ and $D_2$ are the domains in the Preisach plane representing dipoles in an intermediate state, switching to positive and negative saturation respectively. Of course, the temporal and spatial dependences of $D_1$ and $D_2$ are through the local magnetic field $H(x, t)$ and its history $H_{\text{past}}(x, t)$.

To obtain a well-posed boundary value problem, sets (1) and (5) and (2) and (6) must be completed with the suitable boundary conditions (BCs) and initial conditions (ICs), namely

$$\frac{\partial H(x = 0, t)}{\partial x} = 0 \hspace{1cm} \frac{\partial H(x = d, t)}{\partial x} = \sigma \frac{\partial \phi}{\partial t}$$  \hspace{1cm} (9)$$

$$H(x, t = 0) = 0$$

$$\begin{cases}
\eta(\alpha, \beta, t = 0) = +1 & \alpha + \beta < 0 \\
\eta(\alpha, \beta, t = 0) = -1 & \alpha + \beta > 0.
\end{cases}$$  \hspace{1cm} (10)$$

The first BC reflects the symmetry in the lamination. The second BC follows when combining (4) with the symmetry and with the definition of the flux $\phi(t)$ through the lamination. Finally, the IC (10) corresponds to the demagnetized state of the material.

We refer to [1] for the numerical solution of the highly nonlinear parabolic problem (1), (5), (9) and (10) for the CPM. In the next section we deal with a numerical approach to the magnetodynamic problem (2), (6), (9) and (10) for the RPM.

### 3. The numerical method

We present a fully discrete approximation method to the problem (2), (6), (9) and (10), which properly takes into account the rate-dependent hysteresis effects. The method rests upon a modified finite element discretization in the space variable, using quadratic interpolation functions, and a suitable $\theta$-family of finite difference approximations with respect to the time variable. The method includes a proper discretization technique (with respect both to space and to time) for the coefficient functions $k_1$ and $k_2$, which appear in (7) and (8), and which represent the highly nonlinear and hysteretic behaviour of the material.
3.1. Space discretization

Let \( H(t) \), given by

\[
[H(t)]^T = [H_1(t), H_2(t), H_3(t), \ldots, H_{2n+1}(t)]^T
\]
be the vector of the values of the magnetic field in the equidistant nodes \( (x_i)_{i=1}^{2n+1} \) in the FE approximation with piecewise quadratic interpolation functions \( (N_i)_{i=1}^{2n+1} \).

Approximating in each element the coefficient functions \( k_1 \) and \( k_2 \) by their value in the midpoint, we are left with the following system of first-order ODEs with respect to \( t \):

\[
\frac{d}{dt}M_{ij}H + KH = F.
\]  

(12)

Here, \( M \) is the usual mass matrix; the stiffness matrix \( K \) and the force matrix \( F \) respectively are given by

\[
K_{ij} = \frac{1}{\sigma} \int_0^d \frac{dN_i}{d\eta} \frac{dN_j}{d\eta} \, dx
+ \frac{1}{\sigma} \int_0^d \tilde{k}_1(H(x,t), H_{past}(x,t))N_iN_j \, dx
\]

\[
i, j = 1, \ldots, 2n + 1
\]

(13)

\[
F_i = \int_0^d \tilde{k}_2(H(x,t), H_{past}(x,t))N_i \, dx
+ \frac{1}{2\sigma} \tilde{b}_{i,2n+1} \quad i = 1, \ldots, 2n + 1.
\]  

(14)

According to (7) and (8), to evaluate the functions \( \tilde{k}_1 \) and \( \tilde{k}_2 \), we have to approximate the domains \( D_1(x_2l, t) \) and \( D_2(x_2l, t) \) for \( l = 1, \ldots, n \). We discretize the triangle \( \{(\alpha, \beta) \mid -H_m < \alpha < H_m, -H_m < \beta < \alpha \} \) in the Preisach plane by means of an adaptive mixed triangular–rectangular grid with \( N \) elements \( \Delta_\rho \), such that

\[
\int_{\Delta_\rho} \frac{1}{2\sigma} \tilde{b} \, d\alpha \, d\beta < \epsilon_1 \quad \int_{\Delta_\rho} \frac{1}{2\sigma} \tilde{b} \, d\alpha \, d\beta < \epsilon_2
\]

\[
\int_{\Delta_\rho} \frac{1}{2\sigma} \tilde{b} \, d\alpha \, d\beta < \epsilon_2 \quad 1 \leq p \leq N
\]  

(15)

where \( \epsilon_1 \) and \( \epsilon_2 \) are deliberately chosen parameters of the approximation method, see figure 5. Correspondingly, the domains \( D_1(x_2l, t) \) and \( D_2(x_2l, t) \) result from the summation of those elements \( \Delta_\rho \) for which \( (\partial\eta/\partial t)(x_2l, \alpha_{pc}, \beta_{pc}, t) > 0 \) and \( (\partial\eta/\partial t)(x_2l, \alpha_{pc}, \beta_{pc}, t) < 0 \) respectively, \( (\alpha_{pc}, \beta_{pc}) \) being the centre of gravity of \( \Delta_\rho \).

3.2. Time discretization

The system (12) of nonlinear first-order ODEs is discretized by a \( \theta \)-family of finite difference methods (with \( 0 < \theta < 1 \)) for the magnetic field \( H_\rho \simeq [H(t)] \), at the successive time points \( t_s = s\Delta t, s = 1, 2, \ldots, [9] \). Here, according to (10), we start with

\[
H_0 = 0 \quad \text{and} \quad D_1(x_2l, t = 0) = D_2(x_2l, t = 0) = \emptyset
\]

\[l = 1, \ldots, n.
\]  

(17)

By numerous experiments we verify the magnetodynamic model, including either the CPM or the RPM, as well as its FE–FD discretization, outlined in sections 2 and 3. The numerical results obtained for relevant physical quantities, such as the \( B-H \) loops and the iron losses, are in good agreement with the values obtained by measurements.

We consider two materials with different magnetic structures, hereafter referred to as materials 1 and 2. The former is a material with high carbide content and large mechanical stresses. Material 2, known as V-450-50-E in the classification of [11], results from material 1 by the process of de-carbonizing and stress relieving. We compare the quasi-static and the dynamic behaviours of the two materials.

4. Experimental verification

Figure 6 shows the measured Preisach functions for materials 1 and 2. The Preisach function is measured by the technique of [12]. Notice that the Preisach function for material 1 shows two extrema whereas that for material 2 shows only one extremum. As a test for these Preisach functions, figures 7 and 8 show a very good agreement between the quasi-static measured \( B-H \) loops and the loops obtained from the hysteresis model, see (3).

4.1. Quasi-static characterization

Figure 6 shows the measured Preisach functions for materials 1 and 2. The Preisach function is measured by the technique of [12]. Notice that the Preisach function for material 1 shows two extrema whereas that for material 2 shows only one extremum. As a test for these Preisach functions, figures 7 and 8 show a very good agreement between the quasi-static measured \( B-H \) loops and the loops obtained from the hysteresis model, see (3).

4.2. Dynamic characterization

We now consider two types of excitation: we enforce (a) the time-dependent magnetic field strength at the outer boundary of the lamination, denoted by \( H_\rho(t) \), or (b) the average magnetic induction \( B_\rho(t) \) through the lamination.
A magnetodynamic model for iron losses in steel

For both types of excitations the calculated and the measured dynamic $B_a-H_b$ loops are observed to coincide, which, of course, results in a striking good agreement between the numerically obtained and the measured values of the total iron losses.

The total iron losses $E$ during a time interval $[t_1, t_2]$ are evaluated from the $B_a-H_b$ loops by means of the formula

$$E = \int_{t_1}^{t_2} H_b \frac{dB_a}{dt} \, dt.$$  \hspace{1cm} (18)

The electrical conductivity $\sigma$, entering (4), is directly measured. It takes the values $\sigma = 29.2 \times 10^5$ and $30.7 \times 10^5$ for materials 1 and 2 respectively.

4.2.1. Limit cycles

4.2.1.1. H excitation. The enforced magnetic field $H_b(t)$ is taken to be a piecewise linear function of time, namely the piecewise linear interpolant of the extremal values $H_b(t_i)$ in the successive points $t_i, i = 0, 1, 2, \ldots$. For brevity we denote it as

$$H_b(t) = \begin{bmatrix} 0 & H_b(t_1) & H_b(t_2) & \ldots & H_b(t_i) & H_b(t_{i+1}) & \ldots \end{bmatrix}.$$  

4.2.1.2. Material 1. For this material the CPM is found to be sufficiently accurate for describing the dynamic behaviour. Figure 9 shows the $B_a-H_b$ loops, corresponding to the field

$$H_b(t) = \begin{bmatrix} 0 & 1200 & -1200 & 1200 & -1200 & \ldots \end{bmatrix}$$  

whereas figure 10 gives the total iron losses for one cycle as a function of the frequency $f$.

4.2.1.3. Material 2. The hysteresis behaviour of this second material can no longer be described adequately
by the CPM. Instead, this behaviour turns out to be rate-dependent. However, it can be modelled correctly by the RPM, with $k = 55 \text{ mA}^{-1} \text{s}^{-1}$. (Moreover, this value itself is observed to be frequency-independent). Figures 11 and 10 show the $B_a-H_b$ loops and the corresponding total iron losses respectively; the measured values are compared with the numerically obtained values (using both the RPM and the CPM). In this case the used excitation reads

$\begin{bmatrix}
H_b(t) \\
th
\end{bmatrix} = \begin{bmatrix}
0 & 400 & -400 & 400 & -400 & \ldots \\
0 & 47 & 47 & 47 & 47 & \ldots 
\end{bmatrix}$

$f = 500 \text{ Hz}$.

### 4.2.1.4. B excitation.

Of course, in the case of $B$ excitation we expect a similar difference to that given above between the modelling of the two materials. This discrepancy has been confirmed by numerous experiments [13]. In this paper we restrict ourselves to the total iron losses as a function of the frequency, when a sinusoidal average induction $B_a$ (with maximum value 1 T) is enforced (figure 12).

#### 4.2.2. Minor loop excursions.

The complete mathematical model outlined in sections 2 and 3 also provides adequate results for the minor loop $B_a-H_b$ excursions, corresponding to the local extrema of the magnetic field $H(t)$, as described for instance in [14]. Again, for material 1 the dynamic behaviour is found to be accurately modelled when using the CPM; for material 2 the RPM must be used to model the hysteresis behaviour properly. As an example, we consider the $B_a-H_b$ loops for materials 1 and 2 in the case of $H$ excitation, under the enforced fields

$\begin{bmatrix}
H_b(t) \\
th
\end{bmatrix} = \begin{bmatrix}
0 & -400 & 400 & -150 & 200 & -150 \\
0 & 0.02 & 0.03 & 0.0368 & 0.0412 & 0.0455 \\
0.0498 & 0.0542 & 0.0585 & 0.0660 & 0.071 
\end{bmatrix}$

respectively. Figures 13 and 14 show a good agreement between the measured and the numerically obtained loops.
5. Concluding remarks

In this paper we described a complete mathematical approach to the evaluation of the total iron losses in non-oriented steel laminations for arbitrary excitations. The Preisach hysteresis model, both the CPM and the RPM, is coupled with the magnetodynamic field calculations. The discretization of the resulting boundary value problem relies on a modified FE–FD technique.

The model allows one to predict and explain the hysteresis behaviour of most magnetic materials. In the text we considered two materials. Material 1 is a material with high carbide content and large mechanical stresses; material 2 is obtained from material 1 by the process of de-carbonizing and stress-relieving. By comparison of numerical experiments and measurements the hysteresis behaviour of material 1 was found to be modelled accurately when the CPM is used, whereas for material 2 the RPM must be invoked.

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