Definition:

$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) = \frac{\varphi_1(||A \diamondsuit_\Gamma A||, ||B \diamondsuit_\Gamma B||, ||A \diamondsuit_\Gamma B||)}{\varphi_2(||A \diamondsuit_\Gamma A||, ||B \diamondsuit_\Gamma B||, ||A \diamondsuit_\Gamma B||)}$$

for all $A, B \in \mathcal{F}(X)$, with φ_1 and φ_2 two $[0,1]^3 \to \mathbb{R}$ mappings that are increasing in their first and second argument, and $\diamondsuit_\Gamma$ an infix notation for the pointwise extension of a binary aggregation operator Γ.

Potential properties:

$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is } [0,1]\text{-valued} \iff (\forall A,B \in \mathcal{F}(X))(0 \le \mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) \le 1)$$
$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is reflexive} \iff (\forall A,B \in \mathcal{F}(X))(\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) = 1 \Leftarrow A = B)$$
$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is coreflexive} \iff (\forall A,B \in \mathcal{F}(X))(\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) = 1 \Rightarrow A = B)$$
$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is strong reflexive} \iff (\forall A,B \in \mathcal{F}(X))(\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) = 1 \Leftarrow A \subseteq B \vee B \subseteq A)$$
$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is weak coreflexive} \iff (\forall A,B \in \mathcal{F}(X))(\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) = 1 \Rightarrow A \subseteq B \vee B \subseteq A)$$
$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is inclusive} \iff (\forall A,B \in \mathcal{F}(X))(\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) = 1 \Leftarrow A \subseteq B)$$
$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is coinclusive} \iff (\forall A,B \in \mathcal{F}(X))(\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) = 1 \Rightarrow A \subseteq B)$$
$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is } \cap_\mathcal{T}\text{-exclusive} \iff (\forall A,B \in \mathcal{F}(X))(\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) = 0 \Leftarrow A \cap_\mathcal{T} B = \emptyset)$$
$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is } \cap_\mathcal{T}\text{-coexclusive} \iff (\forall A,B \in \mathcal{F}(X))(\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) = 0 \Rightarrow A \cap_\mathcal{T} B = \emptyset)$$
$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is symmetric} \iff (\forall A,B \in \mathcal{F}(X))(\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) = \mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(B,A))$$
$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is left-restrictable} \iff (\forall A,B \in \mathcal{F}(X))(\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) = \mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A/_{\operatorname{supp} A}, B/_{\operatorname{supp} A}))$$
$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is right-restrictable} \iff (\forall A,B \in \mathcal{F}(X))(\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) = \mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A/_{\operatorname{supp} B}, B/_{\operatorname{supp} B}))$$
$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is weak left-restrictable} \iff (\forall A,B \in \mathcal{F}(X))(\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) \le \mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A/_{\operatorname{supp} A}, B/_{\operatorname{supp} A}))$$
$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is weak right-restrictable} \iff (\forall A,B \in \mathcal{F}(X))(\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A,B) \le \mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2}(A/_{\operatorname{supp} B}, B/_{\operatorname{supp} B}))$$

General constraints:

$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is } [0,1]\text{-valued} \Leftarrow (\forall x,y,z \in [0,1])(0 \le \varphi_1(x,y,z) \le \varphi_2(x,y,z))$$
$$\mathcal{M}^{\Gamma}_{\varphi_1,\varphi_2} \text{ is reflexive} \Leftarrow (\forall x \in [0,1])(\varphi_1(x,x,x) = \varphi_2(x,x,x))$$

Constraints for the case $\Gamma = \mathcal{T}$, with $\mathcal{T}$ an arbitrary t-norm:

$$\mathcal{M}^{\mathcal{T}}_{\varphi_1,\varphi_2} \text{ is strong reflexive} \Leftarrow (\forall x,y,z \in [0,1])(\min(x,y) \le z \le \max(x,y) \Rightarrow \varphi_1(x,y,z) = \varphi_2(x,y,z))$$
$$\mathcal{M}^{\mathcal{T}}_{\varphi_1,\varphi_2} \text{ is inclusive} \Leftarrow (\forall x,y,z \in [0,1])(x \le z \le y \Rightarrow \varphi_1(x,y,z) = \varphi_2(x,y,z))$$
$$\mathcal{M}^{\mathcal{T}}_{\varphi_1,\varphi_2} \text{ is } \cap_\mathcal{T}\text{-exclusive} \Leftarrow (\forall x,y \in [0,1])(\varphi_1(x,y,0) = 0)$$
$$\mathcal{M}^{\mathcal{T}}_{\varphi_1,\varphi_2} \text{ is symmetric} \Leftarrow (\forall x,y,z \in [0,1])(\varphi_1(x,y,z) = \varphi_1(y,x,z) \wedge \varphi_2(x,y,z) = \varphi_2(y,x,z))$$
$$\mathcal{M}^{\mathcal{T}}_{\varphi_1,\varphi_2} \text{ is left-restrictable} \Leftarrow (\forall x,z \in [0,1])(\forall u,v \in [0,1])(\varphi_1(x,u,z) = \varphi_1(x,v,z) \wedge \varphi_2(x,u,z) = \varphi_2(x,v,z))$$
$$\mathcal{M}^{\mathcal{T}}_{\varphi_1,\varphi_2} \text{ is right-restrictable} \Leftarrow (\forall y,z \in [0,1])(\forall u,v \in [0,1])(\varphi_1(u,y,z) = \varphi_1(v,y,z) \wedge \varphi_2(u,y,z) = \varphi_2(v,y,z))$$
$$\mathcal{M}^{\mathcal{T}}_{\varphi_1,\varphi_2} \text{ is weak left-restr.} \Leftarrow (\forall x,z \in [0,1])(\forall u,v \in [0,1])(\varphi_1(x,u,z) = \varphi_1(x,v,z))$$
$$\mathcal{M}^{\mathcal{T}}_{\varphi_1,\varphi_2} \text{ is weak right-restr.} \Leftarrow (\forall y,z \in [0,1])(\forall u,v \in [0,1])(\varphi_1(u,y,z) = \varphi_1(v,y,z))$$

Constraints for the case $\Gamma = T_\mathrm{P}$, with T_P the product:

$$\mathcal{M}^{T_\mathrm{P}}_{\varphi_1,\varphi_2} \text{ is } [0,1]\text{-valued} \Leftarrow (\forall x,y,z \in [0,1])(z \le \sqrt{x \cdot y} \Rightarrow 0 \le \varphi_1(x,y,z) \le \varphi_2(x,y,z))$$
$$\mathcal{M}^{T_\mathrm{P}}_{\varphi_1,\varphi_2} \text{ is } \cap_{T_\mathrm{P}}\text{-coexclusive} \Leftarrow (\forall x,y,z \in]0,1])(z \le \sqrt{x \cdot y} \Rightarrow \varphi_1(x,y,z) > 0)$$

Constraints for the case $\Gamma = T_\mathrm{M}$, with T_M the minimum:

$$\mathcal{M}^{T_\mathrm{M}}_{\varphi_1,\varphi_2} \text{ is } [0,1]\text{-valued} \Leftarrow (\forall x,y,z \in [0,1])(z \le \min(x,y) \Rightarrow 0 \le \varphi_1(x,y,z) \le \varphi_2(x,y,z))$$
$$\mathcal{M}^{T_\mathrm{M}}_{\varphi_1,\varphi_2} \text{ is coreflexive} \Leftarrow (\forall x,y,z \in [0,1])(z < \max(x,y) \wedge z \le \min(x,y) \Rightarrow \varphi_1(x,y,z) < \varphi_2(x,y,z))$$
$$\mathcal{M}^{T_\mathrm{M}}_{\varphi_1,\varphi_2} \text{ is strong reflexive} \Leftarrow (\forall x,y \in [0,1])(\varphi_1(x,y,\min(x,y)) = \varphi_2(x,y,\min(x,y)))$$
$$\mathcal{M}^{T_\mathrm{M}}_{\varphi_1,\varphi_2} \text{ is weak coreflexive} \Leftarrow (\forall x,y,z \in [0,1])(z < \min(x,y) \Rightarrow \varphi_1(x,y,z) < \varphi_2(x,y,z))$$
$$\mathcal{M}^{T_\mathrm{M}}_{\varphi_1,\varphi_2} \text{ is inclusive} \Leftarrow (\forall x,y \in [0,1])(x \le y \Rightarrow \varphi_1(x,y,x) = \varphi_2(x,y,x))$$
$$\mathcal{M}^{T_\mathrm{M}}_{\varphi_1,\varphi_2} \text{ is coinclusive} \Leftarrow (\forall x,y,z \in [0,1])(z < x \wedge z \le y \Rightarrow \varphi_1(x,y,z) < \varphi_2(x,y,z))$$
$$\mathcal{M}^{T_\mathrm{M}}_{\varphi_1,\varphi_2} \text{ is } \cap_{T_\mathrm{M}}\text{-coexclusive} \Leftarrow (\forall x,y,z \in]0,1])(z \le \min(x,y) \Rightarrow \varphi_1(x,y,z) > 0)$$