New Mathematics and Natural Computation

A BIPARAMETRIC FAMILY OF CARDINALITY-BASED FUZZY SIMILARITY MEASURES

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We present a systematic way of constructing and analysing fuzzy similarity measures based on cardinality. This is achieved by introducing a general form for such measures, that depends on two parameters. We demonstrate that this general form includes several existing families of fuzzy similarity measures. Moreover, we show that certain properties can be ensured by imposing simple constraints on the parameters. In particular, we present constraints that ensure several forms of restrictability, which allow to reduce the calculation time in practical implementations. To conclude, we illustrate the presented technique by using it to analyse some well-known fuzzy similarity measures.

Keywords: Fuzzy similarity measures; fuzzy cardinality.

1. Introduction

Similarity is an important concept in many research areas. Information retrieval, data mining and machine learning are just a few examples. In particular, it can also play a crucial role in applications that automatically analyse, retrieve or organize multimedia data (e.g. Ref. 1, 2, 3). The exponential growth of the amount of multimedia data in our daily lifes, increases the need for such intelligent multimedia processing applications drastically.

Generally speaking, a similarity measure is a means to express the similarity between two given objects. Such measures are usually constructed by following a feature-based approach. Objects are represented by real-valued feature vectors, and the similarity is calculated by comparing these vectors. More often than not, a traditional distance measure is used for comparing the feature vectors. However, a real-valued vector can be identified with a fuzzy set by means of normalization. Consequently, fuzzy similarity measures can be used to compare the feature vectors. The image quality measures constructed in Ref. 4 are examples of applications that adopt this idea.

In this paper, we present a parametrized family of cardinality-based fuzzy similarity measures. This family allows to construct a fuzzy similarity measure by choosing two mappings that satisfy particular constraints. We also show that certain properties of the generated measures can be ensured by imposing additional constraints. Consequently, our family of cardinality-based fuzzy similarity measures can be used to construct fuzzy similarity measures that meet the requirements of a particular application.

2. Related work

Tversky's ratio model of similarity covers several of the existing rational set-theoretic similarity measures.[5] A fuzzy generalization of this model was presented in Ref. 6. Although it is not strictly necessary, the ratio model is usually based on cardinality. In Ref. 7, a more general way of generating rational cardinality-based similarity measures was presented. Fuzzification schemes for these measures were introduced in Ref. 8. A promising recent advancement concerning this research, is the development of meta-theorems that allow to systematically ensure properties of the generated measures.[9]

Egghe et al. introduced a general form for cardinality-based fuzzy similarity measures that is not restricted to rational measures.[10] In this paper we generalize this form, and we reformulate some of the constraints that ensure certain properties of the constructed measures. Furthermore, we also introduce some additional constraints for other properties. In particalur, we present constraints for ensuring properties that allow to reduce the calculation time in practical implementations. The general form that we present in this paper, covers all of the measures generated by the above-mentioned techniques for constructing rational similarity measures.

3. Preliminary notions from fuzzy set theory

A fuzzy set A in a universe X is a $X \to [0,1]$ mapping that associates with each element x from the universe X a degree of membership $A(x)$. We use the notation $\mathcal{F}(X)$ for the class of fuzzy sets in X. For two fuzzy sets A and B in X, we write $A \subseteq B$ iff $A(x) \leq B(x)$ for all $x \in X$, and $A = B$ iff $A \subseteq B \wedge B \subseteq A$. The support of a fuzzy set A in X is given by: $\operatorname{supp} A = \{x \in X \mid A(x) > 0\}$.

Crisp sets are special fuzzy sets, since they can be represented by a characteristic $X \to \{0,1\}$ mapping. To avoid notational clutter, we reuse the name of a crisp set for its characteristic mapping. In particular, X denotes both the universe and a mapping that associates 1 with each element of the universe. Also, $\emptyset$ denotes the empy set as well as the $X \to [0,1]$ mapping given by: $\emptyset(x) = 0$, for all $x \in X$.

3.1. *Operations on fuzzy sets*

A negator $\mathcal{N}$ is a decreasing $[0,1] \to [0,1]$ mapping satisfying $\mathcal{N}(0) = 1$ and $\mathcal{N}(1) = 0$. An increasing, associative and commutative $[0,1]^2 \to [0,1]$ mapping is called a t-norm $\mathcal{T}$ if it satisfies $\mathcal{T}(x,1) = x$ for all $x \in [0,1]$, and a t-conorm $\mathcal{S}$ if it satisfies $\mathcal{S}(x,0) = x$ for all $x \in [0,1]$. It is well-known that the classical set-theoretic