ISIPTA ’13

Allowing for probability zero in…


Allowing for probability zero in credal networks under epistemic irrelevance

credal networks under epistemic irrelevance


Credal networks under epistemic irrelevance using sets of desirable gambles

…using sets of desirable gambles

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UNIVERSITEIT GENT

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Credal networks: **basic setup**

X_5 X_7

X_3

X_6

X_2

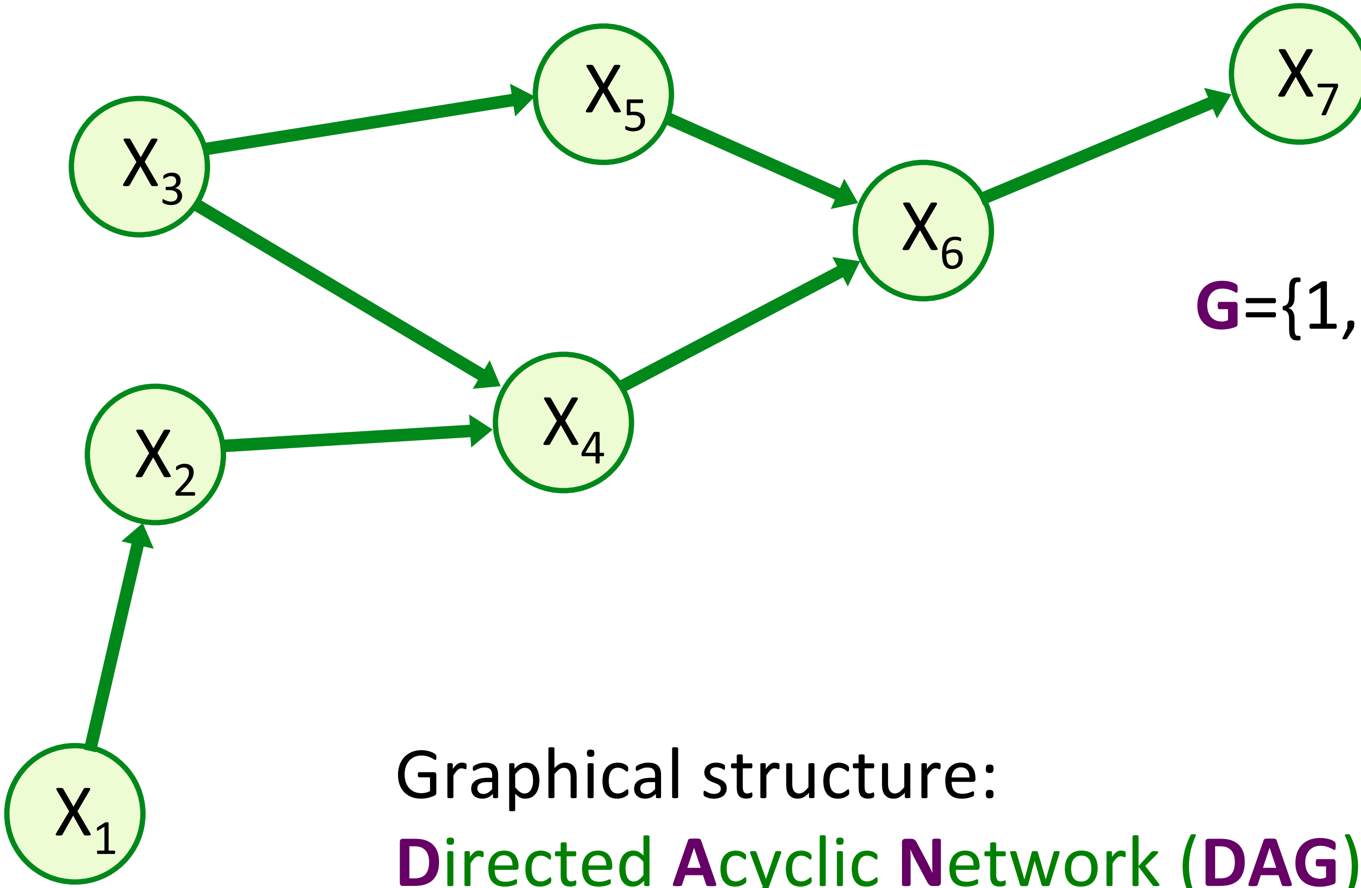
$X_4 = x_4 \in \mathcal{X}_4$

A finite number of **variables**

X_1

Every **variable X_s** takes values x_s in some finite non-empty set $\mathcal{X}_s$

Credal networks: basic setup

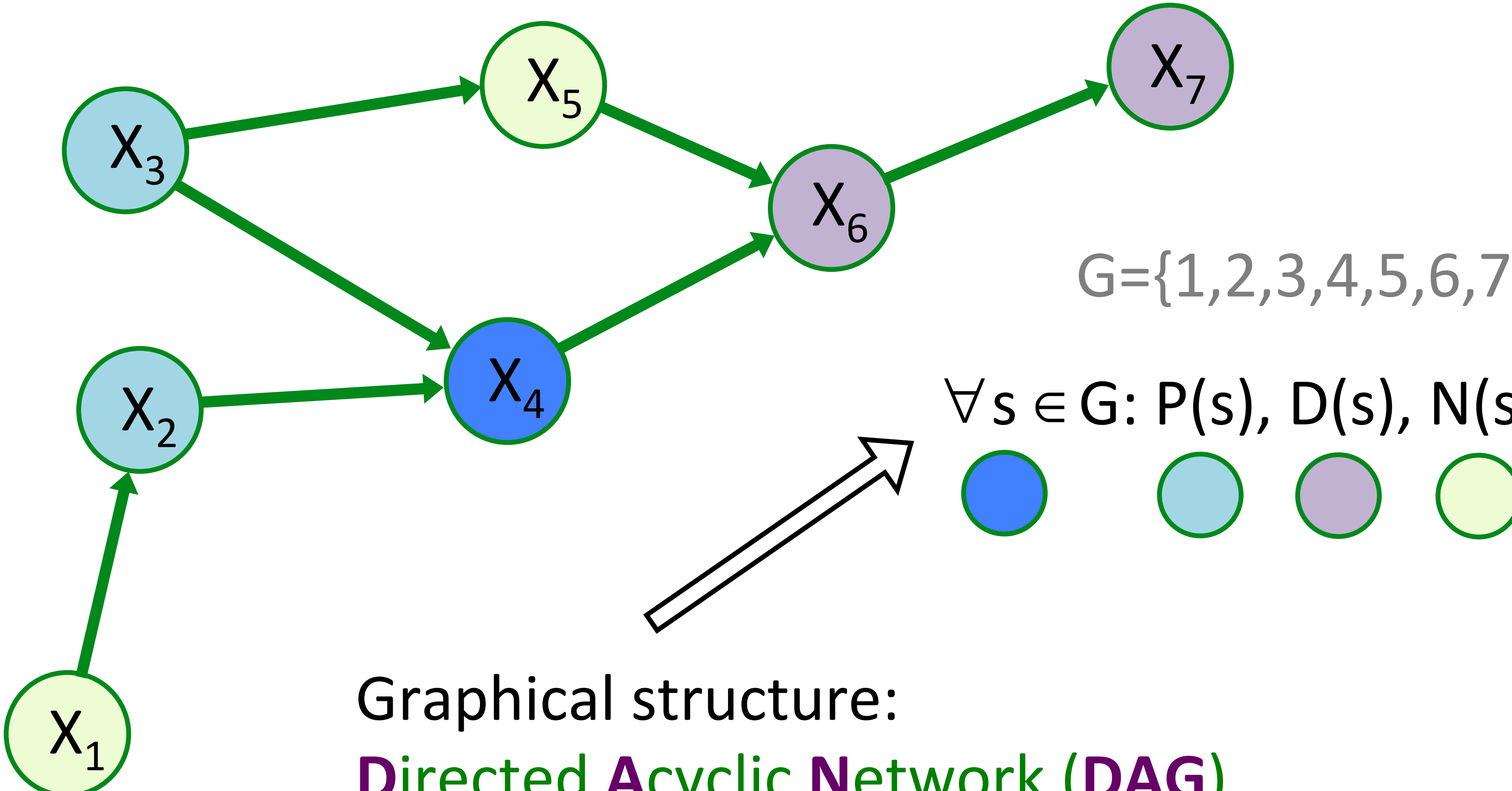


G={1,2,3,4,5,6,7}

Graphical structure:

Directed **A**cyclic **N**etwork (**DAG**)

Credal networks: basic setup



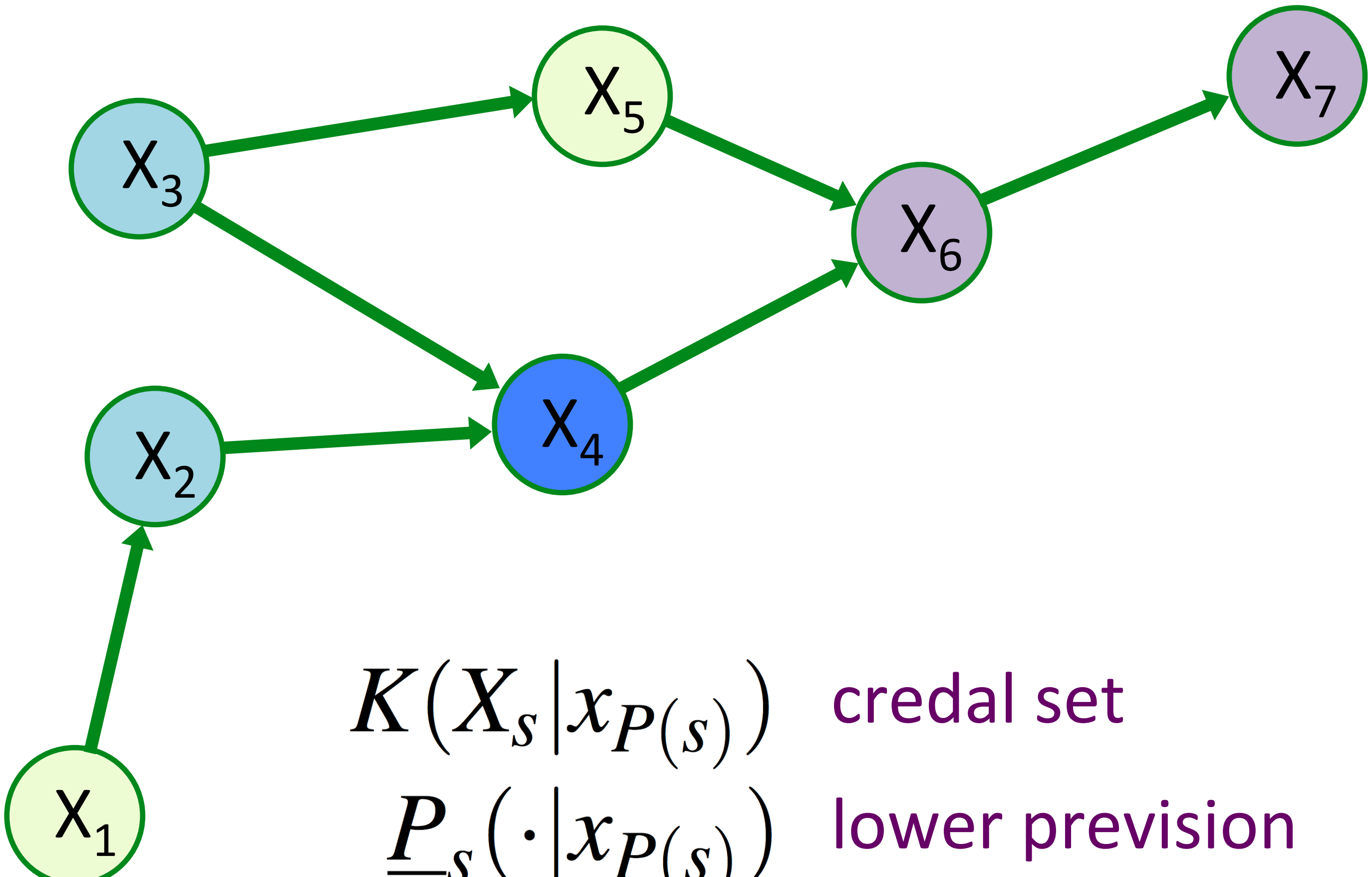
G={1,2,3,4,5,6,7}

$\forall s \in G$: P(s), D(s), N(s)

Graphical structure:

Directed **A**cyclic **N**etwork (**DAG**)

Credal networks: local uncertainty models

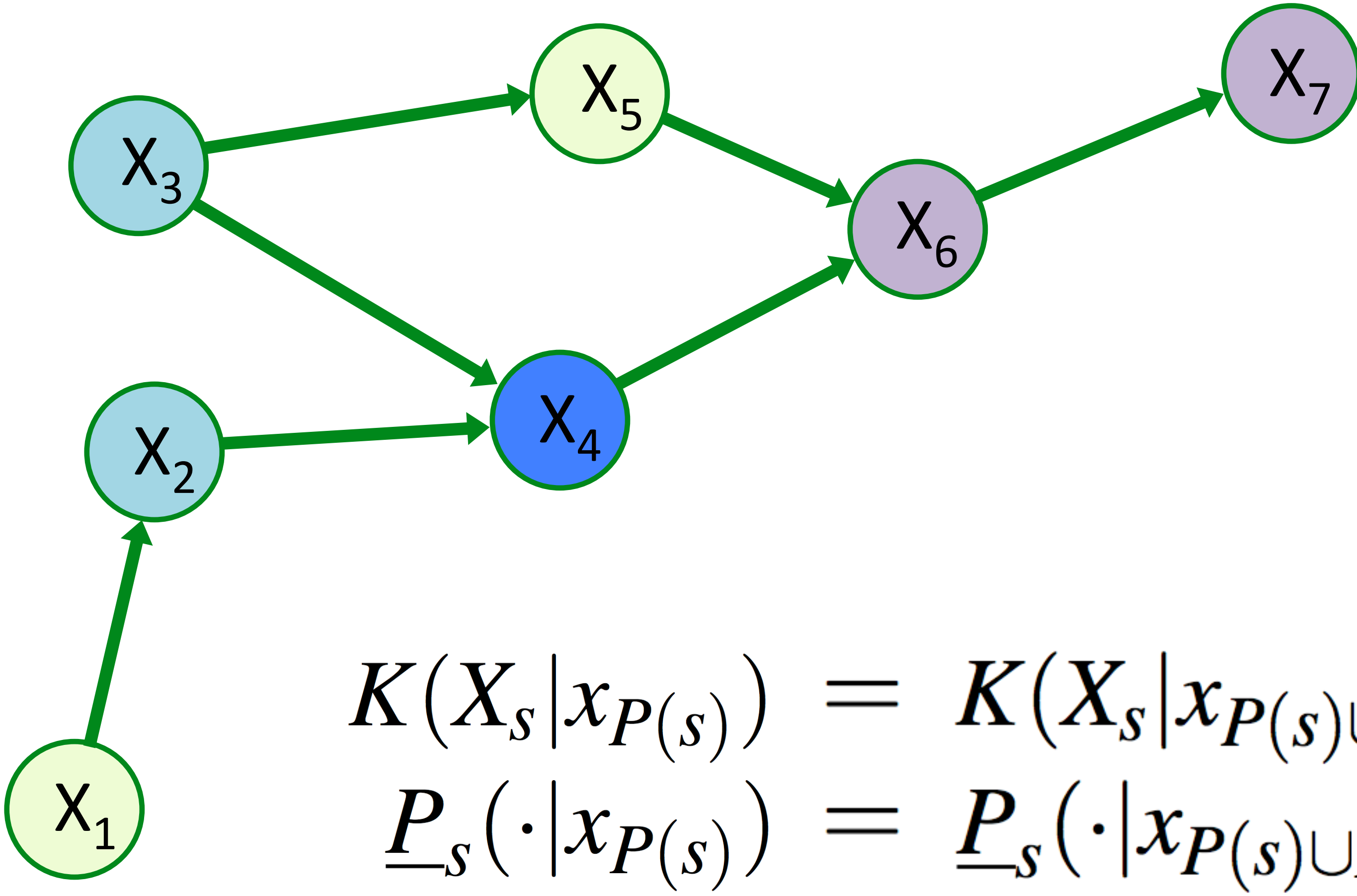


$$K(X_s | x_{P(s)})$$ credal set

$$\underline{P}_s(\cdot | x_{P(s)})$$ lower prevision

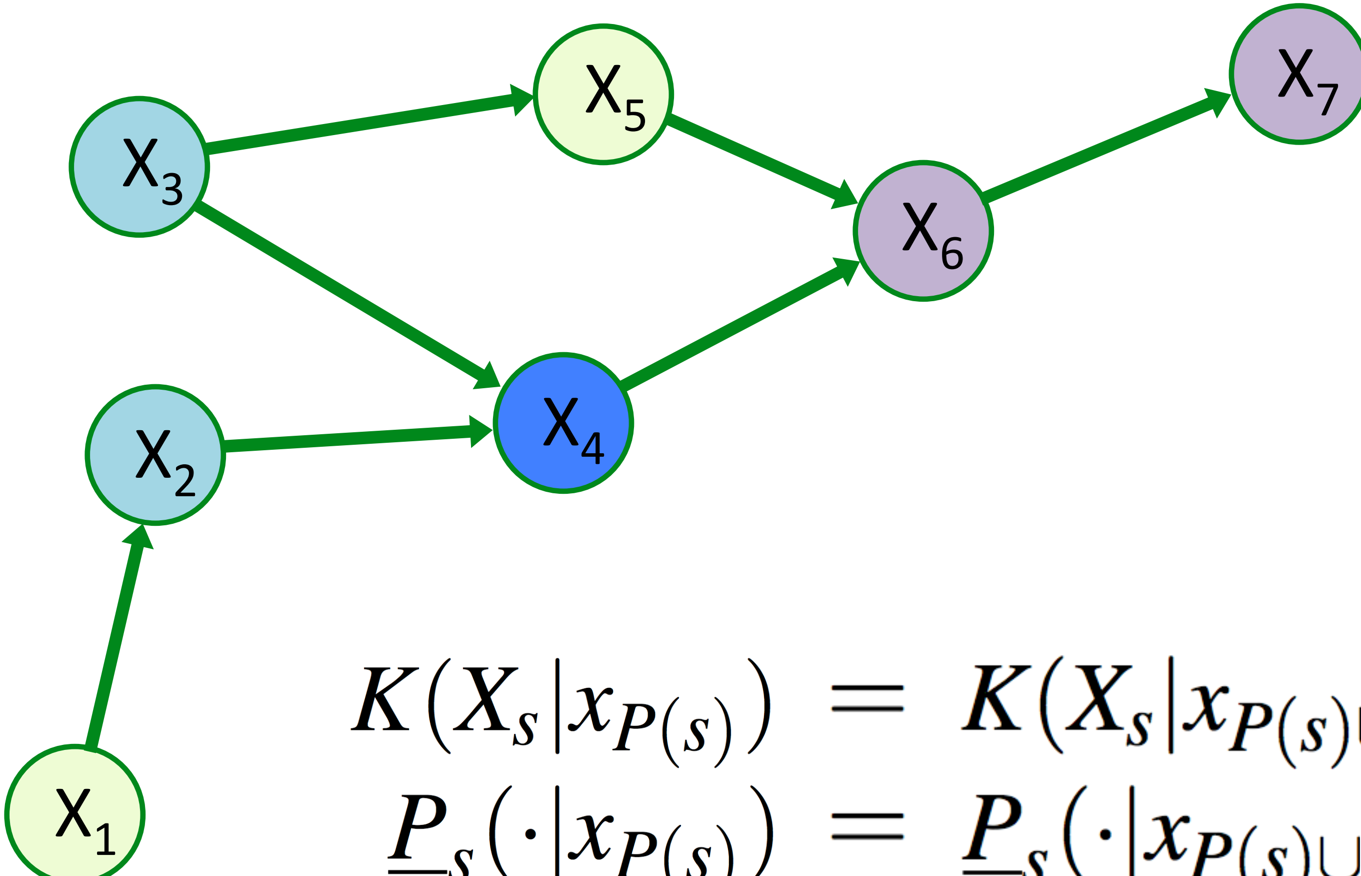
$$\mathscr{D}_{s \rfloor x_{P(s)}}$$ set of desirable gambles

Credal networks: epistemic irrelevance



$$K(X_s | x_{P(s)}) = K(X_s | x_{P(s) \cup N(s)})$$
$$\underline{P}_s(\cdot | x_{P(s)}) = \underline{P}_s(\cdot | x_{P(s) \cup N(s)})$$
$$\mathscr{D}_{s \rfloor x_{P(s)}} = \mathscr{D}_{s \rfloor x_{P(s) \cup N(s)}}$$

Credal networks: a joint model

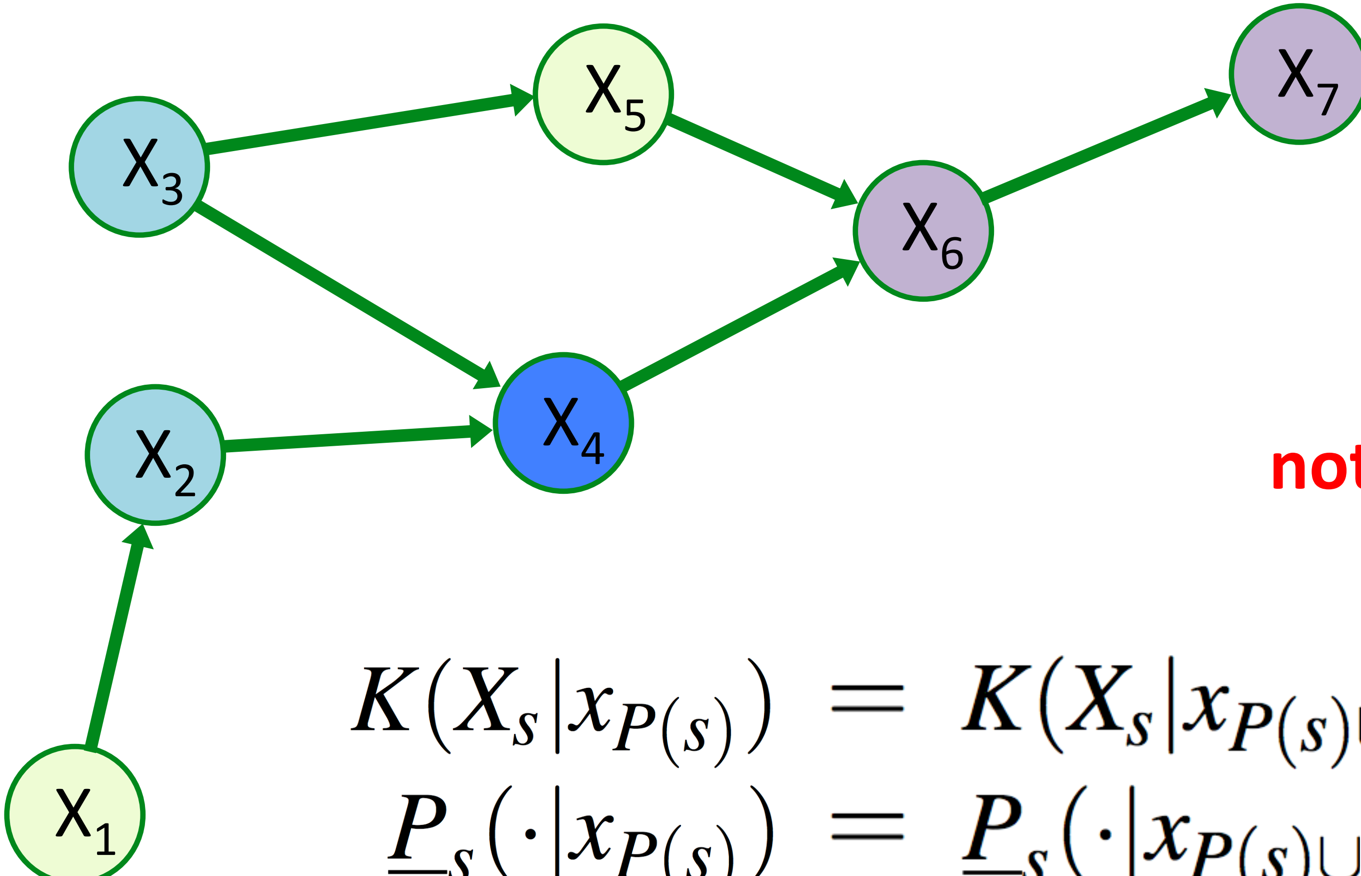


$$K(X_G)$$
$$\underline{P}_G$$
$$\mathscr{D}_G$$

?

$$K(X_s|x_{P(s)}) = K(X_s|x_{P(s)\cup N(s)})$$
$$\underline{P}_s(\cdot|x_{P(s)}) = \underline{P}_s(\cdot|x_{P(s)\cup N(s)})$$
$$\mathscr{D}_{s\rfloor x_{P(s)}} = \mathscr{D}_{s\rfloor x_{P(s)\cup N(s)}}$$

Credal networks: a joint model

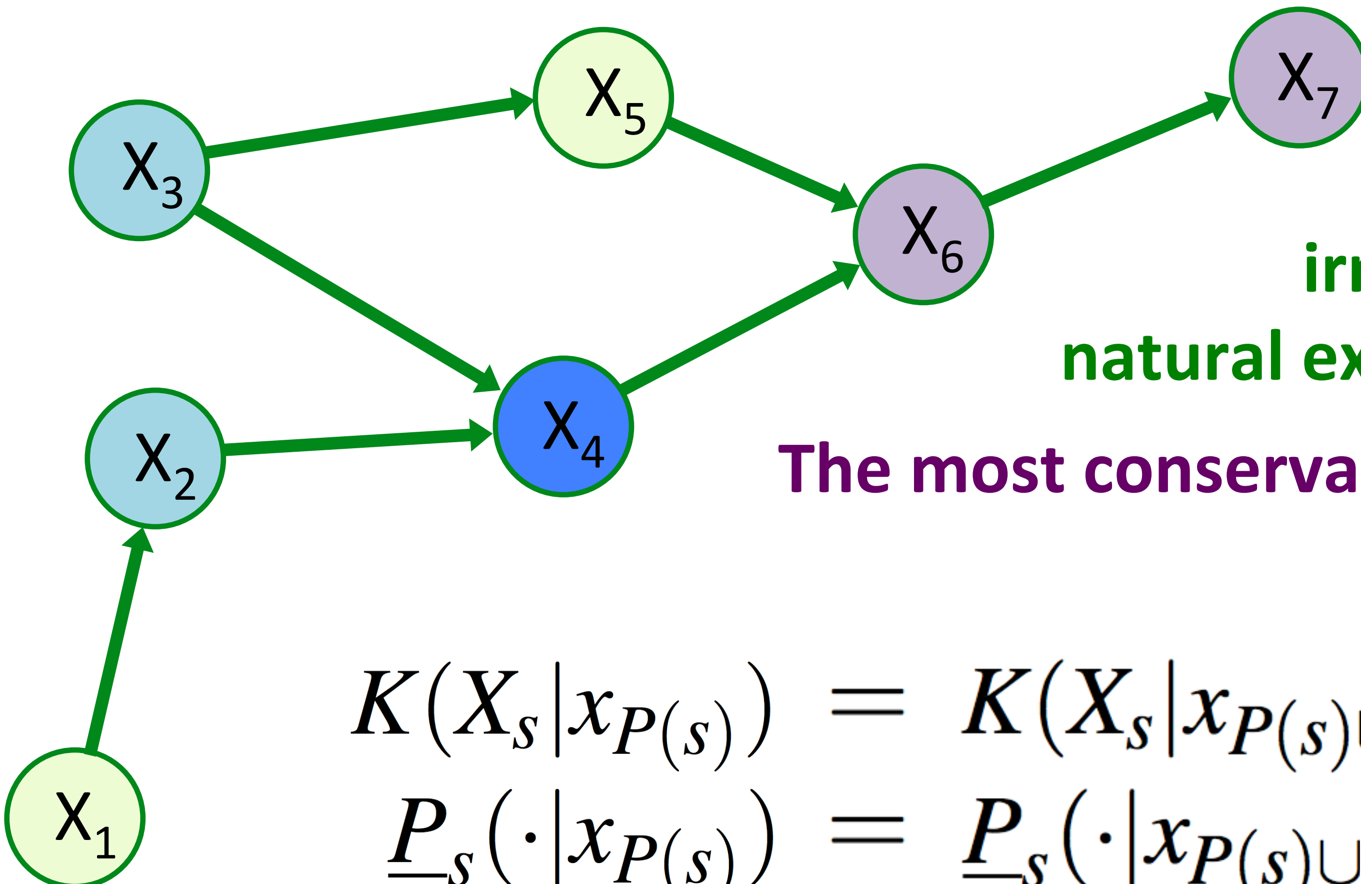


$$K(X_G)$$
$$\underline{P}_G$$
$$\mathscr{D}_G$$

not unique !

$$
\begin{aligned}
K(X_s|x_{P(s)}) &= K(X_s|x_{P(s)\cup N(s)}) \\
\underline{P}_s(\cdot|x_{P(s)}) &= \underline{P}_s(\cdot|x_{P(s)\cup N(s)}) \\
\mathscr{D}_{s\rfloor x_{P(s)}} &= \mathscr{D}_{s\rfloor x_{P(s)\cup N(s)}}
\end{aligned}
$$

Credal networks: a joint model



$K^{\mathrm{irr}}(X_G)$

$\underline{P}_G^{\mathrm{irr}}$

$\mathscr{D}_G^{\mathrm{irr}}$

irrelevant natural extension

The most conservative one !

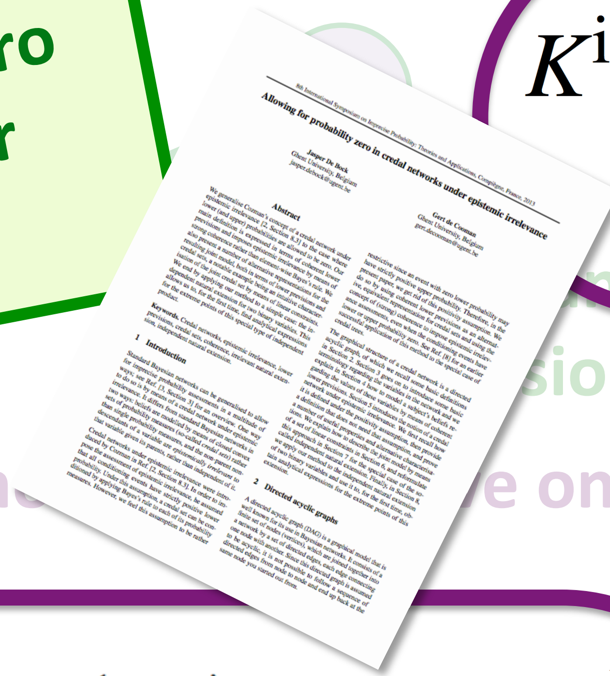
$$K(X_s|x_{P(s)}) = K(X_s|x_{P(s)\cup N(s)})$$

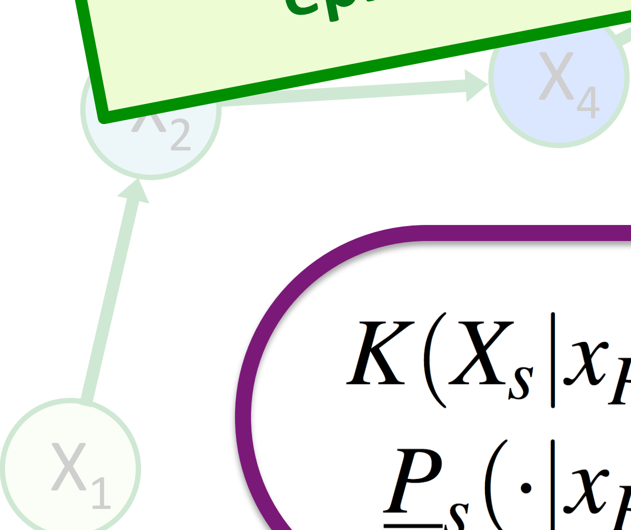
$$\underline{P}_s(\cdot|x_{P(s)}) = \underline{P}_s(\cdot|x_{P(s)\cup N(s)})$$

$$\mathscr{D}_{s\rfloor x_{P(s)}} = \mathscr{D}_{s\rfloor x_{P(s)\cup N(s)}}$$

Credal networks: a joint model

Allowing for probability zero in credal networks under epistemic irrelevance





$K^{\mathrm{irr}}(X_G)$

$\underline{P}^{\mathrm{irr}}_G$

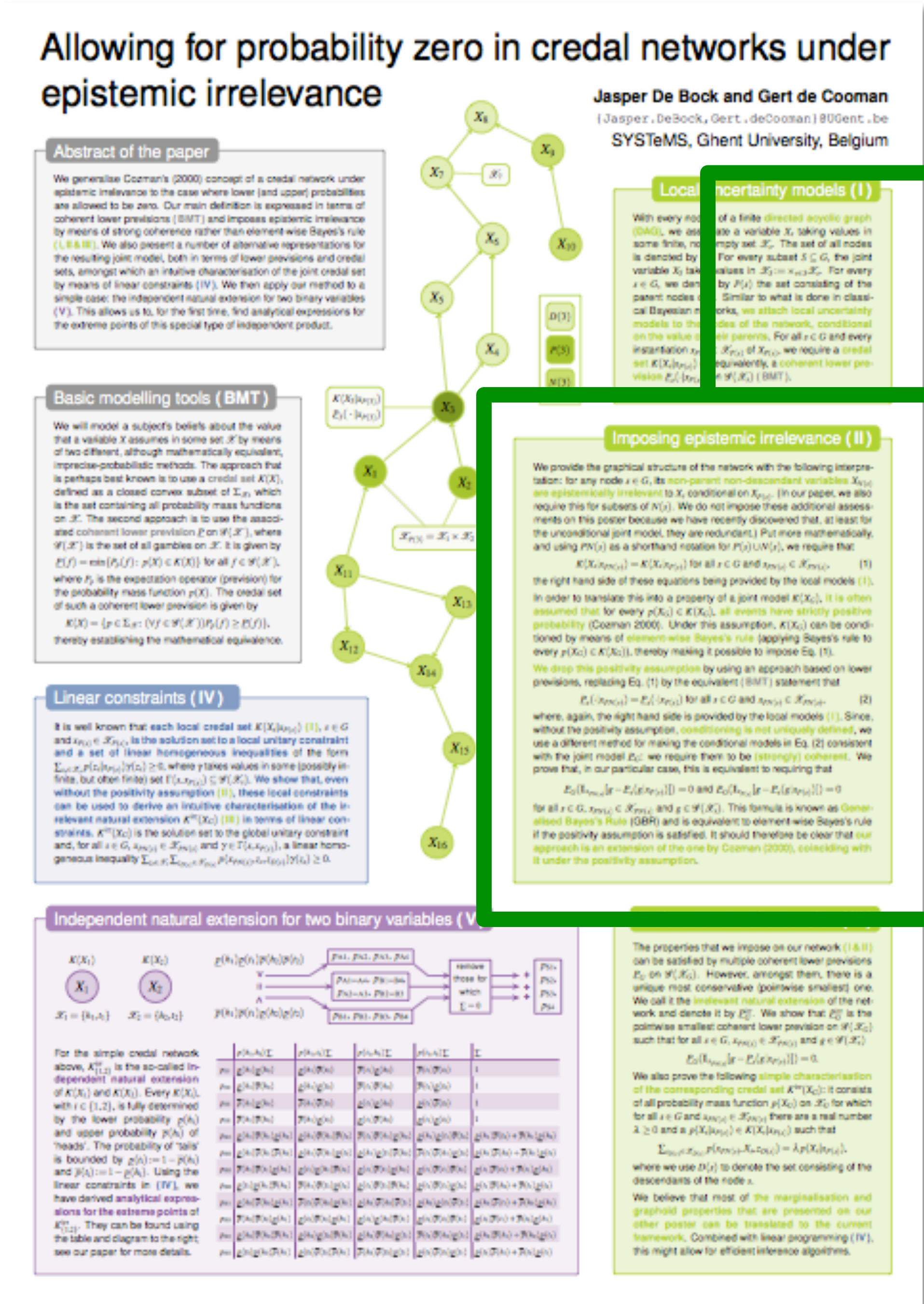
$\mathscr{D}^{\mathrm{irr}}_G$

The m… e one !

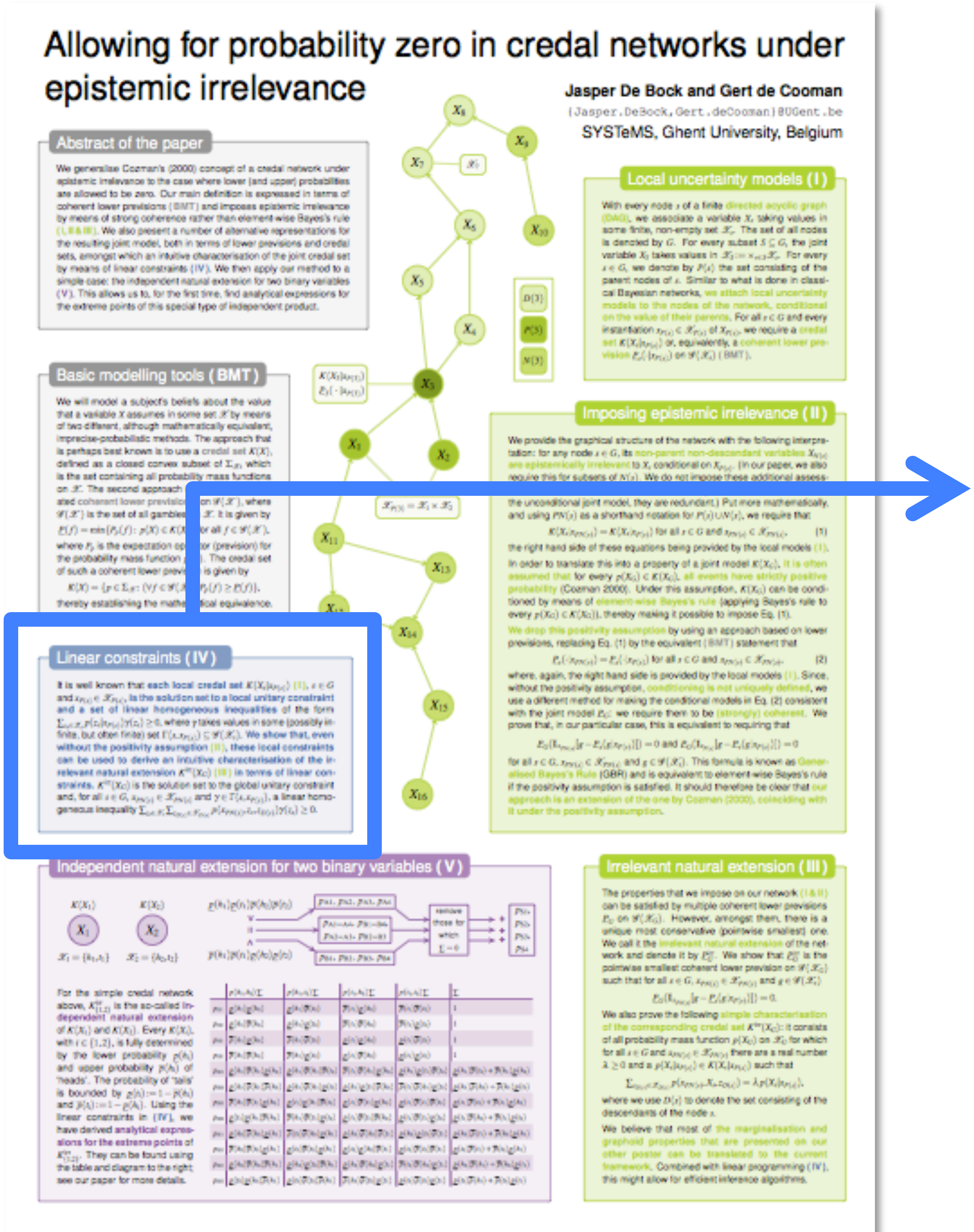
$$K(X_s|x_{P(s)}) = K(X_s|x_{P(s)\cup N(s)})$$

$$\underline{P}_s(\cdot|x_{P(s)}) = \underline{P}_s(\cdot|x_{P(s)\cup N(s)})$$

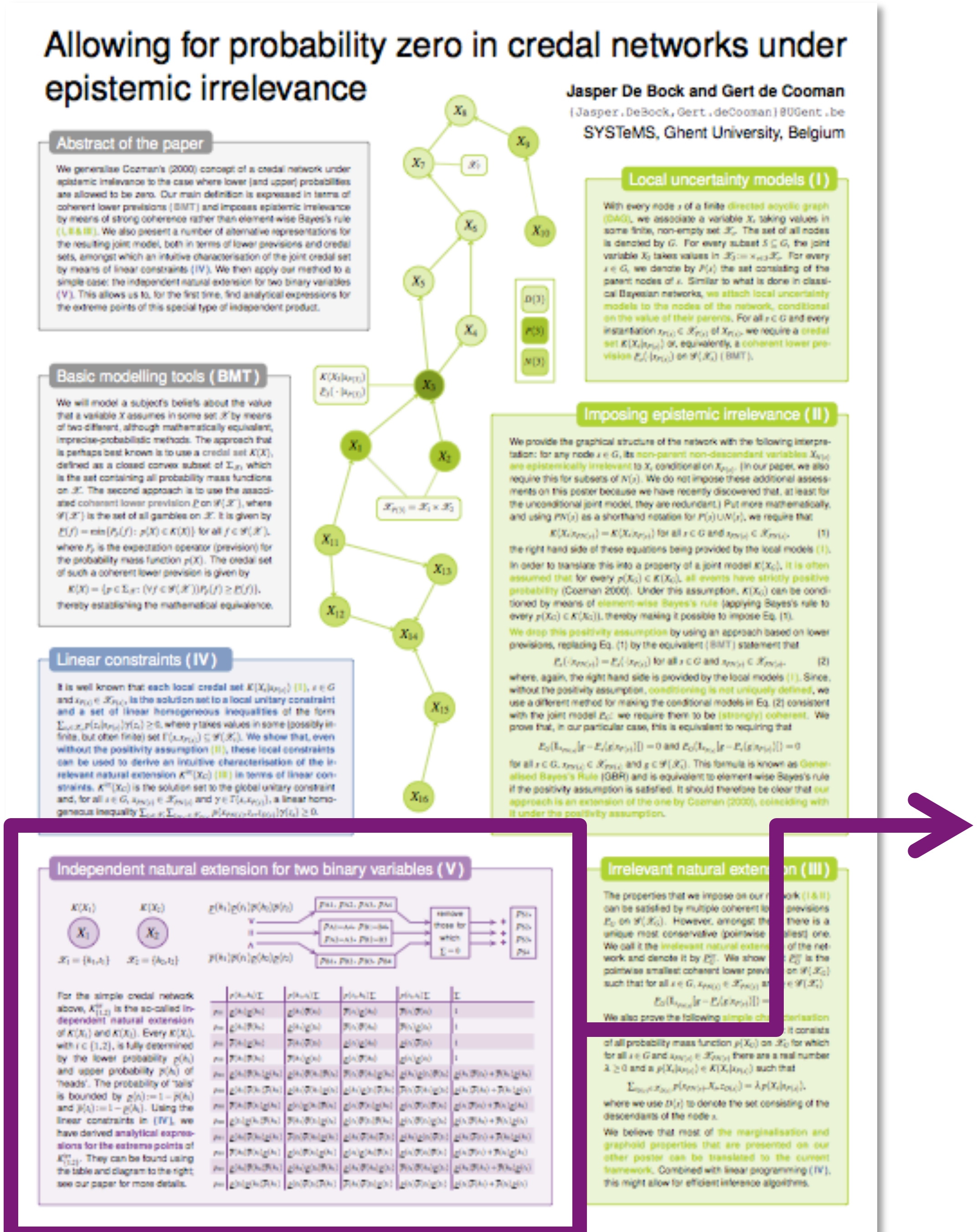
$$\mathscr{D}_s\rfloor x_{P(s)} = \mathscr{D}_s\rfloor x_{P(s)\cup N(s)}$$



- **Definition by means of strong coherence**



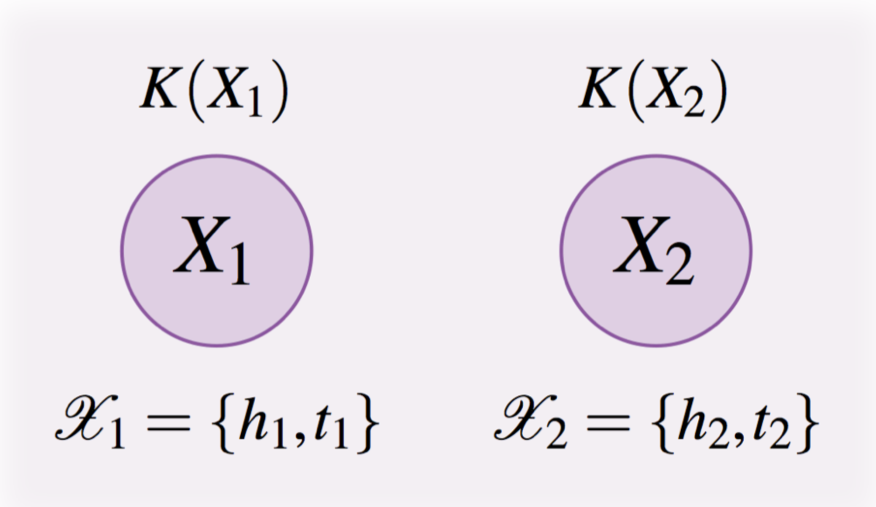
- Definition by means of strong coherence
- Description in terms of linear constraints



- **Definition by means of strong coherence**
- **Description in terms of linear constraints**
- **Independent natural extension for two binary variables**

Independent natural extension
for two binary variables

$$K^{\text{irr}}(X_G) = K(X_1) \otimes K(X_2)$$



$$\overline{p}(t_1)p(h_1,h_2) - \underline{p}(h_1)p(t_1,h_2) \geq 0$$
$$-\underline{p}(t_1)p(h_1,h_2) + \overline{p}(h_1)p(t_1,h_2) \geq 0$$
$$\overline{p}(t_1)p(h_1,t_2) - \underline{p}(h_1)p(t_1,t_2) \geq 0$$
$$-\underline{p}(t_1)p(h_1,t_2) + \overline{p}(h_1)p(t_1,t_2) \geq 0$$
$$\overline{p}(t_2)p(h_1,h_2) - \underline{p}(h_2)p(h_1,t_2) \geq 0$$
$$-\underline{p}(t_2)p(h_1,h_2) + \overline{p}(h_2)p(h_1,t_2) \geq 0$$
$$\overline{p}(t_2)p(t_1,h_2) - \underline{p}(h_2)p(t_1,t_2) \geq 0$$
$$-\underline{p}(t_2)p(t_1,h_2) + \overline{p}(h_2)p(t_1,t_2) \geq 0$$

$$K(X_i) = \{p \in \Sigma_{\mathscr{X}_i} : p(h_i) \in [\underline{p}(h_i), \overline{p}(h_i)]\}$$

$$\overline{p}(t_i) := 1 - \underline{p}(h_i)$$
$$\underline{p}(t_i) := 1 - \overline{p}(h_i)$$

Independent natural extension for two binary variables

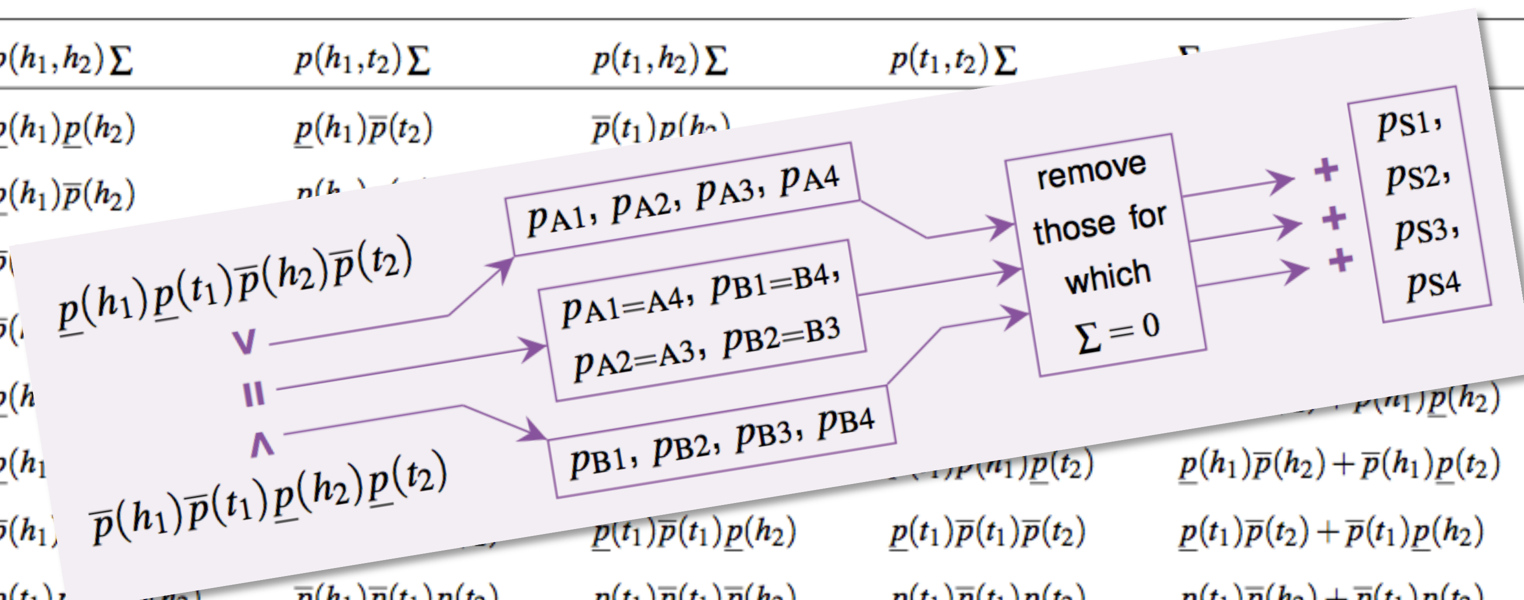
Analytical expressions for the extreme points

	$p(h_1,h_2)\Sigma$	$p(h_1,t_2)\Sigma$	$p(t_1,h_2)\Sigma$	$p(t_1,t_2)\Sigma$	Σ
p_{S1}	$\underline{p}(h_1)\underline{p}(h_2)$	$\underline{p}(h_1)\overline{p}(t_2)$	$\overline{p}(t_1)\underline{p}(h_2)$	$\overline{p}(t_1)\overline{p}(t_2)$	1
p_{S2}	$\underline{p}(h_1)\overline{p}(h_2)$	$\underline{p}(h_1)\underline{p}(t_2)$	$\overline{p}(t_1)\overline{p}(h_2)$	$\overline{p}(t_1)\underline{p}(t_2)$	1
p_{S3}	$\overline{p}(h_1)\underline{p}(h_2)$	$\overline{p}(h_1)\overline{p}(t_2)$	$\underline{p}(t_1)\underline{p}(h_2)$	$\underline{p}(t_1)\overline{p}(t_2)$	1
p_{S4}	$\overline{p}(h_1)\overline{p}(h_2)$	$\overline{p}(h_1)\underline{p}(t_2)$	$\underline{p}(t_1)\overline{p}(h_2)$	$\underline{p}(t_1)\underline{p}(t_2)$	1
p_{A1}	$\underline{p}(h_1)\overline{p}(h_1)\underline{p}(h_2)$	$\underline{p}(h_1)\overline{p}(h_1)\overline{p}(t_2)$	$\overline{p}(t_1)\overline{p}(h_1)\underline{p}(h_2)$	$\underline{p}(h_1)\underline{p}(t_1)\overline{p}(t_2)$	$\underline{p}(h_1)\overline{p}(t_2)+\overline{p}(h_1)\underline{p}(h_2)$
p_{A2}	$\underline{p}(h_1)\overline{p}(h_1)\overline{p}(h_2)$	$\underline{p}(h_1)\overline{p}(h_1)\underline{p}(t_2)$	$\underline{p}(h_1)\underline{p}(t_1)\overline{p}(h_2)$	$\overline{p}(t_1)\overline{p}(h_1)\underline{p}(t_2)$	$\underline{p}(h_1)\overline{p}(h_2)+\overline{p}(h_1)\underline{p}(t_2)$
p_{A3}	$\overline{p}(h_1)\overline{p}(t_1)\underline{p}(h_2)$	$\underline{p}(t_1)\underline{p}(h_1)\overline{p}(t_2)$	$\underline{p}(t_1)\overline{p}(t_1)\underline{p}(h_2)$	$\underline{p}(t_1)\overline{p}(t_1)\overline{p}(t_2)$	$\underline{p}(t_1)\overline{p}(t_2)+\overline{p}(t_1)\underline{p}(h_2)$
p_{A4}	$\underline{p}(t_1)\underline{p}(h_1)\overline{p}(h_2)$	$\overline{p}(h_1)\overline{p}(t_1)\underline{p}(t_2)$	$\underline{p}(t_1)\overline{p}(t_1)\overline{p}(h_2)$	$\underline{p}(t_1)\overline{p}(t_1)\underline{p}(t_2)$	$\underline{p}(t_1)\overline{p}(h_2)+\overline{p}(t_1)\underline{p}(t_2)$
p_{B1}	$\underline{p}(h_2)\overline{p}(h_2)\underline{p}(h_1)$	$\overline{p}(t_2)\overline{p}(h_2)\underline{p}(h_1)$	$\underline{p}(h_2)\overline{p}(h_2)\overline{p}(t_1)$	$\underline{p}(h_2)\underline{p}(t_2)\overline{p}(t_1)$	$\underline{p}(h_2)\overline{p}(t_1)+\overline{p}(h_2)\underline{p}(h_1)$
p_{B2}	$\overline{p}(h_2)\overline{p}(t_2)\underline{p}(h_1)$	$\underline{p}(t_2)\overline{p}(t_2)\underline{p}(h_1)$	$\underline{p}(t_2)\underline{p}(h_2)\overline{p}(t_1)$	$\underline{p}(t_2)\overline{p}(t_2)\overline{p}(t_1)$	$\underline{p}(t_2)\overline{p}(t_1)+\overline{p}(t_2)\underline{p}(h_1)$
p_{B3}	$\underline{p}(h_2)\overline{p}(h_2)\overline{p}(h_1)$	$\underline{p}(h_2)\underline{p}(t_2)\overline{p}(h_1)$	$\underline{p}(h_2)\overline{p}(h_2)\underline{p}(t_1)$	$\overline{p}(t_2)\overline{p}(h_2)\underline{p}(t_1)$	$\underline{p}(h_2)\overline{p}(h_1)+\overline{p}(h_2)\underline{p}(t_1)$
p_{B4}	$\underline{p}(t_2)\underline{p}(h_2)\overline{p}(h_1)$	$\underline{p}(t_2)\overline{p}(t_2)\overline{p}(h_1)$	$\overline{p}(h_2)\overline{p}(t_2)\underline{p}(t_1)$	$\underline{p}(t_2)\overline{p}(t_2)\underline{p}(t_1)$	$\underline{p}(t_2)\overline{p}(h_1)+\overline{p}(t_2)\underline{p}(t_1)$

Independent natural extension for two binary variables

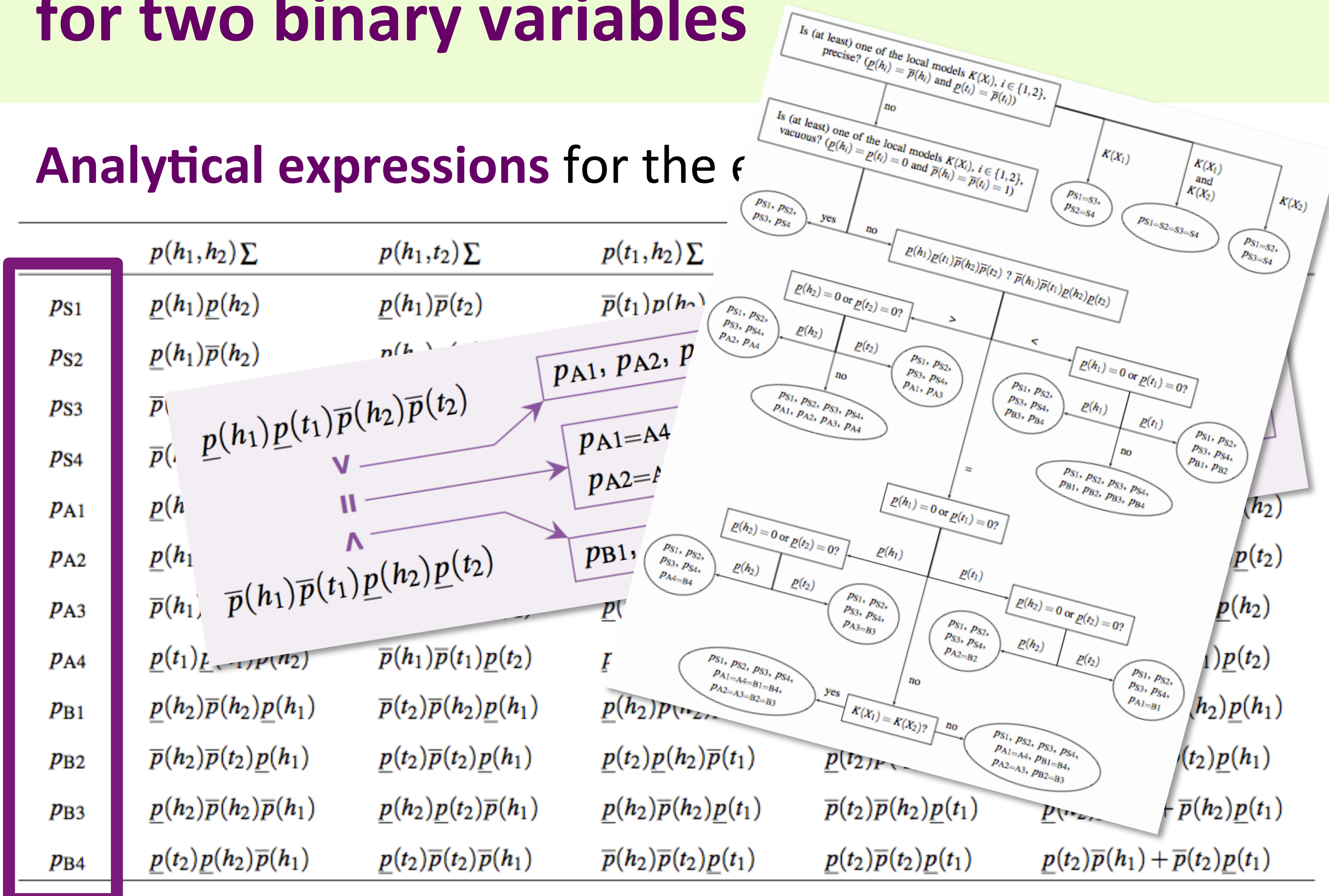
Analytical expressions for the extreme points

	$p(h_1,h_2)\Sigma$	$p(h_1,t_2)\Sigma$	$p(t_1,h_2)\Sigma$	$p(t_1,t_2)\Sigma$	Σ
p_{S1}	$\underline{p}(h_1)\underline{p}(h_2)$	$\underline{p}(h_1)\overline{p}(t_2)$	[illegible]	[illegible]	[illegible]
p_{S2}	$\underline{p}(h_1)\overline{p}(h_2)$	[illegible]	[illegible]	[illegible]	[illegible]
p_{S3}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
p_{S4}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
p_{A1}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
p_{A2}	[illegible]	[illegible]	[illegible]	[illegible]	$\underline{p}(h_1)\overline{p}(h_2)+\overline{p}(h_1)\underline{p}(t_2)$
p_{A3}	[illegible]	[illegible]	$\underline{p}(t_1)\overline{p}(t_1)\underline{p}(h_2)$	$\underline{p}(t_1)\overline{p}(t_1)\overline{p}(t_2)$	$\underline{p}(t_1)\overline{p}(t_2)+\overline{p}(t_1)\underline{p}(h_2)$
p_{A4}	[illegible]	$\overline{p}(h_1)\overline{p}(t_1)\underline{p}(t_2)$	$\underline{p}(t_1)\overline{p}(t_1)\overline{p}(h_2)$	$\underline{p}(t_1)\overline{p}(t_1)\underline{p}(t_2)$	$\underline{p}(t_1)\overline{p}(h_2)+\overline{p}(t_1)\underline{p}(t_2)$
p_{B1}	$\underline{p}(h_2)\overline{p}(h_2)\underline{p}(h_1)$	$\overline{p}(t_2)\overline{p}(h_2)\underline{p}(h_1)$	$\underline{p}(h_2)\overline{p}(h_2)\overline{p}(t_1)$	$\underline{p}(h_2)\underline{p}(t_2)\overline{p}(t_1)$	$\underline{p}(h_2)\overline{p}(t_1)+\overline{p}(h_2)\underline{p}(h_1)$
p_{B2}	$\overline{p}(h_2)\overline{p}(t_2)\underline{p}(h_1)$	$\underline{p}(t_2)\overline{p}(t_2)\underline{p}(h_1)$	$\underline{p}(t_2)\underline{p}(h_2)\overline{p}(t_1)$	$\underline{p}(t_2)\overline{p}(t_2)\overline{p}(t_1)$	$\underline{p}(t_2)\overline{p}(t_1)+\overline{p}(t_2)\underline{p}(h_1)$
p_{B3}	$\underline{p}(h_2)\overline{p}(h_2)\overline{p}(h_1)$	$\underline{p}(h_2)\underline{p}(t_2)\overline{p}(h_1)$	$\underline{p}(h_2)\overline{p}(h_2)\underline{p}(t_1)$	$\overline{p}(t_2)\overline{p}(h_2)\underline{p}(t_1)$	$\underline{p}(h_2)\overline{p}(h_1)+\overline{p}(h_2)\underline{p}(t_1)$
p_{B4}	$\underline{p}(t_2)\underline{p}(h_2)\overline{p}(h_1)$	$\underline{p}(t_2)\overline{p}(t_2)\overline{p}(h_1)$	$\overline{p}(h_2)\overline{p}(t_2)\underline{p}(t_1)$	$\underline{p}(t_2)\overline{p}(t_2)\underline{p}(t_1)$	$\underline{p}(t_2)\overline{p}(h_1)+\overline{p}(t_2)\underline{p}(t_1)$

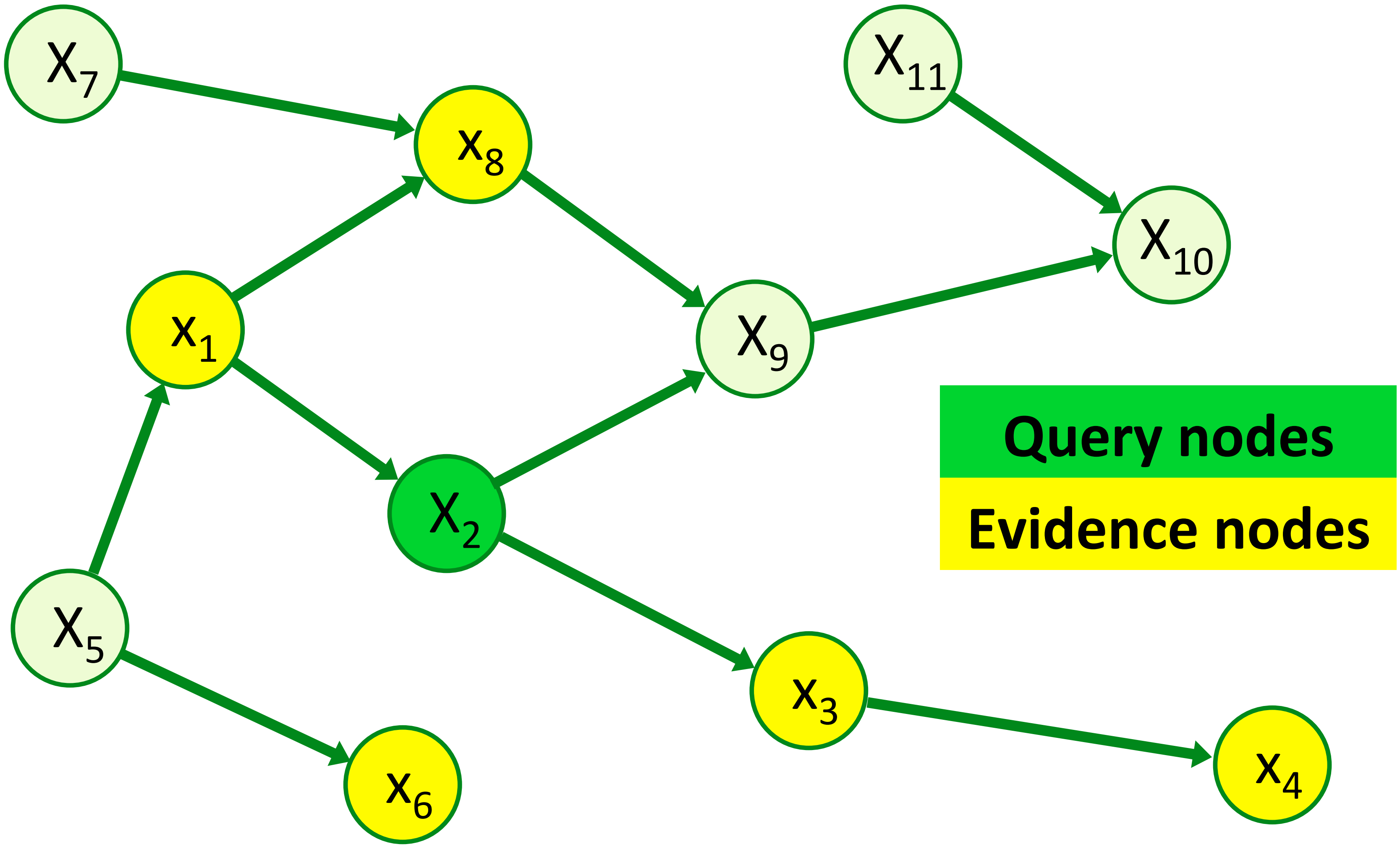


Independent natural extension for two binary variables

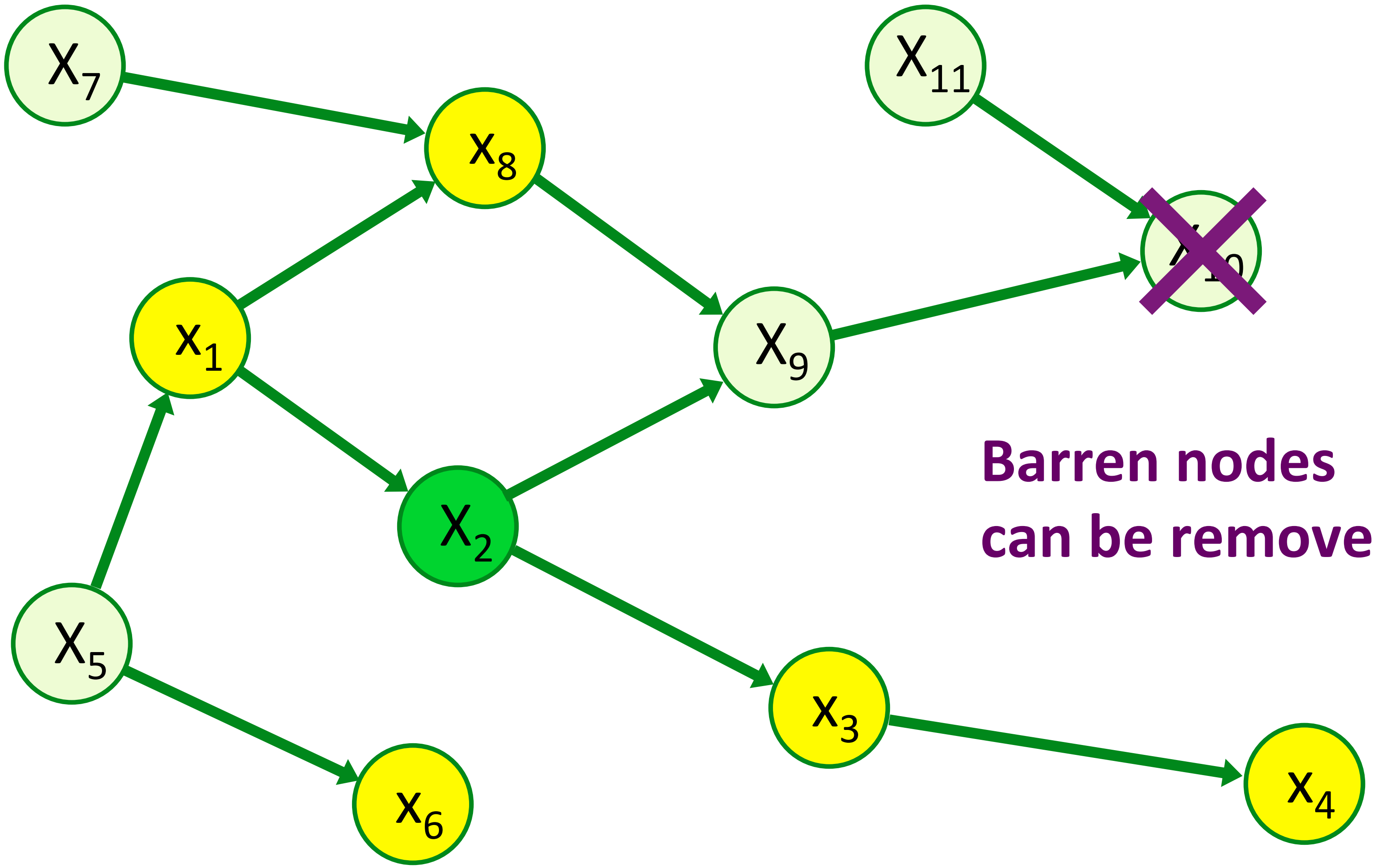
Analytical expressions for the e



Bayesian networks: useful properties

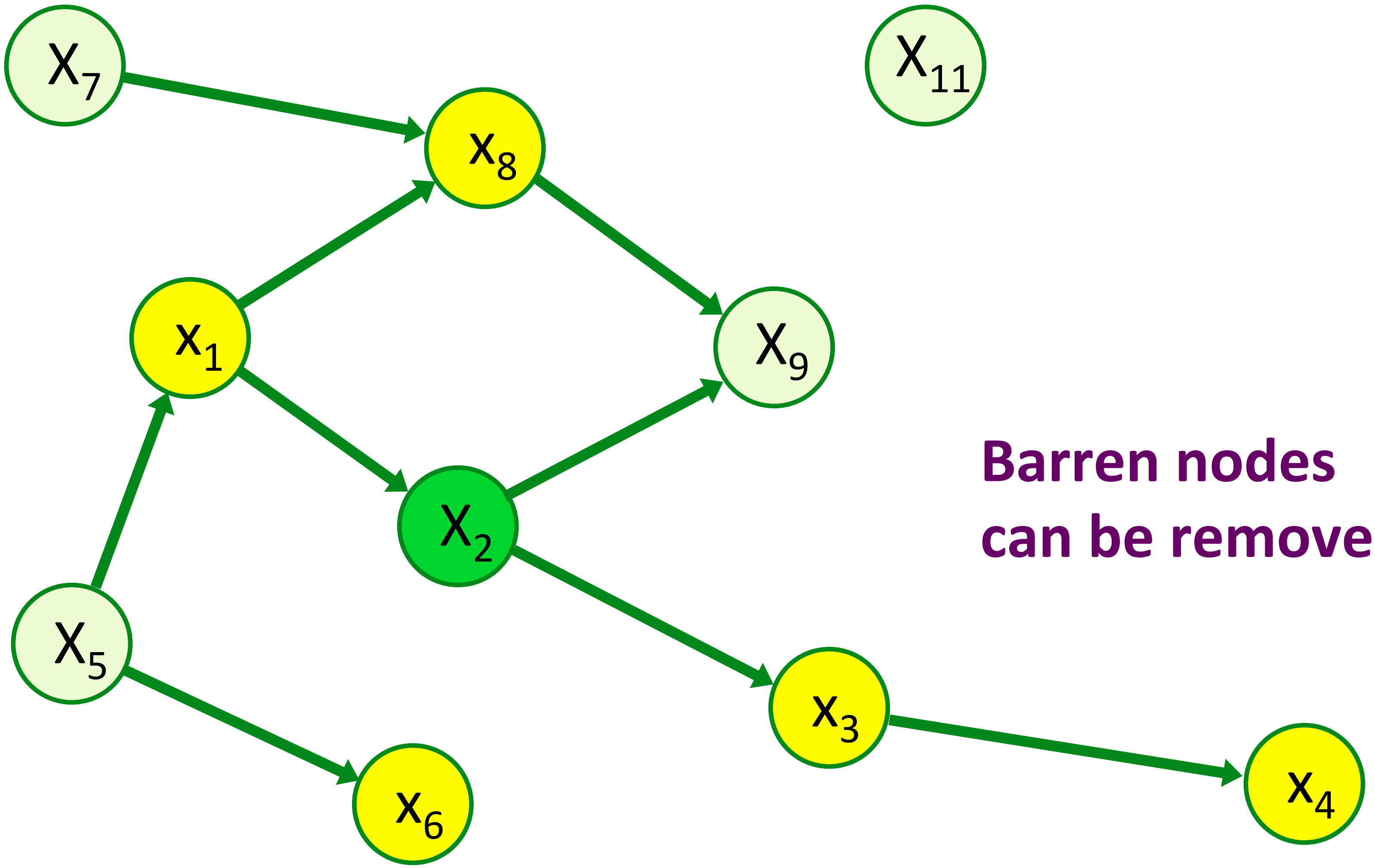


Bayesian networks: useful properties



Barren nodes can be removed

Bayesian networks: useful properties



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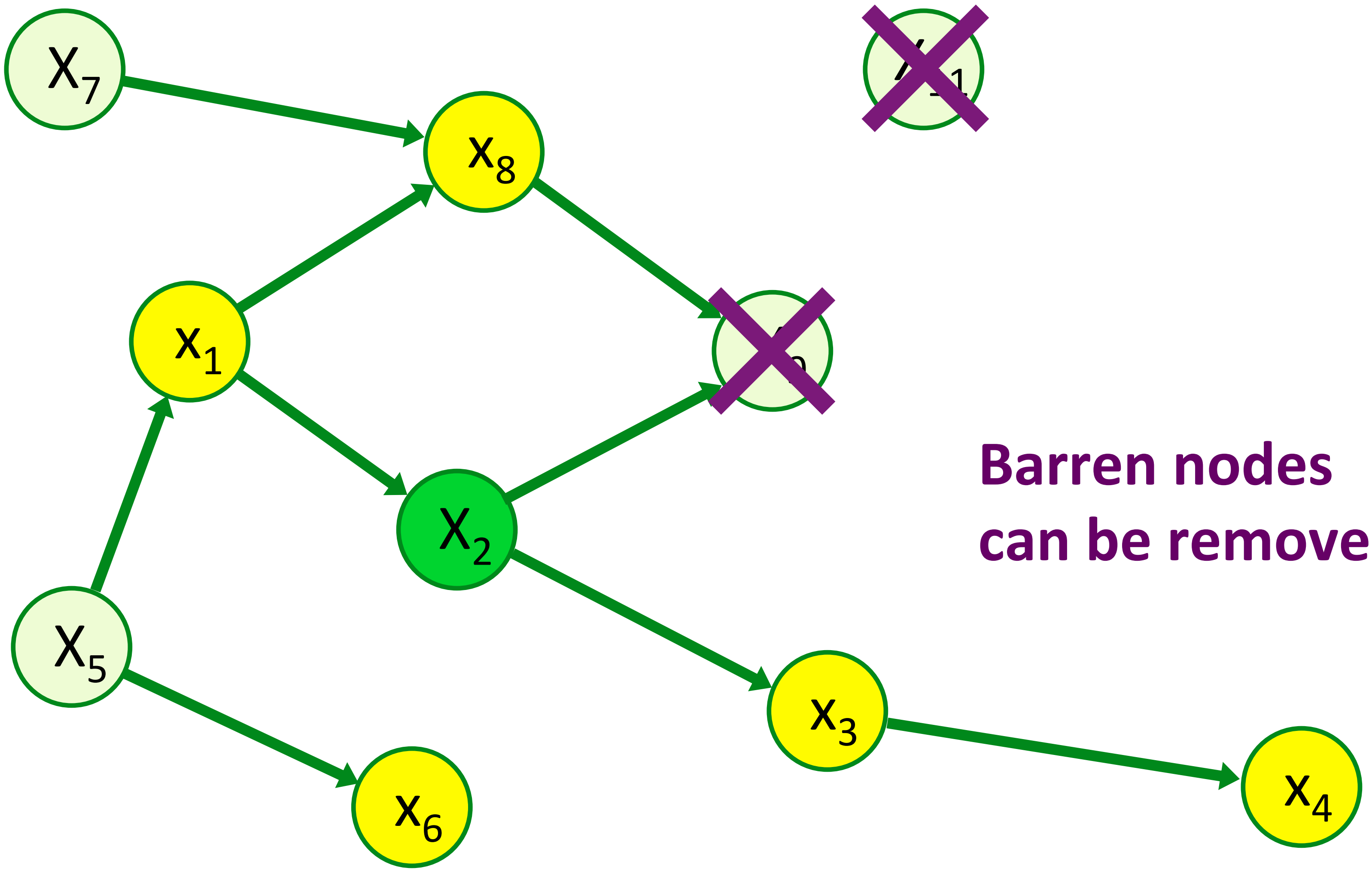

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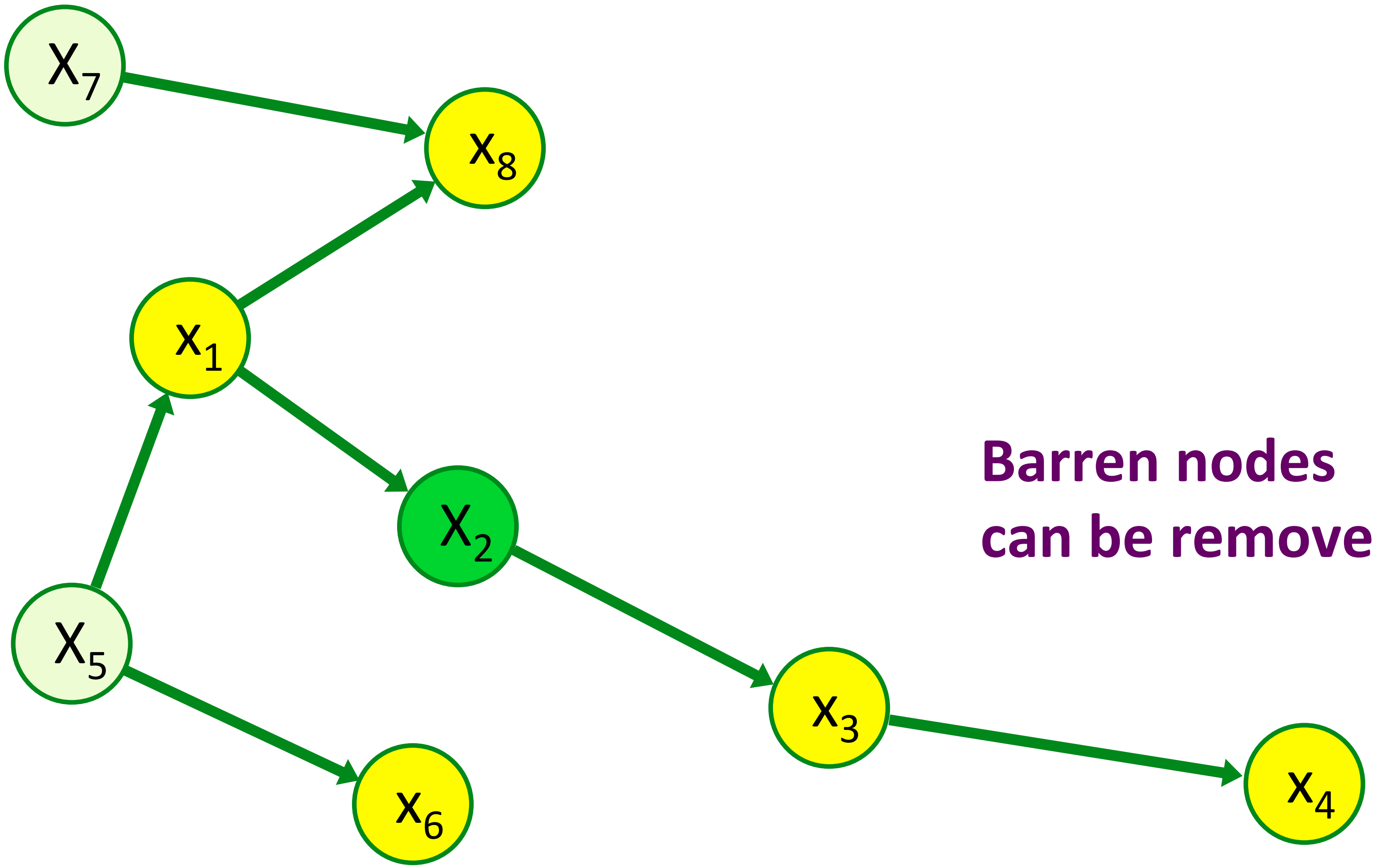

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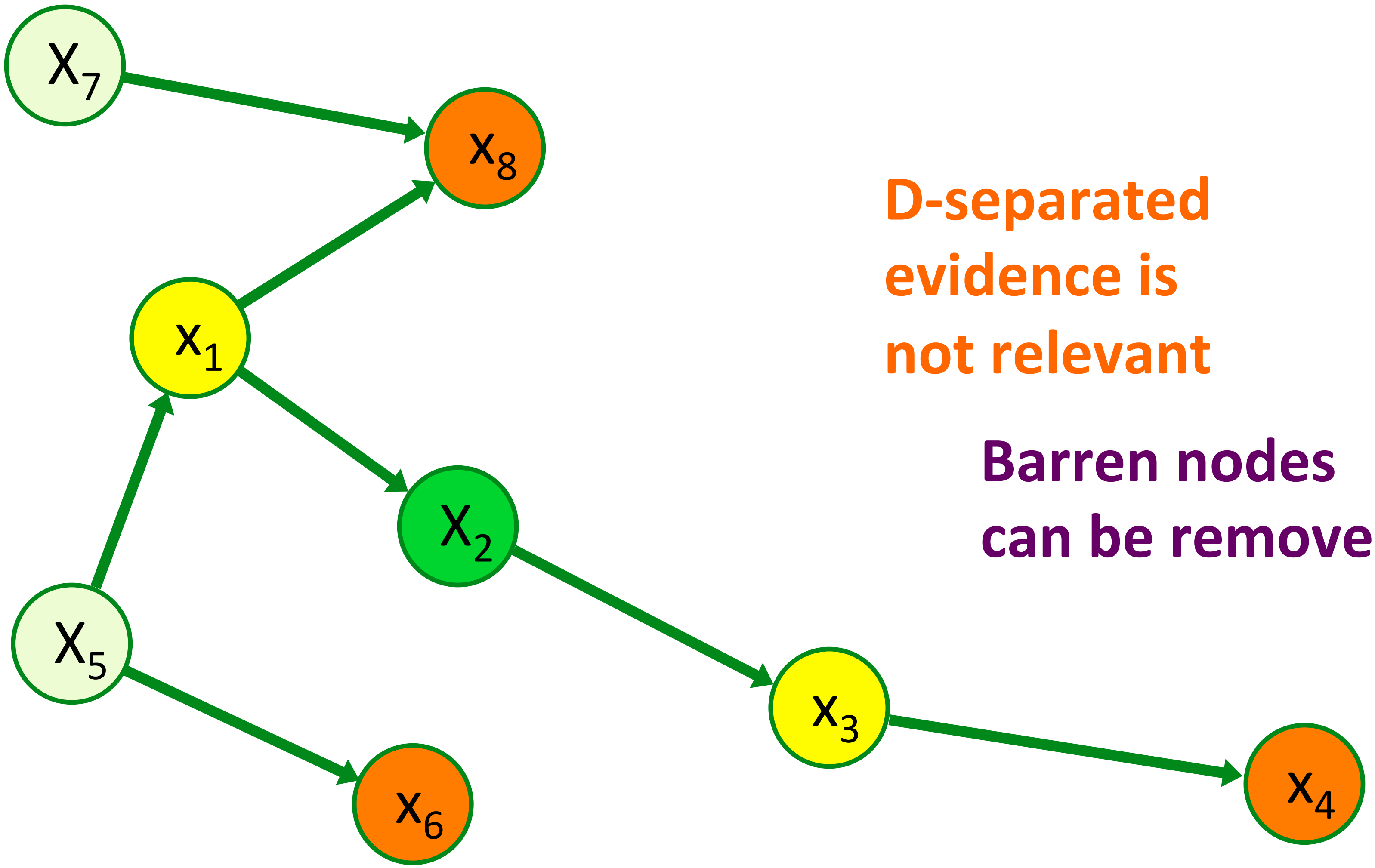
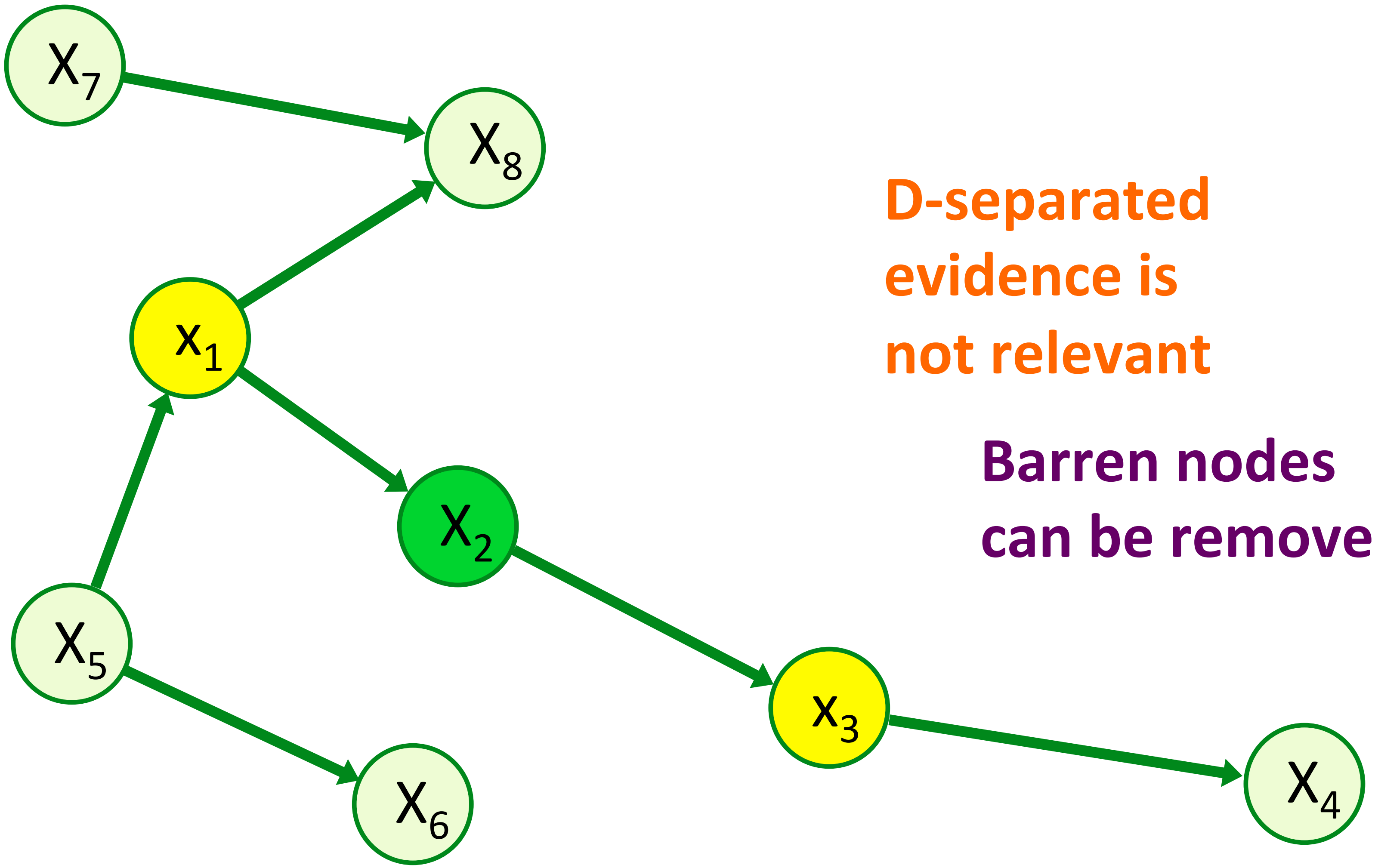


D-separated evidence is not relevant

Barren nodes can be removed

Bayesian networks: useful properties



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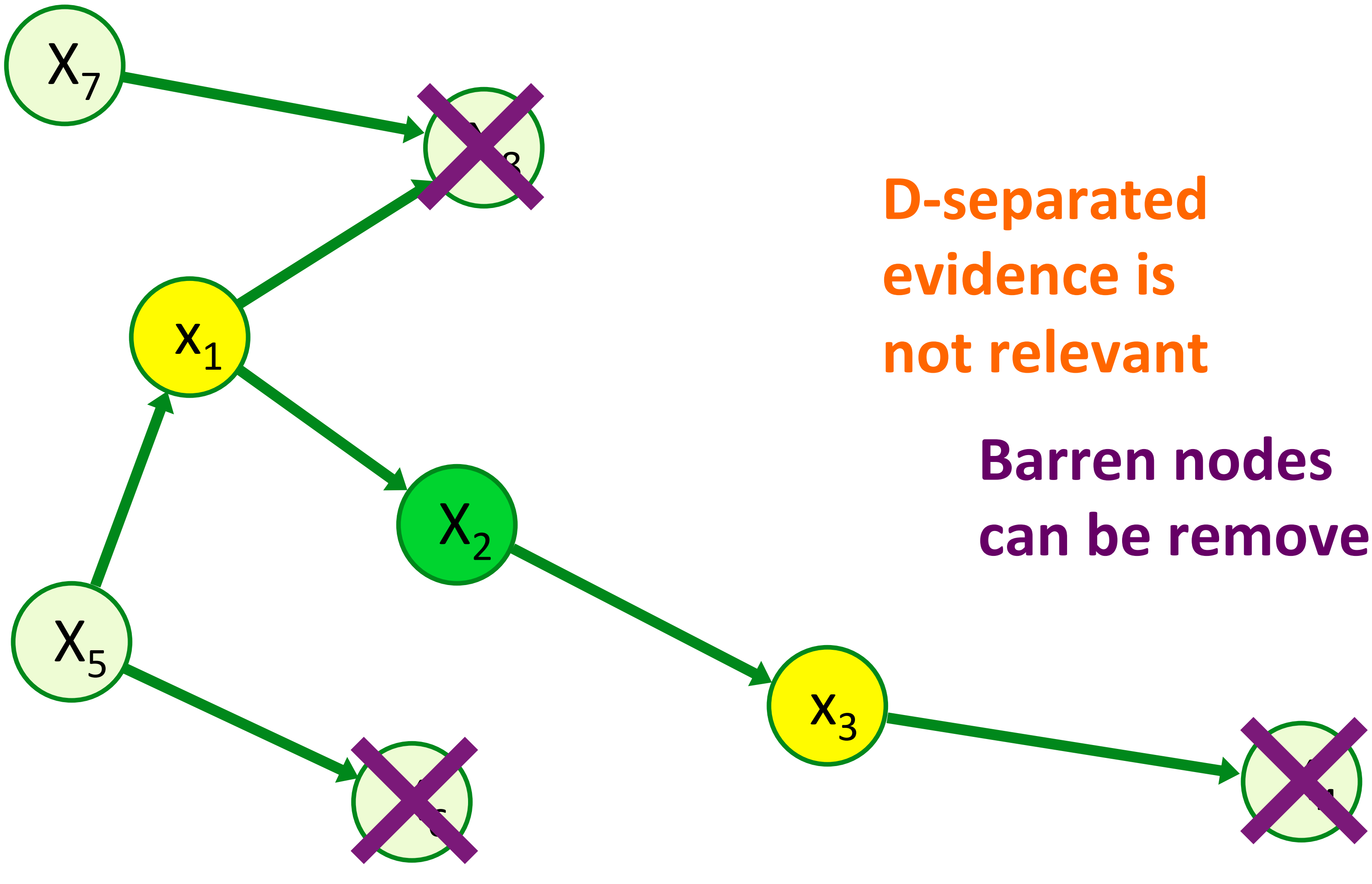

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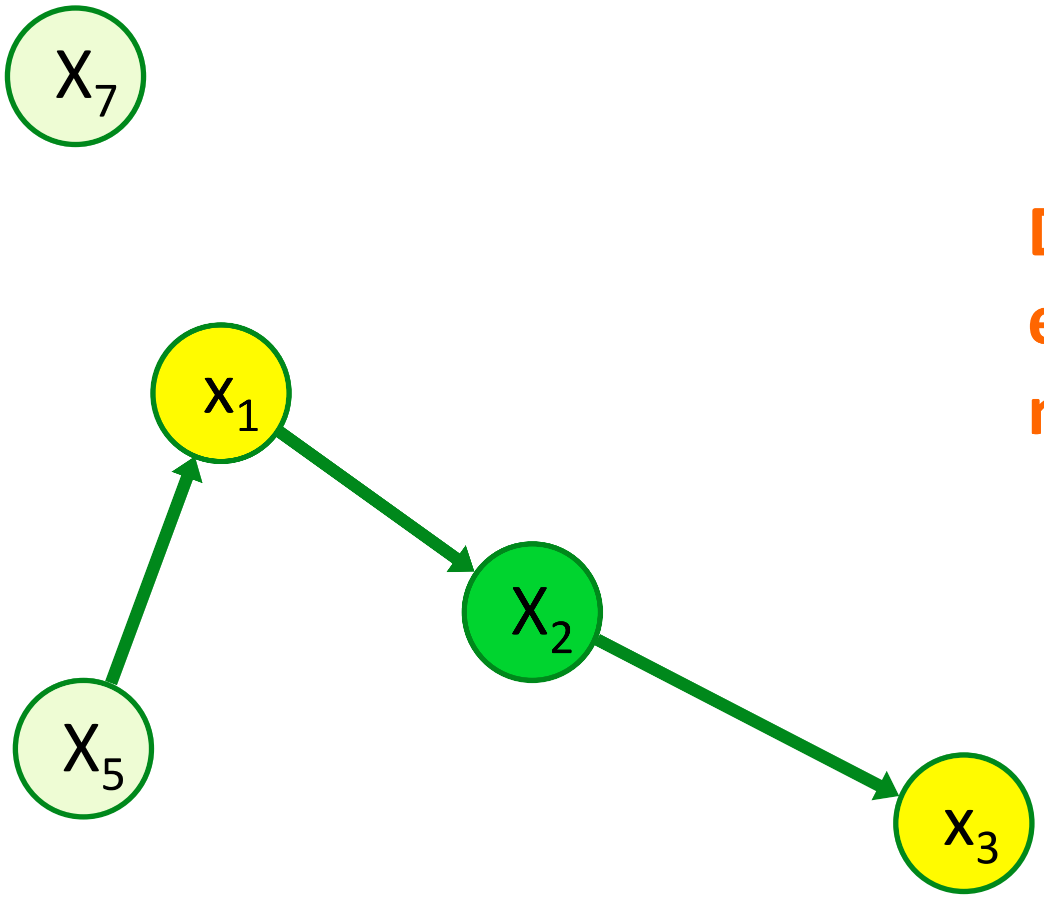

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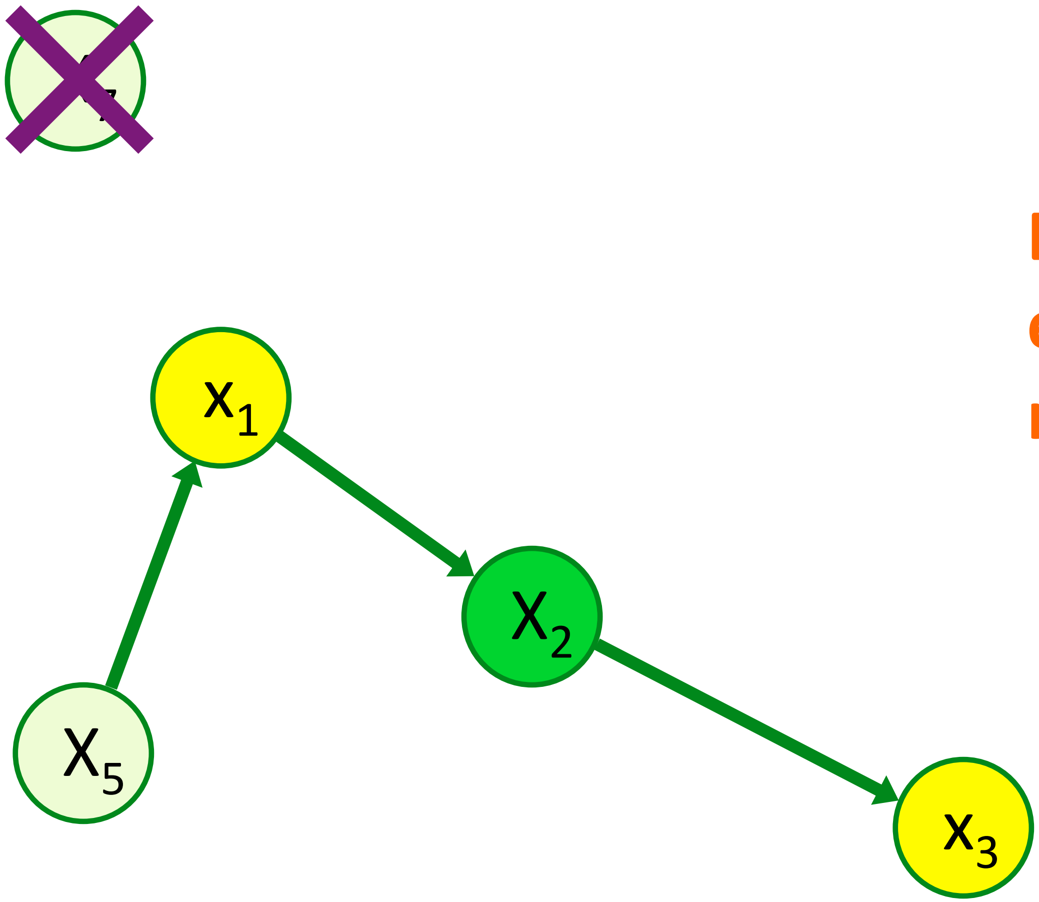

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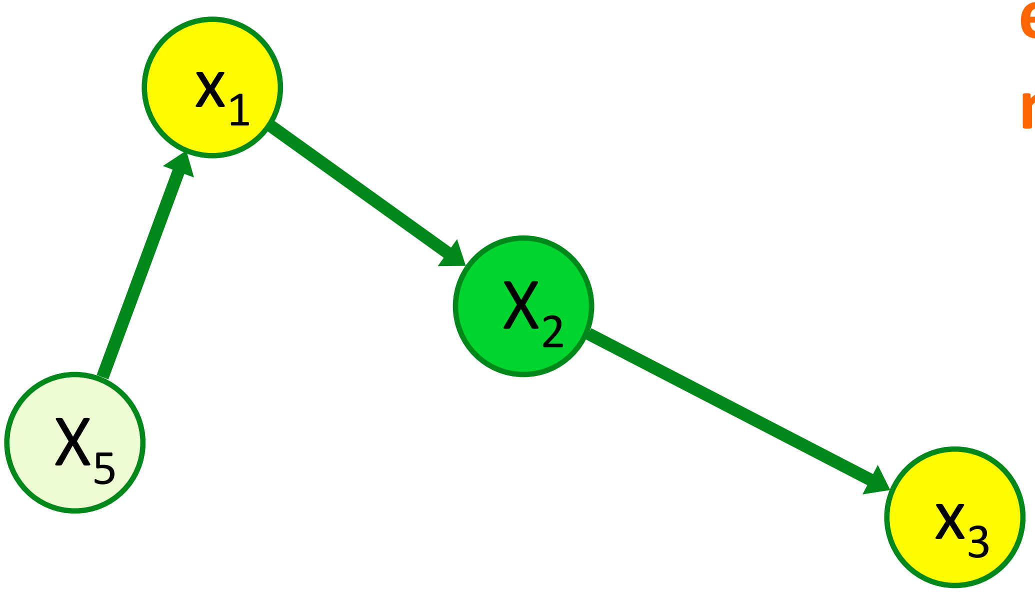

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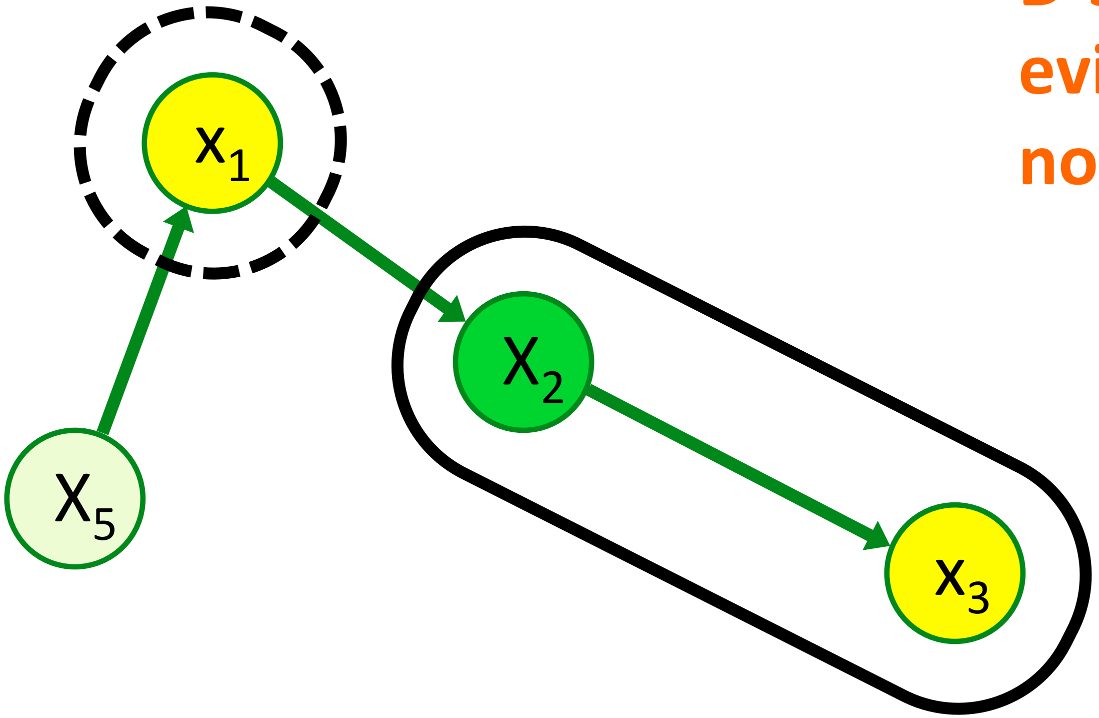
Barren nodes can be removed

Bayesian networks: useful properties

Conditional marginalisation properties

D-separated evidence is not relevant

Barren nodes can be removed

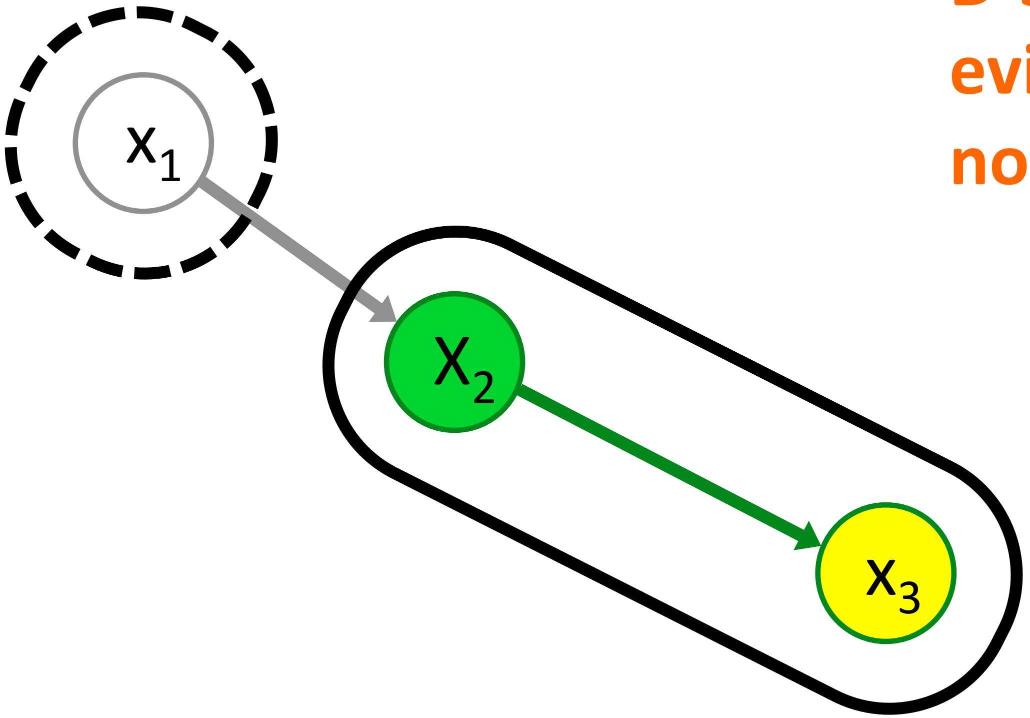


Bayesian networks: useful properties

Conditional marginalisation properties

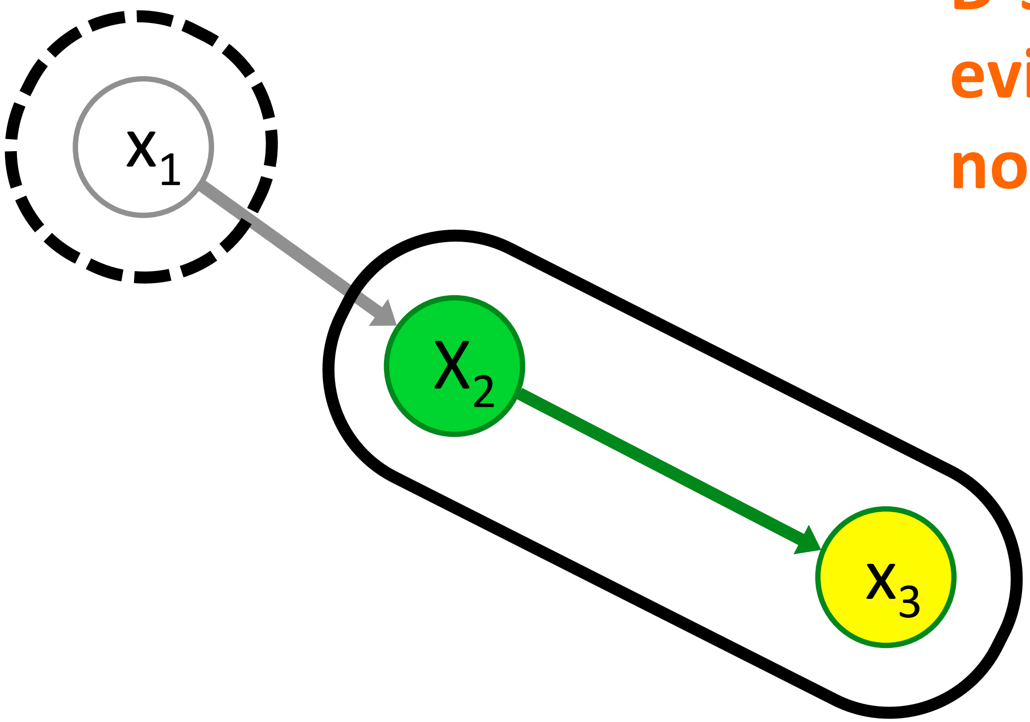
D-separated evidence is not relevant

Barren nodes can be removed



Credal networks: useful properties ?

Conditional marginalisation properties



D-separated evidence is not relevant

Barren nodes can be removed

Credal networks: useful properties ?

? Conditional marginalisation properties

? D-separated evidence is not relevant

Barren nodes can be removed

✔

Credal networks under epistemic irrelevance

Credal networks: useful properties ?

? Conditional marginalisation properties

? D-separated evidence is not relevant

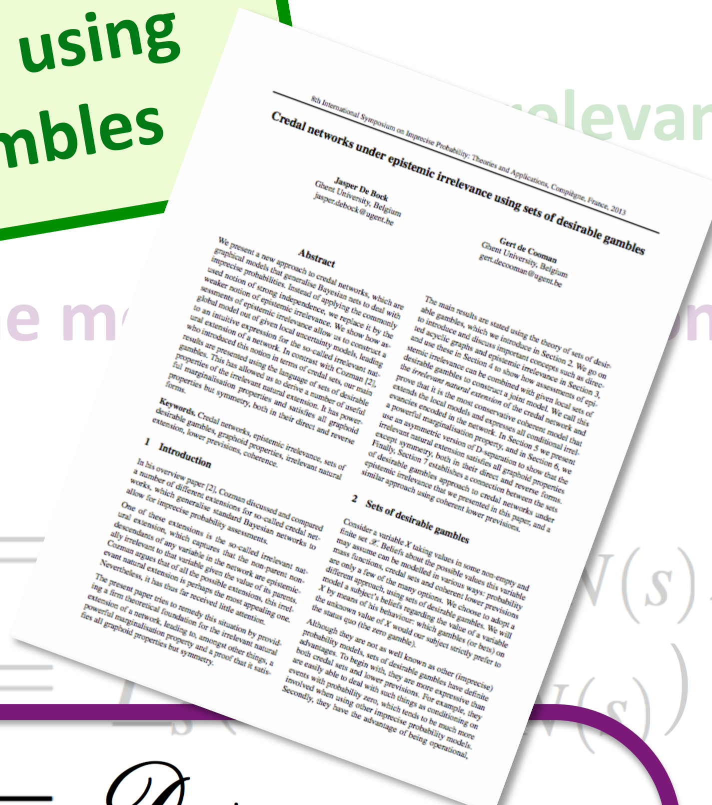
Barren nodes can be removed ✔

graphoid axioms

Credal networks under epistemic irrelevance

Credal networks: a joint model

Credal networks under epistemic irrelevance using sets of desirable gambles



$K^{\mathrm{irr}}(X_G)$

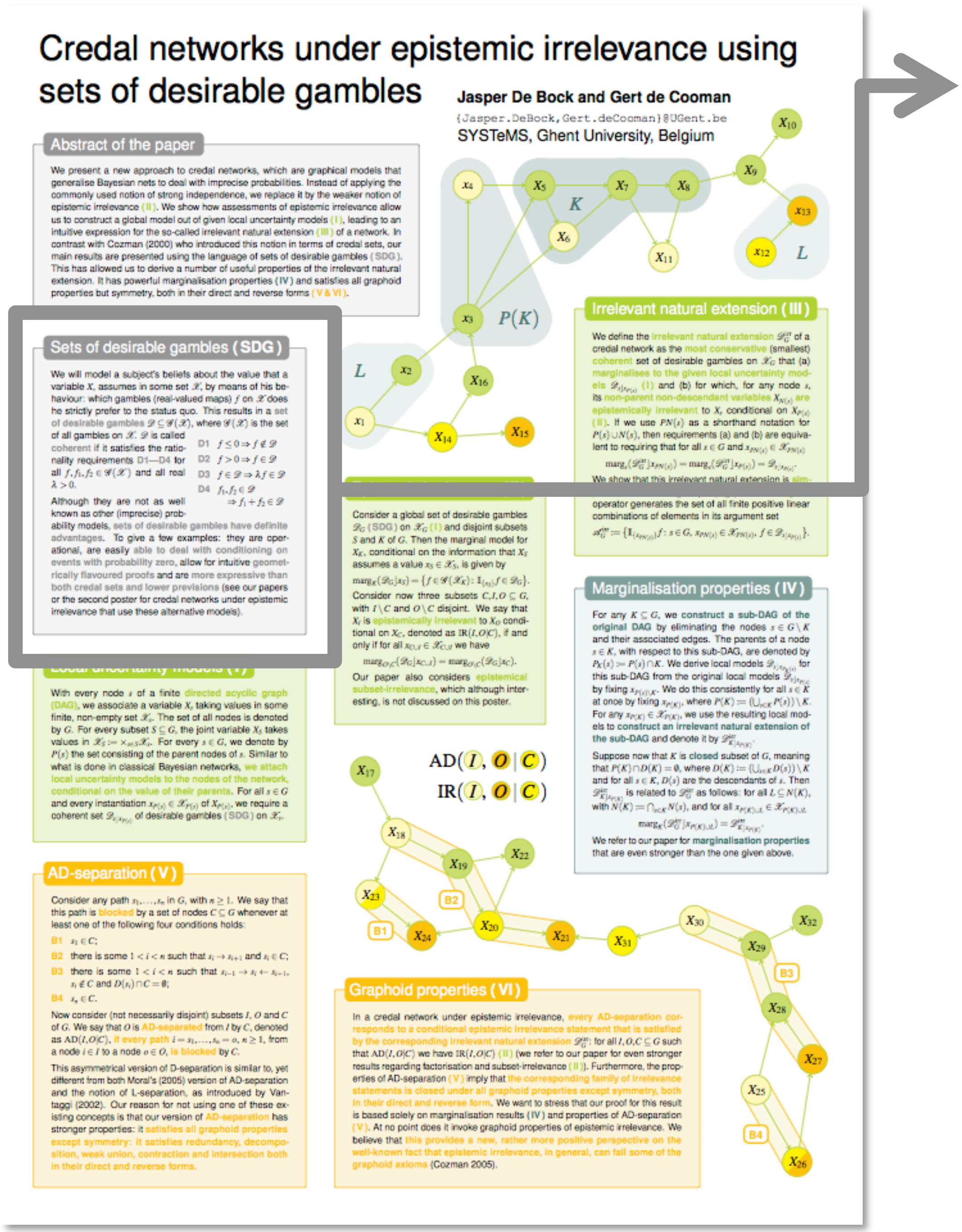
$\underline{P}^{\mathrm{irr}}_G$

$\mathscr{D}^{\mathrm{irr}}_G$

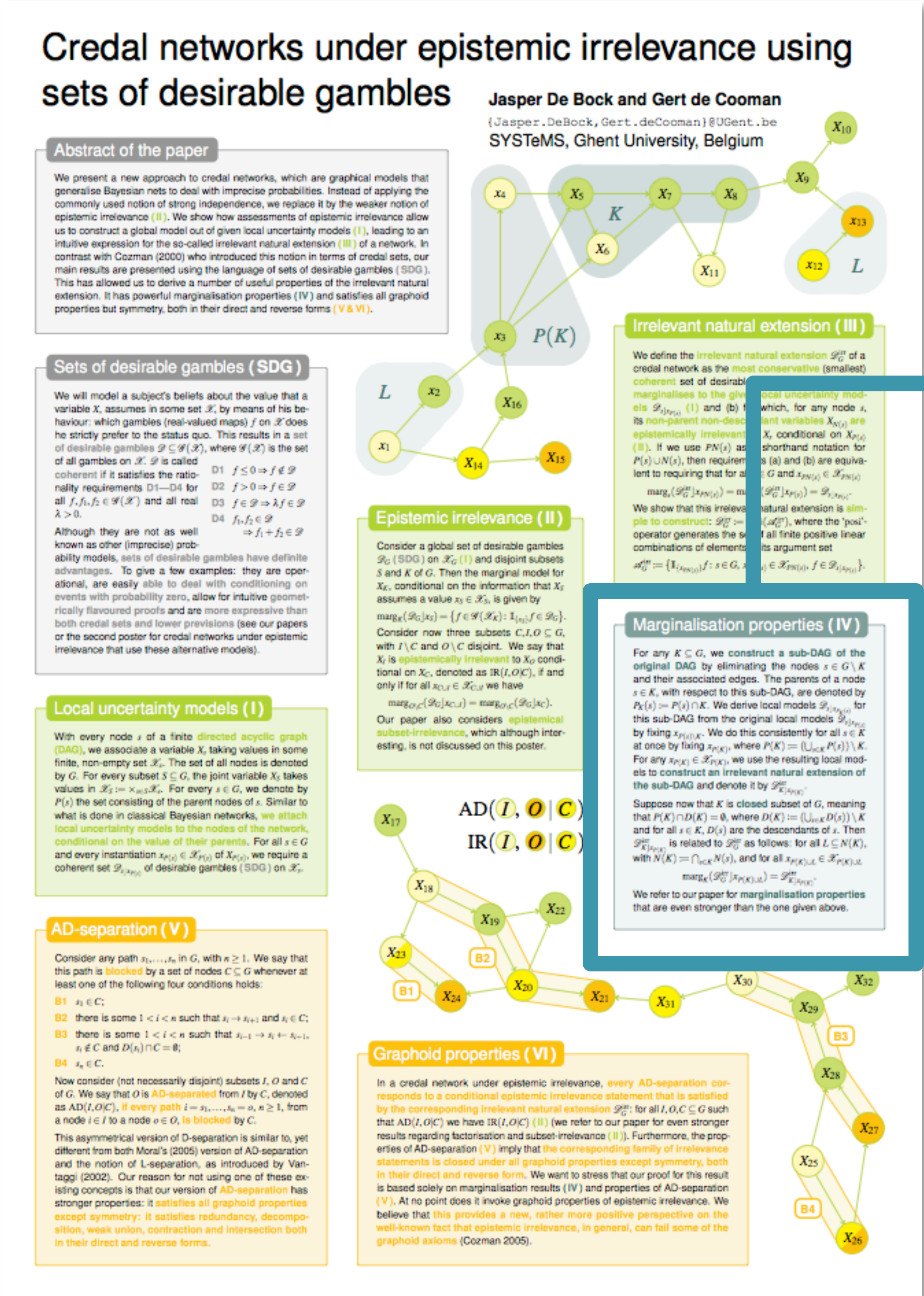
$K(X_s \rfloor x_{P(s)}) = \ldots N(s))$

$\underline{P}_s(\cdot \rfloor x_{P(s)}) = \ldots N(s))$

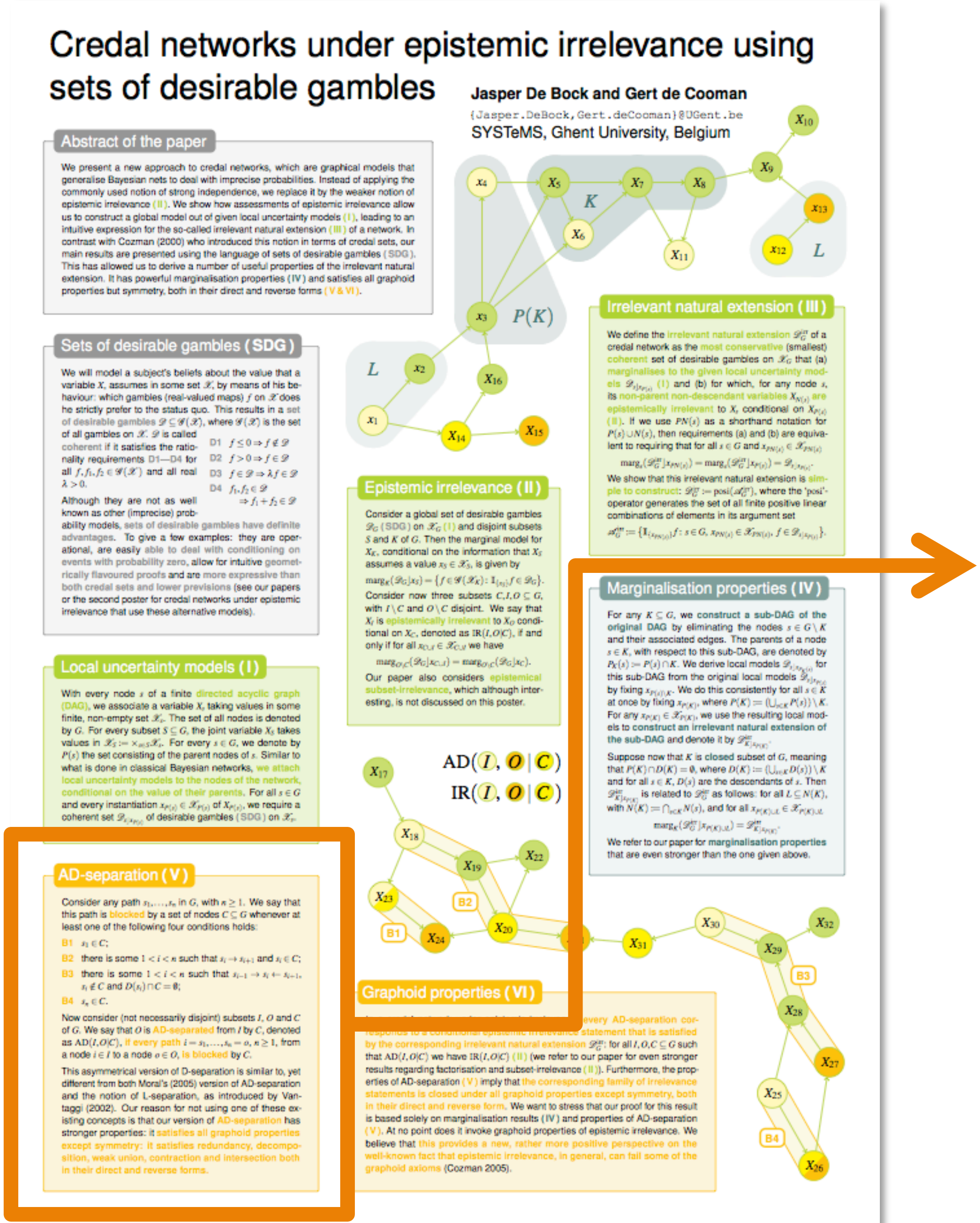
$$\mathscr{D}_{s\rfloor x_{P(s)}} = \mathscr{D}_{s\rfloor x_{P(s)\cup N(s)}}$$



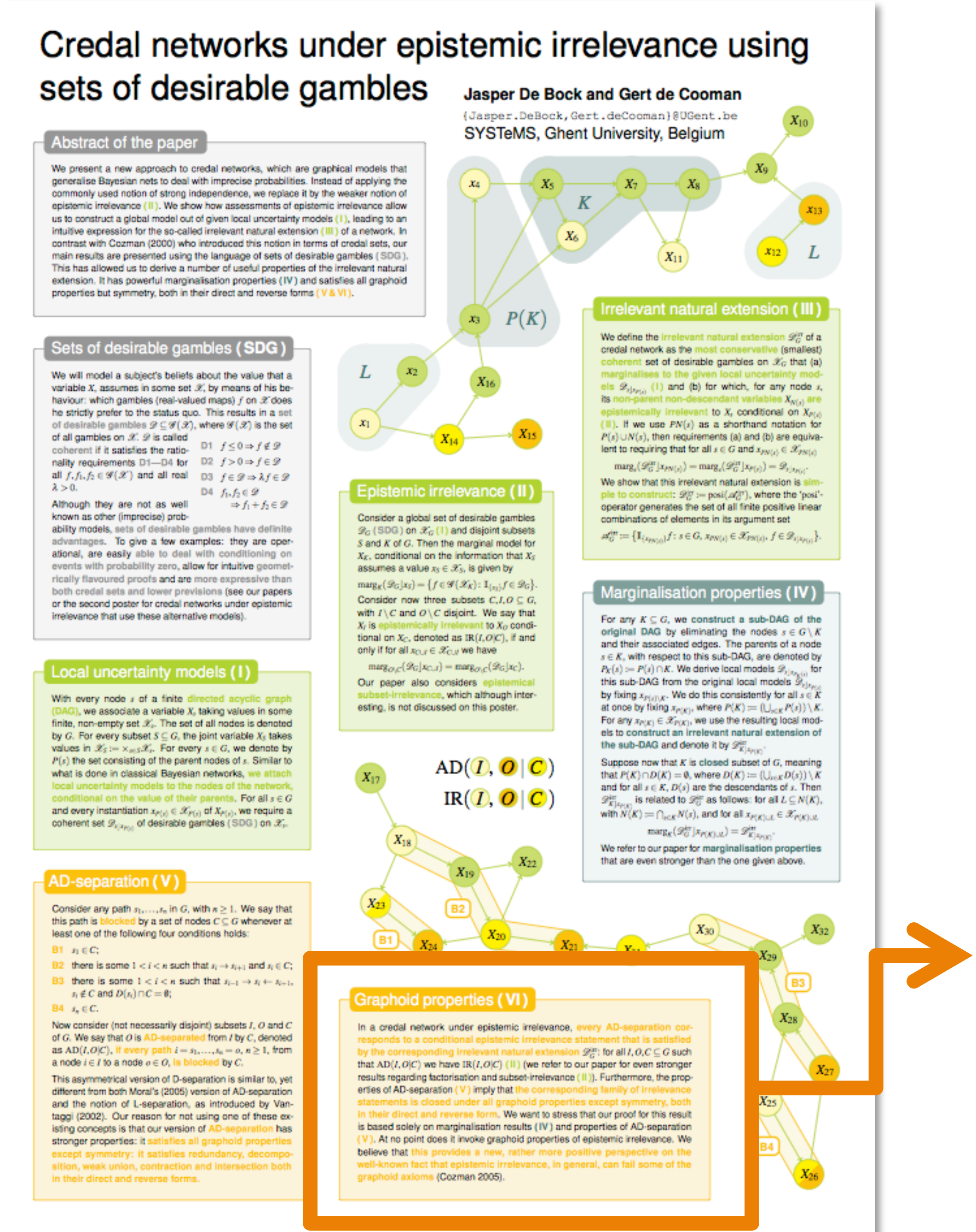
- Advantages of sets of desirable gambles



- Advantages of sets of desirable gambles
- **Marginalisation properties**



- Advantages of sets of desirable gambles
- Marginalisation properties
- AD-separation satisfies every graphoid property except symmetry



- Advantages of sets of desirable gambles
- Marginalisation properties
- AD-separation satisfies every graphoid property except symmetry
- AD-separation implies epistemic irrelevance

Credal networks: useful properties ✔

✔ Conditional marginalisation properties

~~graphoid axioms~~ ~~→~~ ✔ **AD-separated** evidence is not relevant

Barren nodes can be removed ✔

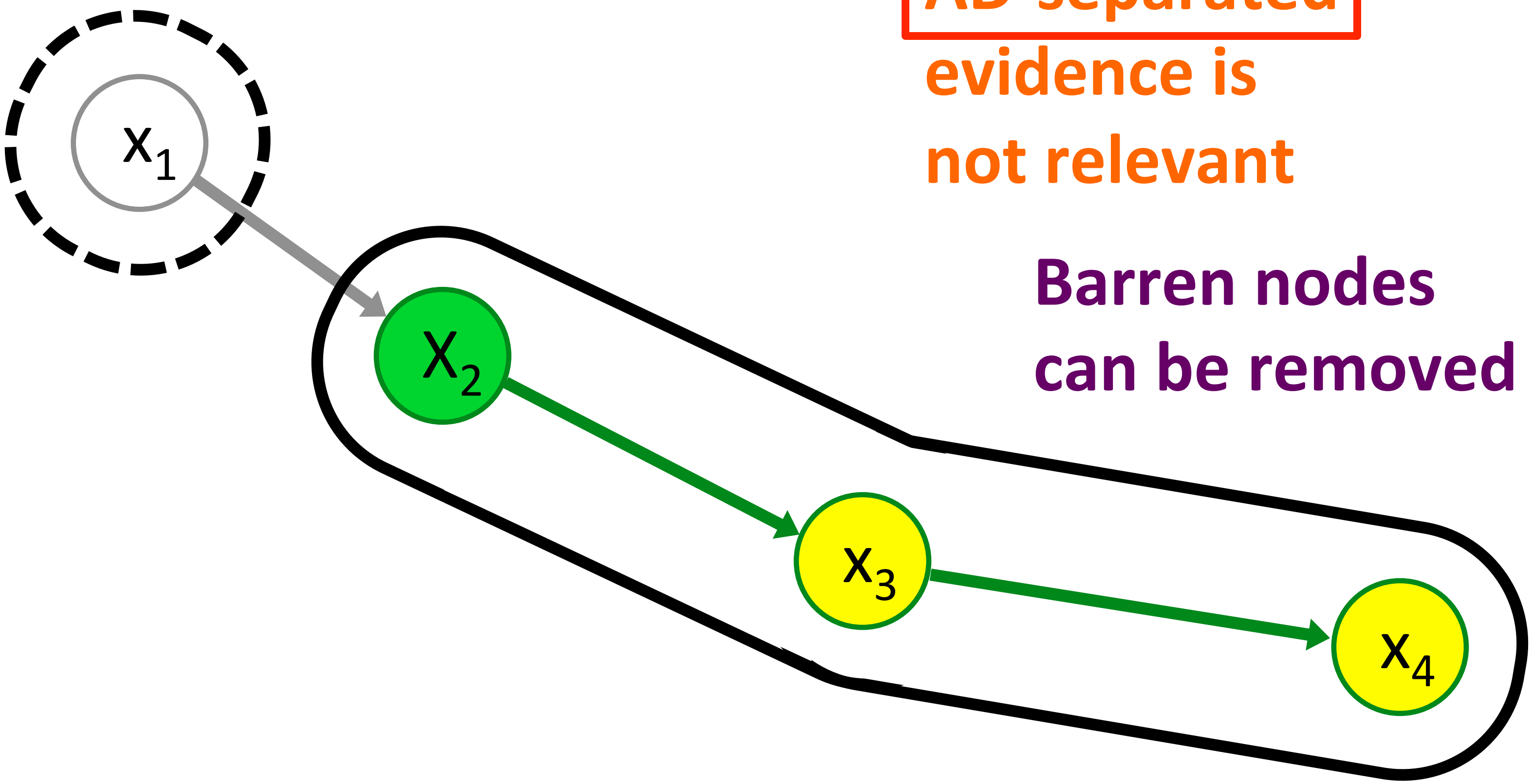
Credal networks under epistemic irrelevance

Credal networks: useful properties ✔

Conditional marginalisation properties

AD-separated evidence is not relevant

Barren nodes can be removed



Hope to see you at the poster session!