SYSTeMS dialogue

Imprecise Bernoulli processes

Jasper De Bock & Gert de Cooman

26 April 2012

Classical Bernoulli processes

Classical Bernoulli processes

An infinite sequence of binary random variables

$$X_1 , X_2 , \dots , X_n , \dots$$

assuming values in the set

$$\mathscr{X} = \{ a , b \}$$

Classical Bernoulli processes

An infinite sequence of binary random variables

$$X_1 , X_2 , \ldots , X_n , \ldots$$

defining properties

IID

Independent

Identically **D**istributed

Classical Bernoulli processes

An infinite sequence of binary random variables

$$X_1 , X_2 , \dots , X_n , \dots$$

! IMPLICIT ASSUMPTION !

a single **Bernoulli experiment X_i** has a **precise** and **precisely known** **probability mass function**

Classical Bernoulli processes

$$P(X_i = a) = \theta \qquad P(X_i = b) = 1 - \theta$$

with a fixed $\theta \in [0, 1]$

a single **Bernoulli experiment X_i** has a
precise and **precisely known**
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Classical Bernoulli processes

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Classical Bernoulli processes

$X_1 , X_2 , \ldots , X_n$

BINOMIAL DISTRIBUTION
with parameters θ and n

Classical Bernoulli processes

$X_1 , X_2 , \ldots , X_n$

BINOMIAL DISTRIBUTION
with parameters θ and n

For every x = (x_1, …, x_n) in $\mathscr{X}^n$:

Probability of occurence p(x) = $\theta^{n(a)}(1- \theta)^{n(b)}$

Classical Bernoulli processes

$X_1, X_2, \ldots, X_n$

BINOMIAL DISTRIBUTION
with parameters θ and n

For every $x = (x_1, \ldots, x_n)$ in $\mathscr{X}^n$:

Probability of occurence $p(x) = \theta^{n(a)}(1-\theta)^{n(b)}$

For every gamble (real valued map) f on $\mathscr{X}^n$:

Expected value

$$E(f) = Bn^n(f \mid \theta) = \sum_{x \in \mathcal{X}^n} f(x)p(x)$$

Sensitivity analysis in Bernoulli processes

Sensitivity analysis in Bernoulli processes

An infinite sequence of binary random variables

$$X_1 , X_2 , \dots , X_n , \dots$$

defining properties

IID

Independent

Identically **D**istributed

Sensitivity analysis in Bernoulli processes

An infinite sequence of binary random variables

$$X_1, X_2, \ldots, X_n, \ldots$$

! Introducing imprecision !

a single **Bernoulli experiment X_i** has a

precise and ~~**precisely known**~~

probability mass function

Sensitivity analysis in Bernoulli processes

$$\boxed{P(X_i = a) = \theta \qquad P(X_i = b) = 1 - \theta}$$

θ varies over an interval $[\underline{\theta}, \overline{\theta}]$

a single **Bernoulli experiment X_i** has a
precise and **~~precisely known~~**
probability mass function

Sensitivity analysis in Bernoulli processes

$$P(X_i = a) = \theta \qquad P(X_i = b) = 1 - \theta$$

θ varies over an interval $[\underline{\theta}, \overline{\theta}]$

IID → **I**ndependent

→ **I**dentically **D**istributed

Sensitivity analysis in Bernoulli processes

For a fixed $\theta \in [0, 1]$:

For every gamble f on $\mathscr{X}^n$:

Expected value: E(f) = Bn^n(f | θ) = $\sum_{x \in \mathcal{X}^n}$ f(x)p(x)

Sensitivity analysis in Bernoulli processes

For a fixed $\theta \in [0, 1]$:

For every gamble f on $\mathscr{X}^n$:

Expected value: E(f) = Bn^n(f | θ) = $\sum_{x \in \mathcal{X}^n}$ f(x)p(x)

If θ varies over an interval $[\underline{\theta}, \overline{\theta}]$:

Sensitivity analysis in Bernoulli processes

For a fixed $\theta \in [0, 1]$:

For every gamble f on $\mathscr{X}^n$:

Expected value: E(f) = Bn^n(f | θ) = $\sum_{x \in \mathcal{X}^n}$ f(x)p(x)

If θ varies over an interval $[\underline{\theta}, \overline{\theta}]$:

Lower and upper expected value:

$$\overline{E}(f) = \max\{ \text{ Bn}^n(f|\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$

$$\underline{E}(f) = \min\{ \text{ Bn}^n(f|\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$

Imprecise Bernoulli processes

Imprecise Bernoulli processes

An infinite sequence of binary random variables

$$X_1 , X_2 , \ldots , X_n , \ldots$$

! dropping both assumptions !

a single **Bernoulli experiment X_i** has a
~~**precise**~~ and ~~**precisely known**~~
probability mass function

Imprecise Bernoulli processes

An infinite sequence of binary random variables

$$X_1, X_2, \dots, X_n, \dots$$

a single **Bernoulli experiment X_i** is regarded as **inherently imprecise**

We do not assume the existence of an underlying precise probability distribution

Imprecise Bernoulli processes

Sets of Desirable gambles

Serafin Moral

?

Peter Walley

Imprecise Bernoulli processes

No underlying precise probability distribution!

$\boxed{\text{A set } \mathcal{D} \text{ of desirable gambles}}$

We model a **subject's beliefs** regarding the possible **outcomes Ω of an experiment** by looking at the **gambles he is willing to accept**

Imprecise Bernoulli processes

No underlying precise probability distribution!

A set $\mathcal{D}$ of desirable gambles

Rationality criteria:

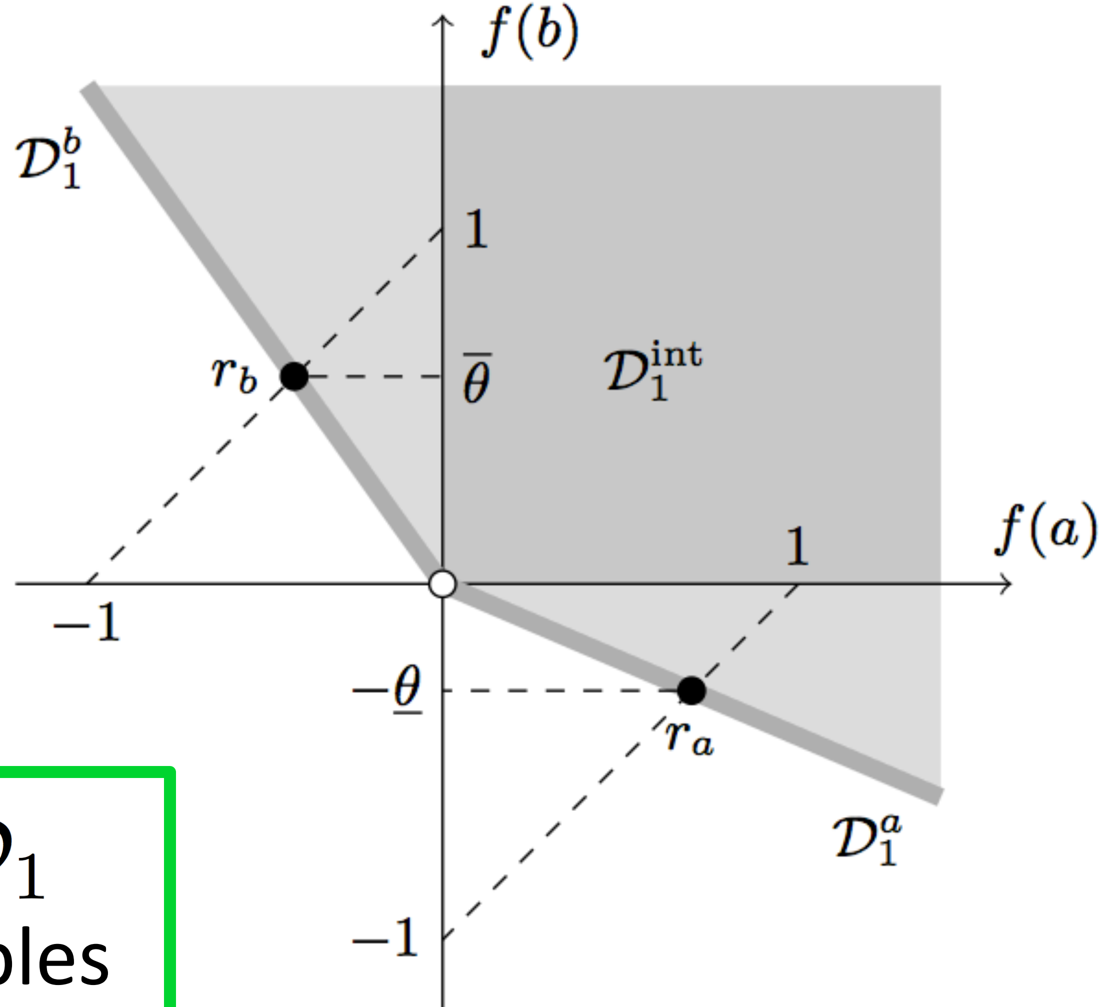
COHERENT

C1. *if* $f = 0$ *then* $f \notin \mathcal{D}$;
C2. *if* $f > 0$ *then* $f \in \mathcal{D}$;
C3. *if* $f \in \mathcal{D}$ *then* $\lambda f \in \mathcal{D}$ *[scaling]*;
C4. *if* $f_1, f_2 \in \mathcal{D}$ *then* $f_1 + f_2 \in \mathcal{D}$ *[combination]*.

($f > 0$ iff $f \geq 0$ and $f \neq 0$)

Imprecise Bernoulli processes

a single
Bernoulli experiment

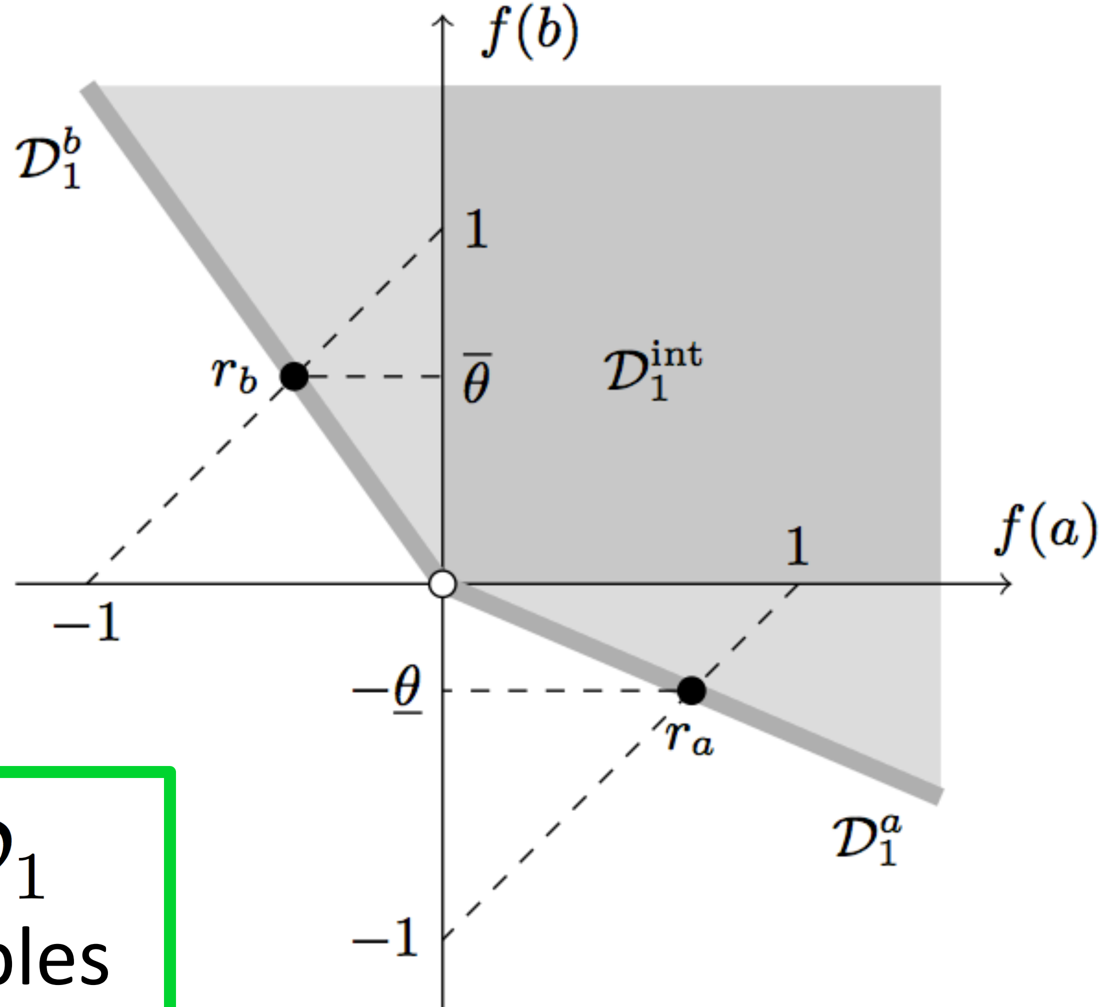


A coherent set $\mathcal{D}_1$ of desirable gambles

Imprecise Bernoulli processes

Due to coherence:

$$0 \leq \underline{\theta} \leq \overline{\theta} \leq 1$$



A coherent set $\mathcal{D}_1$ of desirable gambles

Imprecise Bernoulli processes

Due to coherence:

$$0 \cancel{\leq} \underline{\theta} \cancel{\leq} \overline{\theta} \cancel{\leq} 1$$

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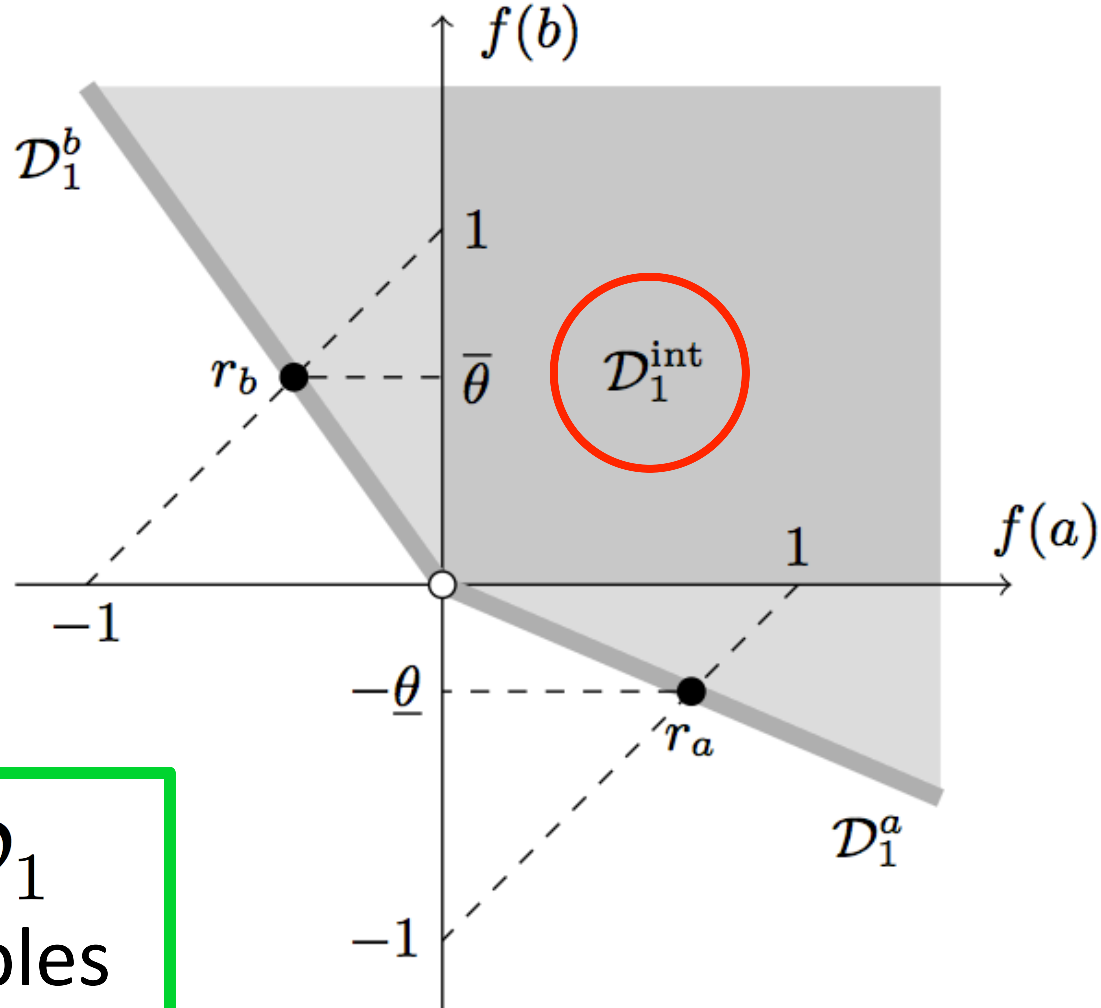
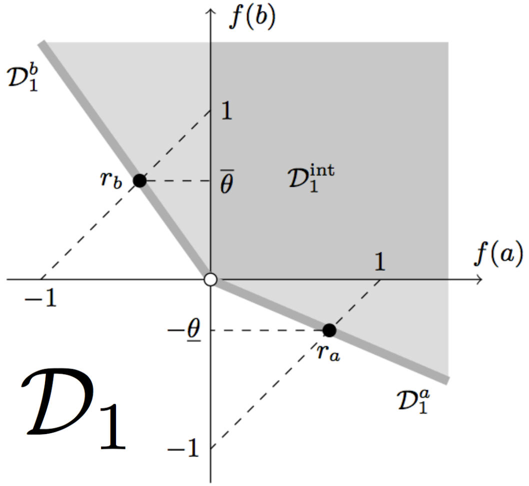


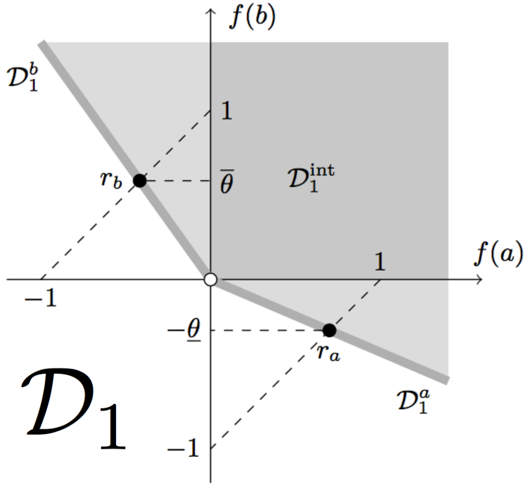
A coherent set $\mathcal{D}_1$ of desirable gambles

Imprecise Bernoulli processes

Single **Bernoulli experiment** $\mathcal{D}_1$



Imprecise Bernoulli processes



Single **Bernoulli experiment** $\mathcal{D}_1$

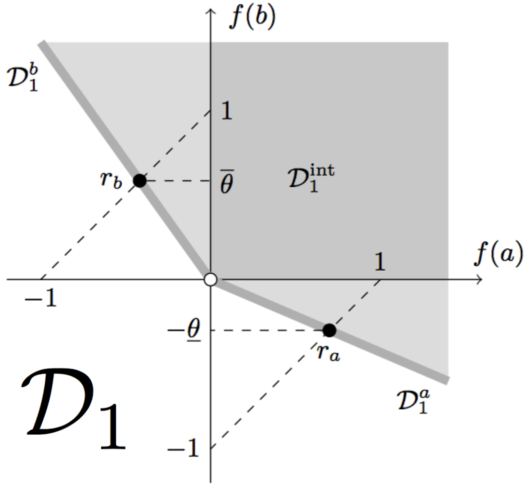
Imprecise Bernoulli process

Infinite sequence $X_1, X_2, \dots, X_n, \dots$

Family of coherent sets $\mathcal{D}_n$ of desirable gambles $\mathscr{X}^n$

(for all $n \in \mathbb{N}_0$)

Imprecise Bernoulli processes



Single **Bernoulli experiment** $\mathcal{D}_1$

Imprecise Bernoulli process

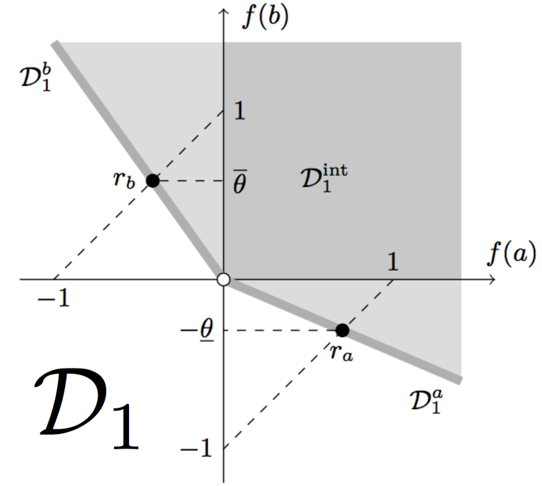
Infinite sequence $X_1, X_2, \ldots, X_n, \ldots$

Family of coherent sets $\mathcal{D}_n$ of desirable gambles $\mathscr{X}^n$

time consistent!

(for all $n \in \mathbb{N}_0$)

Imprecise Bernoulli processes



Single **Bernoulli experiment** $\mathcal{D}_1$

- **Exchangeability**
- **Epistemic independence**

Imprecise Bernoulli process

Infinite sequence $X_1, X_2, \dots, X_n, \dots$

Family of coherent sets $\mathcal{D}_n$ of desirable gambles $\mathscr{X}^n$

time consistent!

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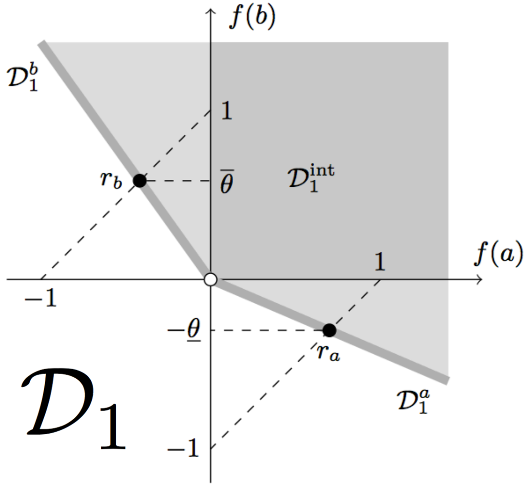
Imprecise Bernoulli processes



Single **Bernoulli experiment** $\mathcal{D}_1$

- $\boxed{\textbf{Exchangeability}}$
- **Epistemic independence**

Imprecise Bernoulli process

Infinite sequence $X_1, X_2, \ldots, X_n, \ldots$

Family of coherent sets $\mathcal{D}_n$ of desirable gambles $\mathscr{X}^n$

(for all $n \in \mathbb{N}_0$)

time consistent!

Imprecise Bernoulli processes

Exchangeability for Sets of Desirable gambles

Gert de Cooman

Erik Quaeghebeur

Exchangeability

Consider any permutation **π** of the set of indices **$\{1, 2, \dots, n\}$**

For any **$x = (x_1, x_2, \dots, x_n)$** in $\mathscr{X}^n$ we let **$\pi x := (x_{\pi(1)}, x_{\pi(2)}, \dots, x_{\pi(n)})$**

For any **gamble f** on $\mathscr{X}^n$ we let **$\pi^t f := f \circ \pi$**, so **$(\pi^t f)(x) = f(\pi x)$**

Imprecise Bernoulli processes

Exchangeability

Consider any permutation π of the set of indices **{1, 2, … , n}**

For any $\mathbf{x} = (x_1, x_2, \dots, x_n)$ in $\mathscr{X}^n$ we let $\pi\mathbf{x} := (x_{\pi(1)}, x_{\pi(2)}, \dots, x_{\pi(n)})$

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$X_1, X_2, \dots, X_n$ is assessed to be exchangeable

$\Leftrightarrow$ You are willing to exchange **f** for $\pi^t f$

Exchangeability

Consider any permutation π of the set of indices $\{1, 2, \dots, n\}$

For any $x = (x_1, x_2, \dots, x_n)$ in $\mathscr{X}^n$ we let $\pi x := (x_{\pi(1)}, x_{\pi(2)}, \dots, x_{\pi(n)})$

For any **gamble f** on $\mathscr{X}^n$ we let $\pi^t f := f \circ \pi$, so $(\pi^t f)(x) = f(\pi x)$

$X_1, X_2, \dots, X_n$ is assessed to be exchangeable

⟺ You are willing to exchange **f** for $\pi^t f$

$\mathcal{D}_n$ is exchangeable

⟺ $f - \pi^t f$ is (weakly) desirable $\approx f - \pi^t f \in \mathcal{D}_n$

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence $X_1, X_2, \ldots, X_n, \ldots$

Family of coherent sets $\mathcal{D}_n$ of desirable gambles $\mathscr{X}^n$

time consistent! (for all $n \in \mathbb{N}_0$)

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence $X_1, X_2, \ldots, X_n, \ldots$

Family of coherent sets $\mathcal{D}_n$ of desirable gambles $\mathscr{X}^n$

time consistent! (for all $n \in \mathbb{N}_0$)

Each $\mathcal{D}_n$ should be exchangeable !

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence $X_1, X_2, \ldots, X_n, \ldots$

Family of coherent sets $\mathcal{D}_n$ of desirable gambles $\mathscr{X}^n$

time consistent! (for all $n \in \mathbb{N}_0$)

$\boxed{\text{Each } \mathcal{D}_n \text{ should be exchangeable !}}$

How to impose this property?

Imprecise Bernoulli processes

Exchangeability

BINOMIAL DISTRIBUTION (θ and n)

For every gamble f on $\mathscr{X}^n$:

E(f) = $Bn^n(f \mid \theta) = \sum_{x \in \mathcal{X}^n} f(x)p(x)$

$\theta^{n(a)}(1- \theta)^{n(b)}$

Imprecise Bernoulli processes

Exchangeability

BINOMIAL DISTRIBUTION (θ and n)

For every gamble f on $\mathscr{X}^n$:

$$E(f) = \boxed{Bn^n(f \mid \theta)} = \sum_{x \in \mathcal{X}^n} f(x)p(x)$$

$\theta^{n(a)}(1-\theta)^{n(b)}$

Polynomial function of θ

$Bn^n(f) := Bn^n(f \mid \theta)$ $(\deg(p) \leq n)$

Exchangeability

gamble f on $\mathscr{X}^n$

$$\Big\downarrow \mathrm{Bn}^n$$

Polynomial function of θ

$\mathrm{Bn}^n(\ f\ ) := \mathrm{Bn}^n(\ f \mid \theta\ )$ $\quad$ $(\deg(p) \leq n)$

Exchangeability

2^n-dimensional space

gamble f on $\mathscr{X}^n$

Bn^n

(n+1)-dimensional space

Polynomial function of θ

$\mathrm{Bn}^n(\, f\,) := \mathrm{Bn}^n(\, f \mid \theta\,)$ (deg(p) $\leq$ n)

Imprecise Bernoulli processes

Exchangeability

2^n-dimensional space

gamble f on $\mathscr{X}^n$

Bn^n

$\boxed{\mathrm{Bn}^n(\, \boldsymbol{\pi} f \,) = \mathrm{Bn}^n(\, f \,)}$

(n+1)-dimensional space

Polynomial function of θ

$\mathrm{Bn}^n(\, f \,) := \mathrm{Bn}^n(\, f \mid \theta \,)$ (deg(p) ≤ n)

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence $X_1, X_2, \ldots, X_n, \ldots$

Family of coherent sets $\mathcal{D}_n$ of desirable gambles $\mathscr{X}^n$

time consistent! (for all $n \in \mathbb{N}_0$)

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence $X_1, X_2, \ldots, X_n, \ldots$

Family of coherent sets $\mathcal{D}_n$ of desirable gambles $\mathscr{X}^n$

time consistent! (for all $n \in \mathbb{N}_0$)

Bn^n $\big\downarrow$

Set $\mathcal{H}$ of polynomial functions

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence $X_1, X_2, \dots, X_n, \dots$

Family of coherent sets $\mathcal{D}_n$ of desirable gambles $\mathscr{X}^n$

time consistent! (for all $n \in \mathbb{N}_0$)

$\mathrm{Bn}^n \ \Big\downarrow \qquad \cancel{\Big\uparrow}\ (\mathrm{Bn}^n)^{-1}$

$\boxed{\text{Set } \mathcal{H} \text{ of polynomial functions}}$

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence $X_1, X_2, \dots, X_n, \dots$

Family of coherent sets $\mathcal{D}_n$ of desirable gambles $\mathscr{X}^n$

time consistent! (for all $n \in \mathbb{N}_0$)

$\mathrm{Bn}^n \downarrow \quad \uparrow (\mathrm{Bn}^n)^{-1}$

Each $\mathcal{D}_n$ should be exchangeable !

Set $\mathcal{H}$ of polynomial functions

Imprecise Bernoulli processes

Exchangeability

Infinite exchangeable sequence $X_1, X_2, \dots, X_n, \dots$

Bernstein coherent:

B1. *if* $p = 0$ *then* $p \notin \mathcal{H}$;
B2. *if* $p \in \mathcal{V}^+$, *then* $p \in \mathcal{H}$;
B3. *if* $p \in \mathcal{H}$ *then* $\lambda p \in \mathcal{H}$;
B4. *if* $p_1, p_2 \in \mathcal{H}$ *then* $p_1 + p_2 \in \mathcal{H}$.

Set $\mathcal{H}$ of polynomial functions

Imprecise Bernoulli processes

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B4. *if* $p_1, p_2 \in \mathcal{H}$ *then* $p_1 + p_2 \in \mathcal{H}$.

$p(\theta) > 0$
$\forall \theta \in]0,1[$

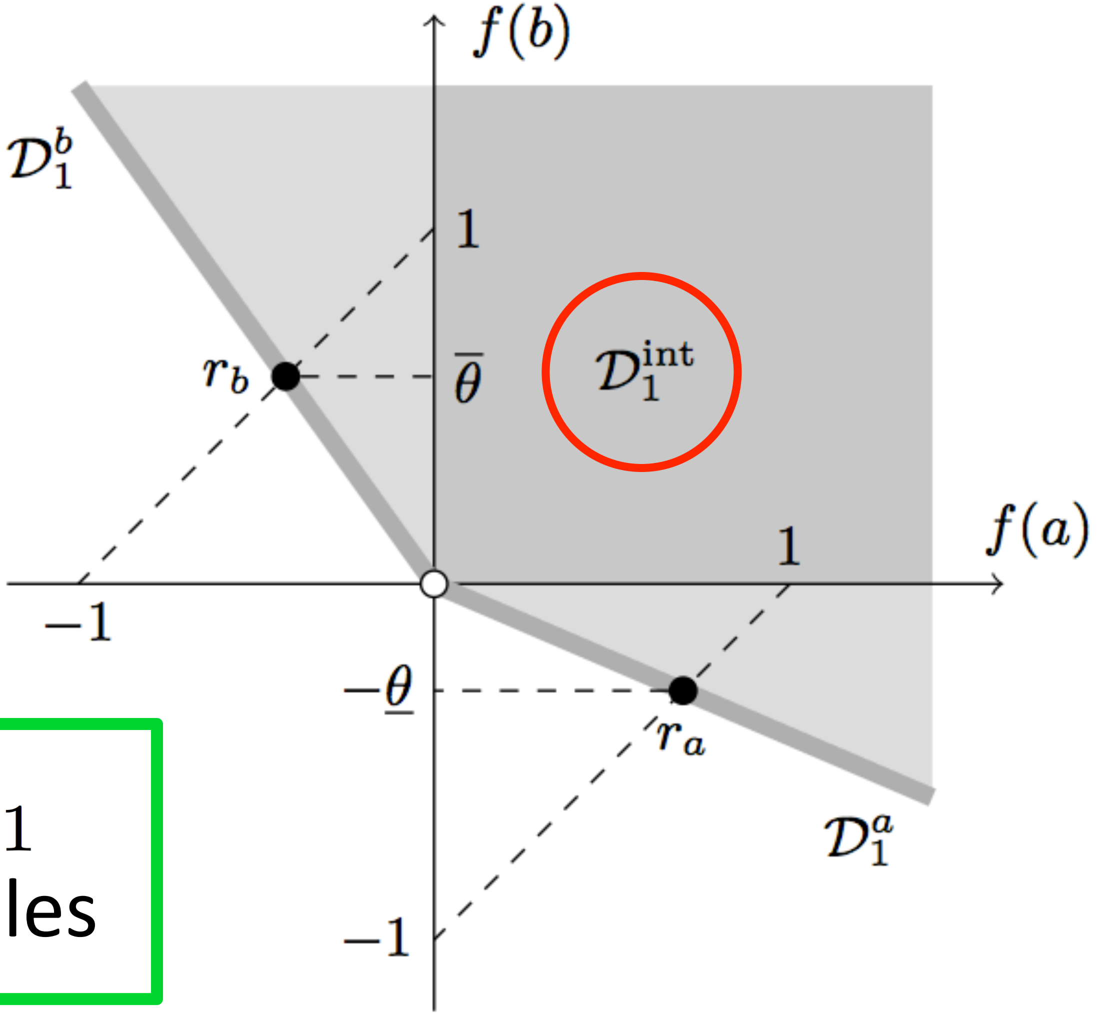
Set $\mathcal{H}$ of polynomial functions

Imprecise Bernoulli processes

Exchangeability

$0 \cancel{\leq} \underline{\theta} \cancel{\leq} \overline{\theta} \cancel{\leq} 1$

$<\quad<\quad<$

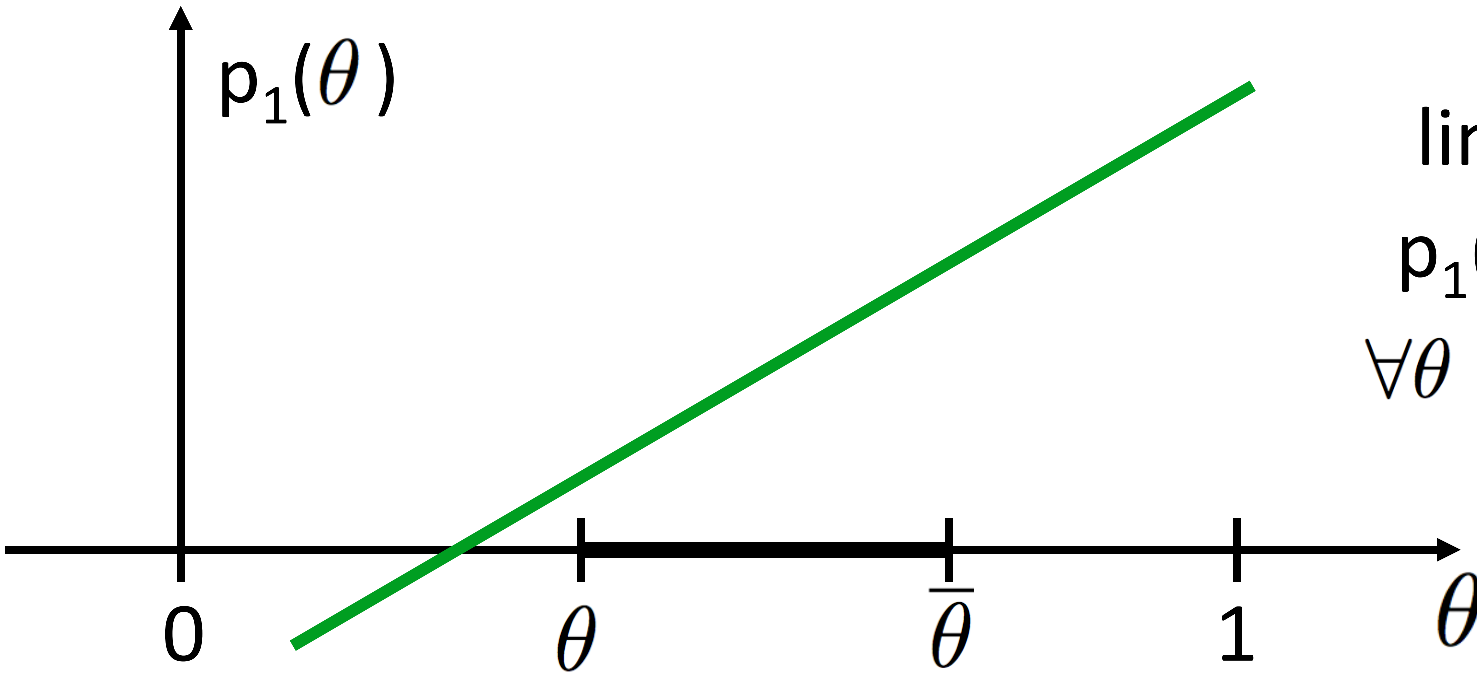


A coherent set $\mathcal{D}_1$ of desirable gambles

Imprecise Bernoulli processes

Exchangeability

$\mathcal{H}_1$



linear p_1

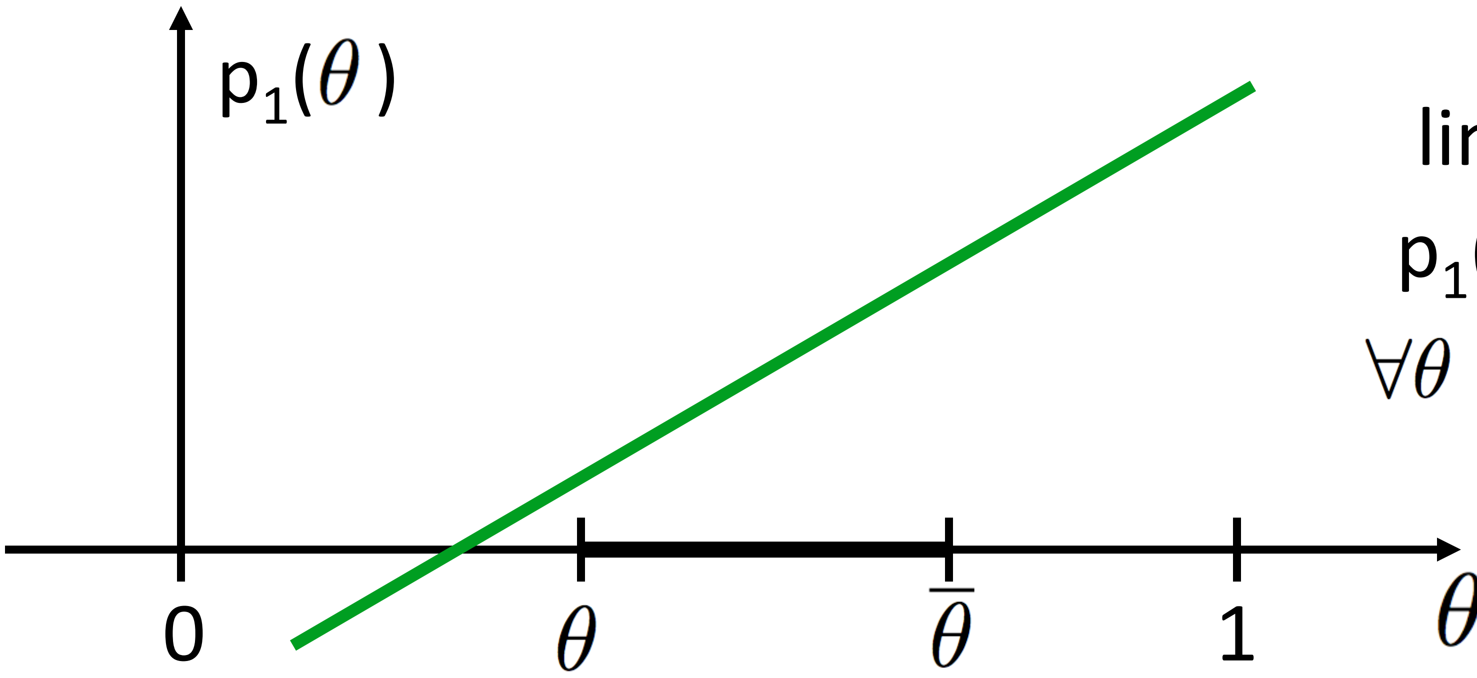
$p_1(\theta) > 0$

$\forall\theta \in [\underline{\theta}, \overline{\theta}]$

Imprecise Bernoulli processes

Exchangeability

$\mathcal{H}_1$



linear p_1

$p_1(\theta) > 0$

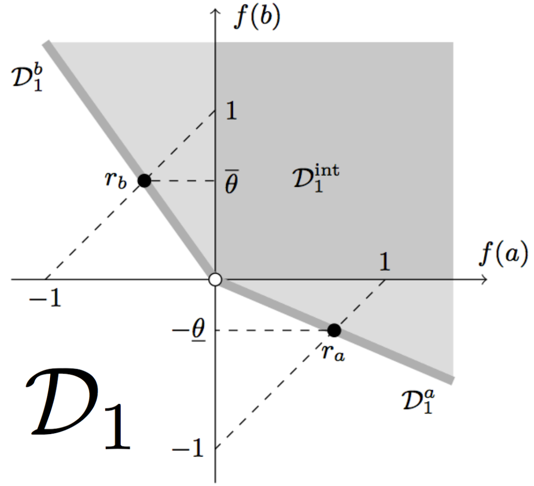
$\forall\theta \in [\underline{\theta}, \overline{\theta}]$

Set $\mathcal{H}$ of polynomial functions

?

Imprecise Bernoulli processes

Single **Bernoulli experiment** $\mathcal{D}_1$



- **Exchangeability**
- **Epistemic independence**

Imprecise Bernoulli process

Imprecise Bernoulli processes

Epistemic independence

Infinite sequence $X_1, X_2, \ldots, X_n, \ldots$

assessment of epistemic independence

Imprecise Bernoulli processes

Epistemic independence

Infinite sequence $X_1, X_2, \ldots, X_n, \ldots$

assessment of epistemic independence

$\Updownarrow$

Learning the value of any **finite number of variables does not change our beliefs about** any finite subset of the **remaining, unobserved ones**.

Imprecise Bernoulli processes

Epistemic independence

Infinite sequence $X_1 , X_2 , \dots , X_n , \dots$

assessment of epistemic independence

$\boxed{\text{Set } \mathcal{H} \text{ of polynomial functions}}$ **?**

Imprecise Bernoulli processes

Epistemic independence

Infinite sequence $X_1, X_2, \ldots, X_n, \ldots$

assessment of epistemic independence

$$p \in \mathcal{H} \Leftrightarrow \theta p \in \mathcal{H}$$

$$p \in \mathcal{H} \Leftrightarrow (1-\theta) p \in \mathcal{H}$$

$\boxed{\text{Set } \mathcal{H} \text{ of polynomial functions}}$?

Imprecise Bernoulli processes

Exchangeability:

Set $\mathcal{H}$ of polynomial functions

Bernstein coherent:

B1. *if* $p = 0$ *then* $p \notin \mathcal{H}$;
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Epistemic independence:

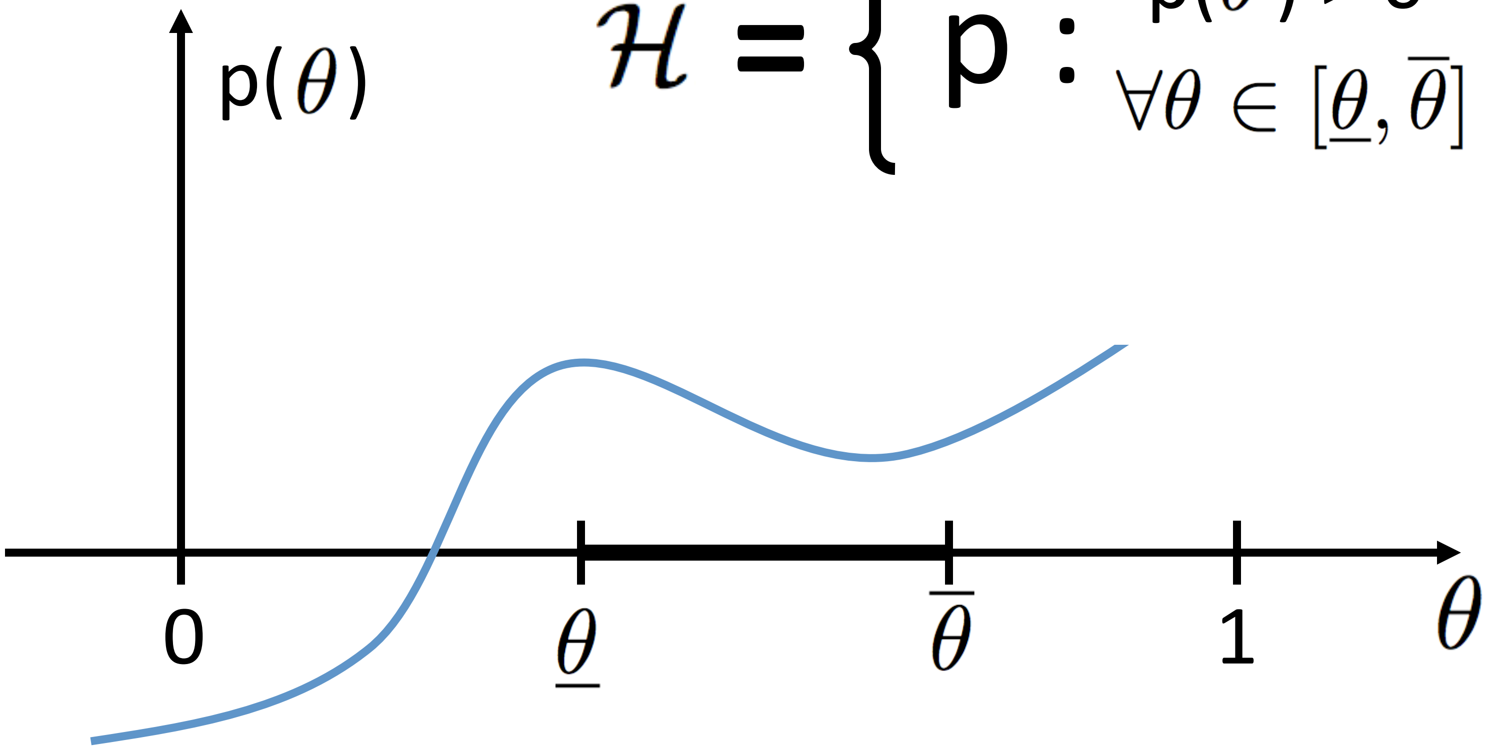
$$p \in \mathcal{H} \Leftrightarrow \theta p \in \mathcal{H}$$

$$p \in \mathcal{H} \Leftrightarrow (1-\theta) p \in \mathcal{H}$$

We are looking for the smallest such set $\mathcal{H}$
(most conservative inferences) **that contains $\mathcal{H}_1$**

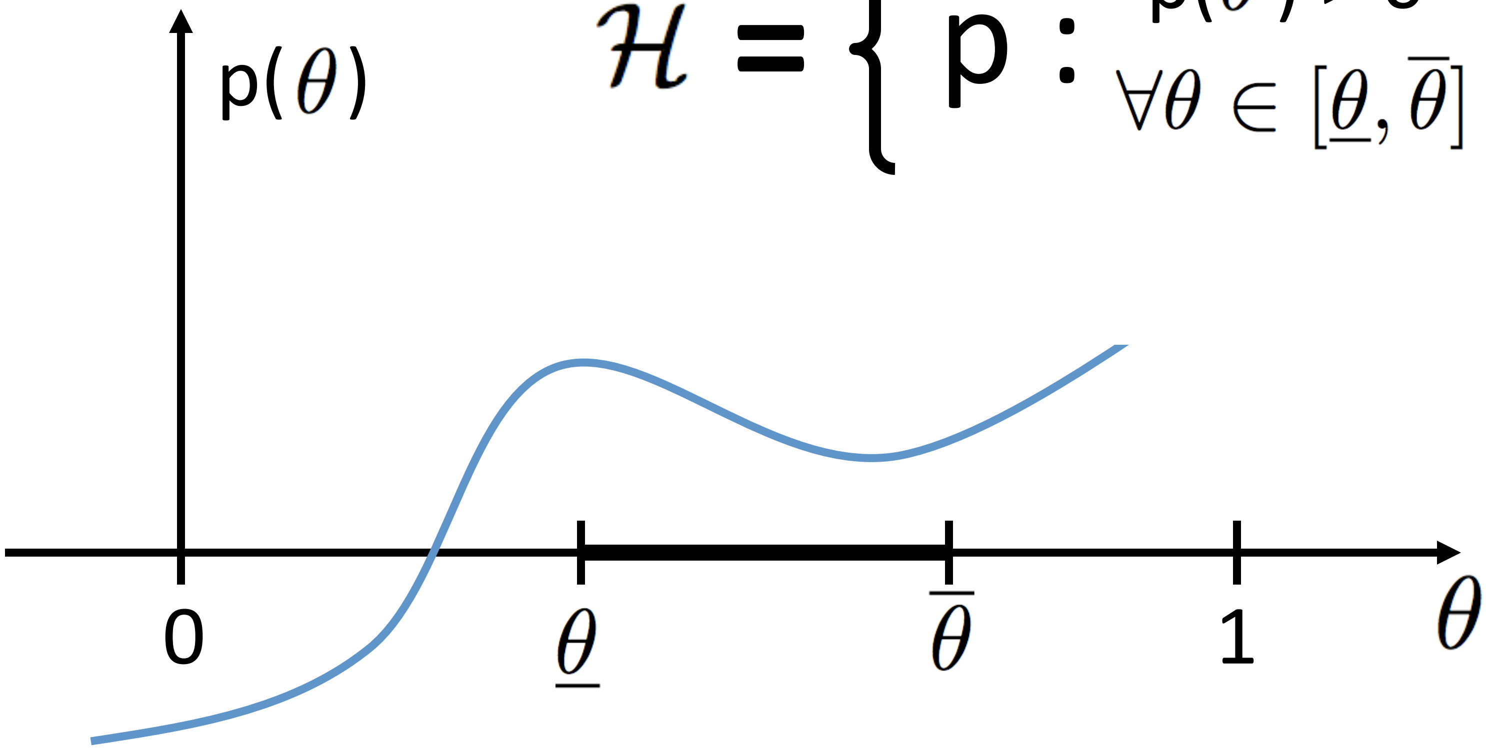
Imprecise Bernoulli processes

$$\mathcal{H} = \left\{ \mathrm{p} : \begin{array}{c} \mathrm{p}(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\} \textbf{?}$$



Imprecise Bernoulli processes

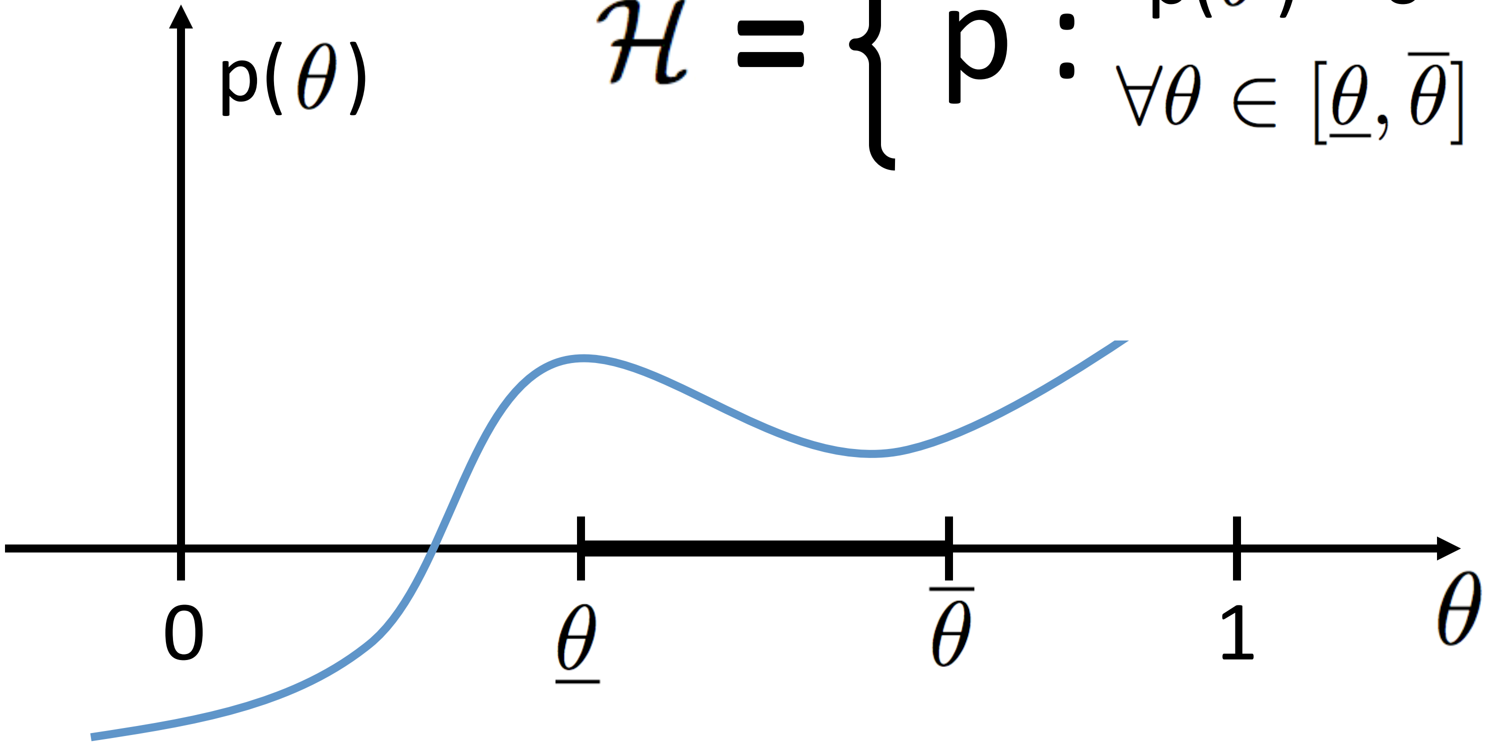
$$\mathcal{H} = \left\{ p : \begin{array}{c} p(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\} \; ?$$



B1. *if* $p = 0$ *then* $p \notin \mathcal{H}$ ✔

Imprecise Bernoulli processes

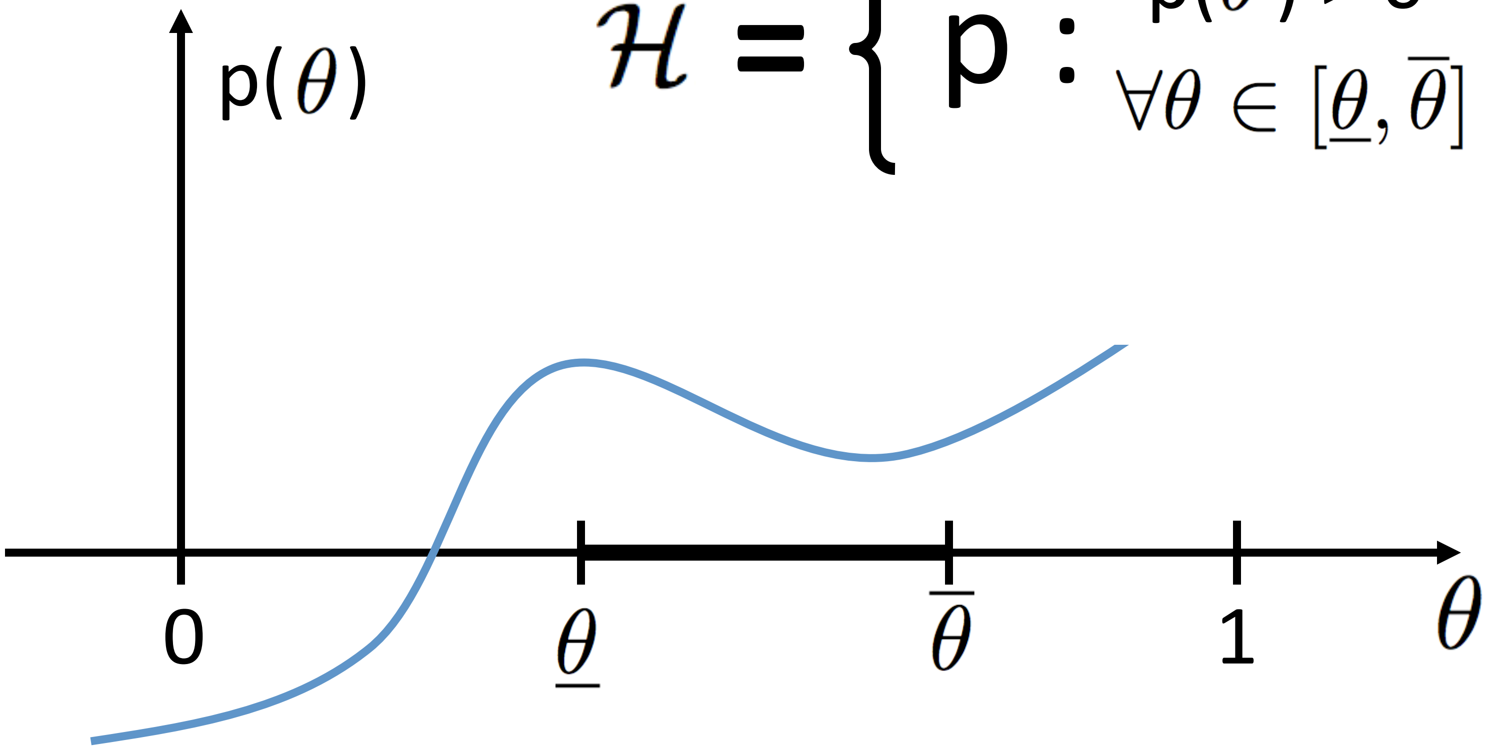
$$\mathcal{H} = \left\{ p : \begin{array}{c} p(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\} \; ?$$



B2. *if* $p \in \mathcal{V}^+$, *then* $p \in \mathcal{H}$ ✔

Imprecise Bernoulli processes

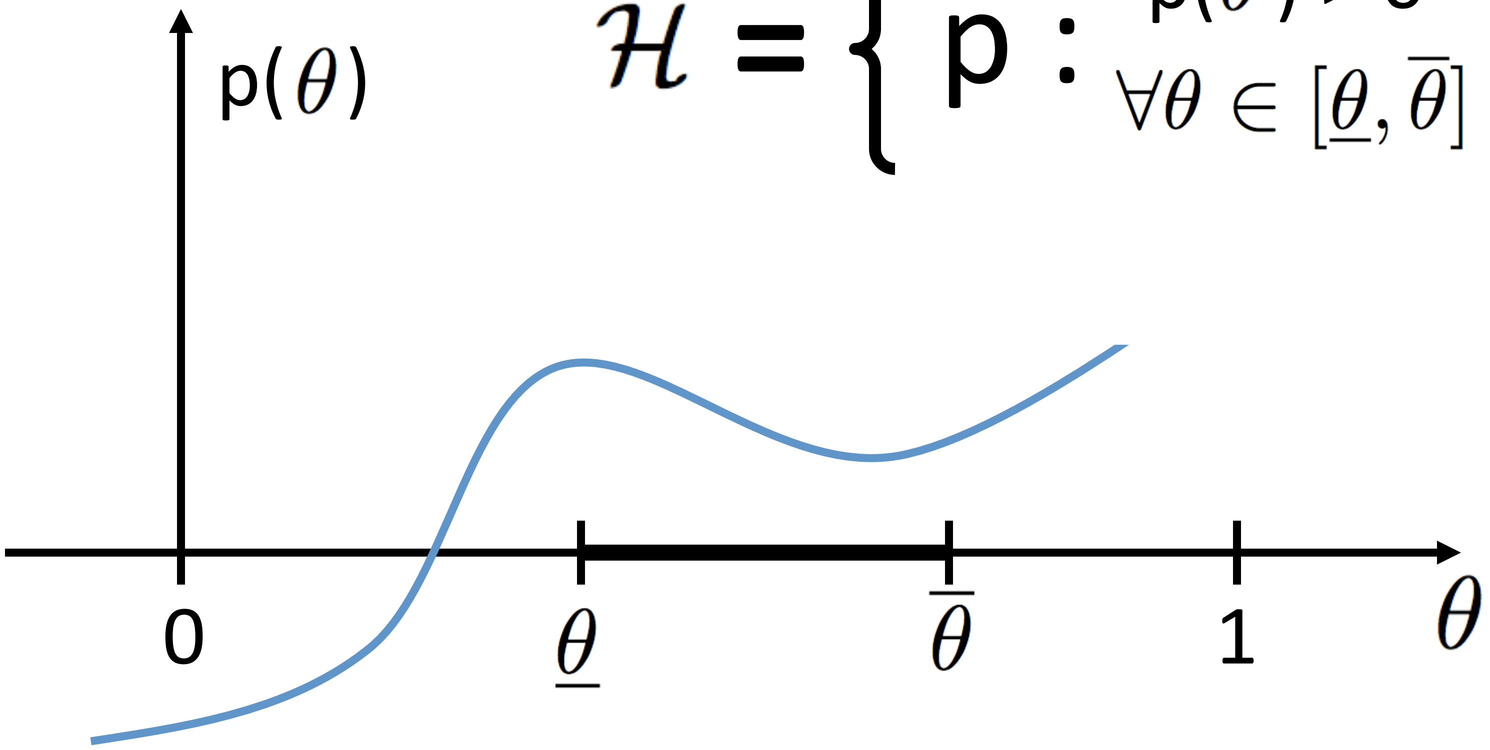
$$\mathcal{H} = \left\{ \mathrm{p} : \begin{array}{c} \mathrm{p}(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\} ?$$



B3. *if* $p \in \mathcal{H}$ *then* $\lambda p \in \mathcal{H}$ ✔

Imprecise Bernoulli processes

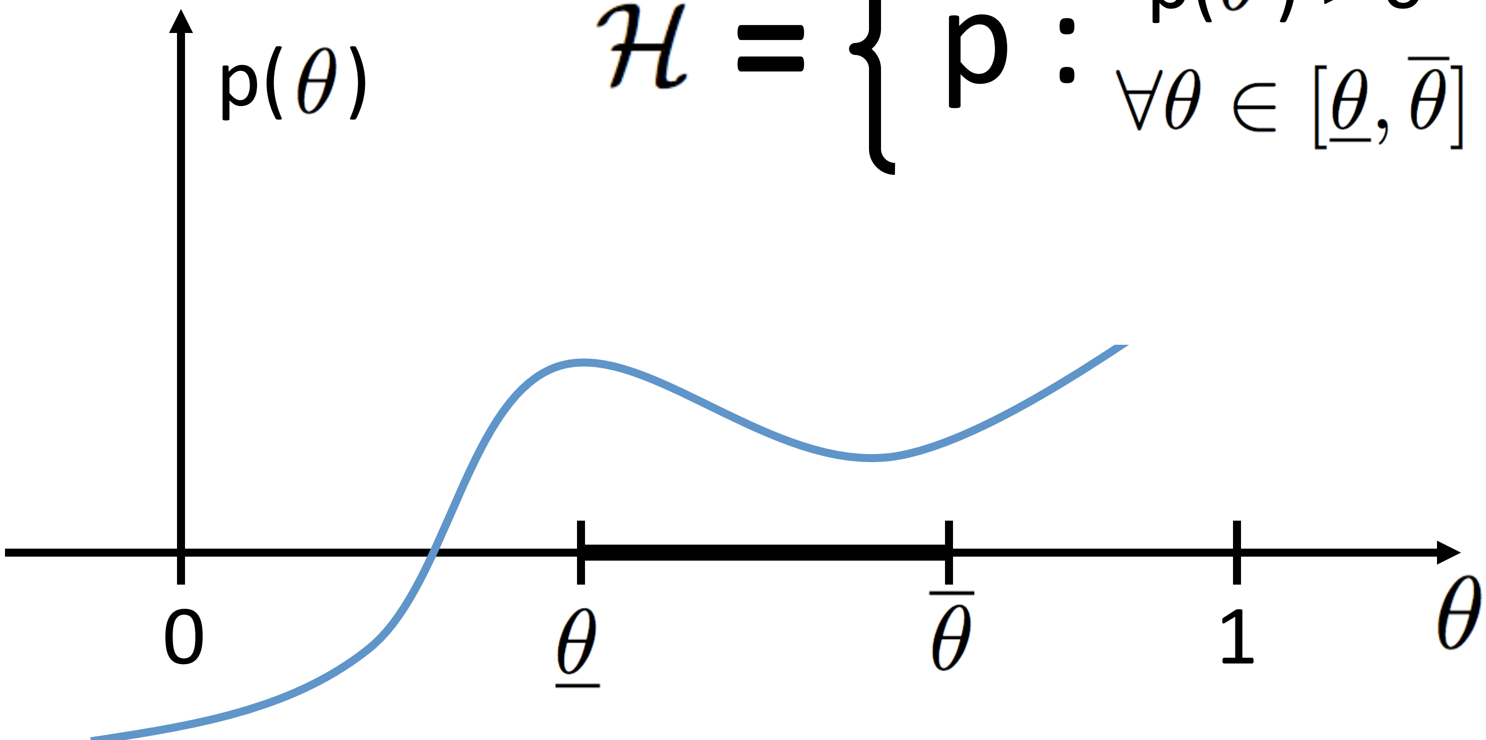
$$\mathcal{H} = \left\{ p : \begin{array}{c} p(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\} \ ?$$



B4. *if* $p_1, p_2 \in \mathcal{H}$ *then* $p_1 + p_2 \in \mathcal{H}$ ✔

Imprecise Bernoulli processes

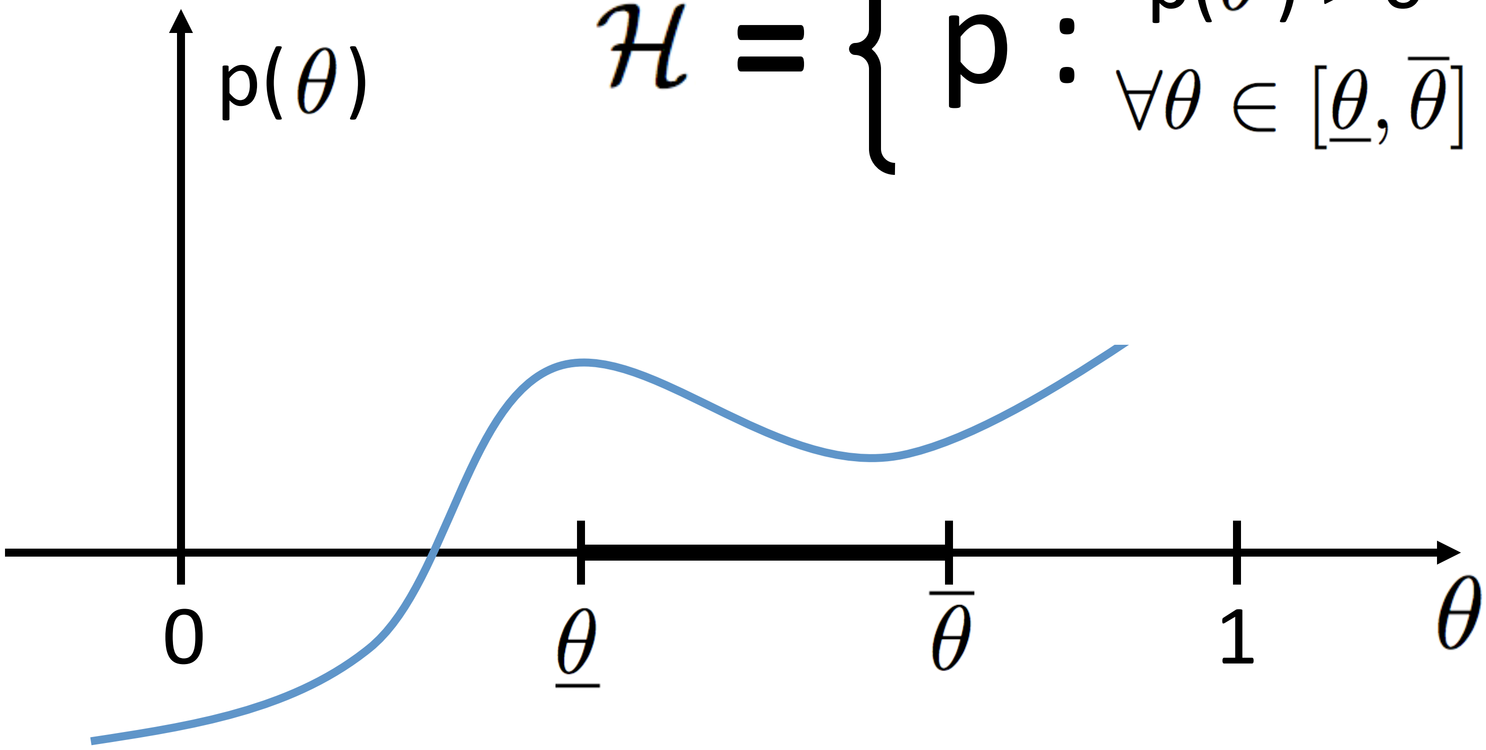
$$\mathcal{H} = \left\{ \mathrm{p} : \begin{matrix} \mathrm{p}(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{matrix} \right\} \; ?$$



$$p \in \mathcal{H} \begin{matrix} \Leftrightarrow \theta p \in \mathcal{H} \\ \Leftrightarrow (1-\theta) p \in \mathcal{H} \end{matrix} \quad ✔$$

Imprecise Bernoulli processes

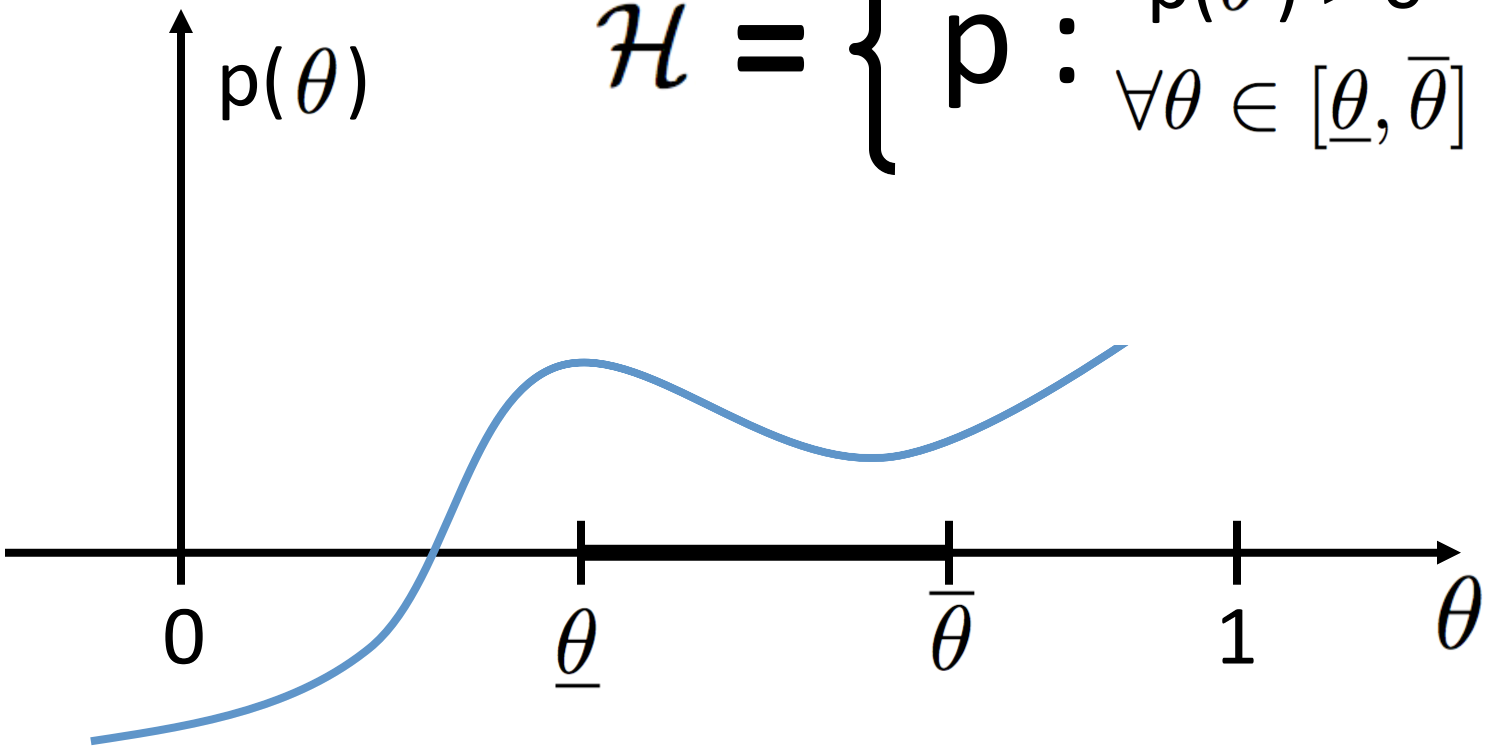
$$\mathcal{H} = \left\{ \mathrm{p} : \begin{array}{c} \mathrm{p}(\theta) > 0 \\ \forall\theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\} \text{?}$$



contains $\mathcal{H}_1$ ✔

Imprecise Bernoulli processes

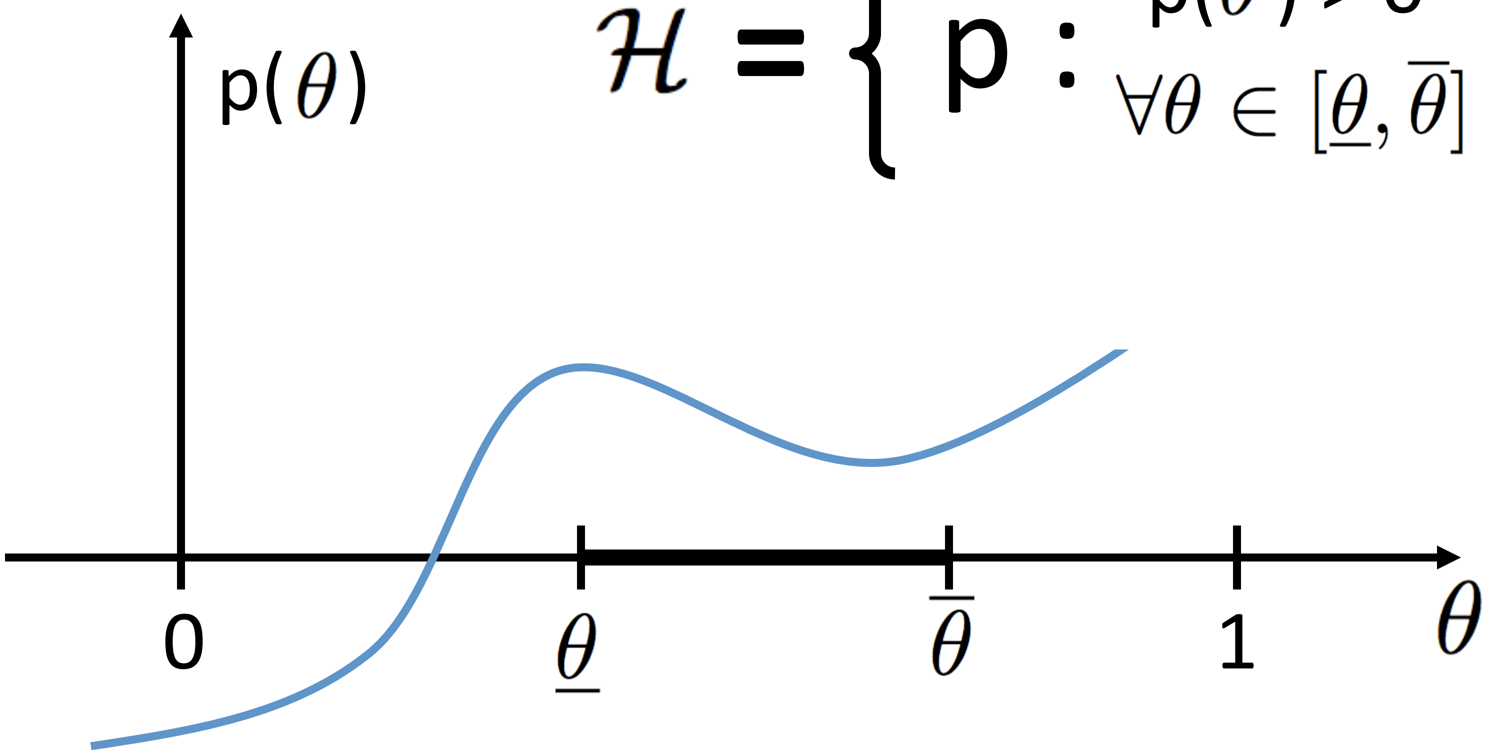
$$\mathcal{H} = \left\{ p : \begin{array}{c} p(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\} \textbf{?}$$



Smallest such set ?

Imprecise Bernoulli processes

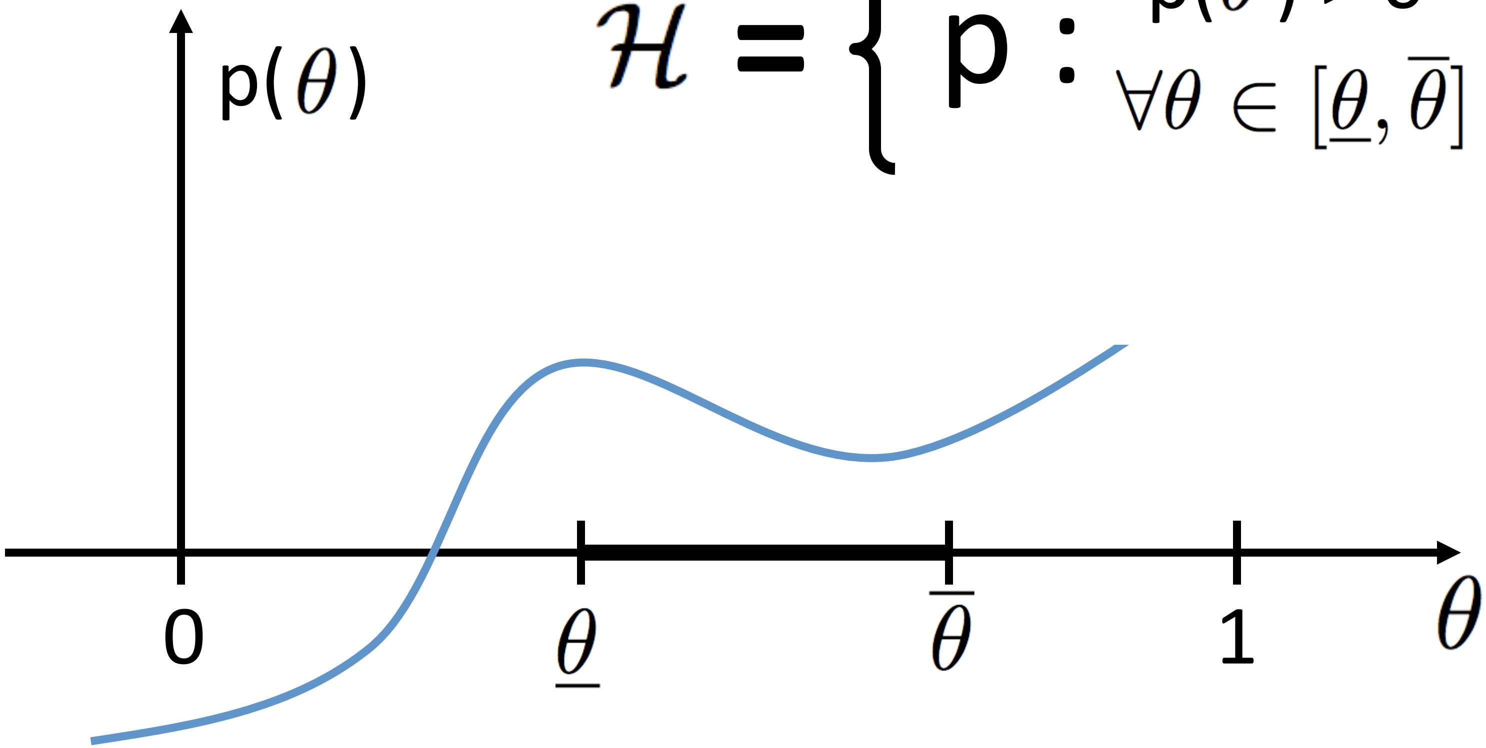
$$\mathcal{H} = \left\{ \mathrm{p} : \begin{array}{c} \mathrm{p}(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\} \ ?$$



Smallest such set ✔

Imprecise Bernoulli processes

$$\mathcal{H} = \left\{ p : \begin{array}{c} p(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\} \ !$$



Smallest such set ✔

Imprecise Bernoulli processes

$$\mathcal{D}_1$$

Imprecise Bernoulli processes

$$\mathcal{D}_1 \rightarrow \mathcal{H}_1$$

$$\mathrm{Bn}^n(\,f\,) := \mathrm{Bn}^n(\,f \mid \theta\,)$$

Imprecise Bernoulli processes

$\mathcal{D}_1$

$\mathrm{Bn}^n(\,f\,) := \mathrm{Bn}^n(\,f \mid \theta\,)$

$\mathcal{H}_1$

smallest epistemic independent set of polynomials

$$\mathcal{H} = \left\{ p : \begin{array}{c} p(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\}$$

Imprecise Bernoulli processes

$\mathcal{D}_1$

$\mathrm{Bn}^n(f) := \mathrm{Bn}^n(f \mid \theta)$

$\mathcal{H}_1$

smallest epistemic independent set of polynomials

$$\mathcal{H} = \left\{ p : \begin{array}{c} p(\theta) > 0 \\ \forall \theta \in [\underline{\theta}, \overline{\theta}] \end{array} \right\}$$

exchangeability

Family of coherent sets $\mathcal{D}_n$ of desirable gambles $\mathscr{X}^n$

time consistent! (for all $n \in \mathbb{N}_0$)

Imprecise Bernoulli processes

Link with sensitivity analysis

For every gamble f on $\mathscr{X}^n$:

$$\underline{E}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\}$$

Supremum acceptable buying price

Imprecise Bernoulli processes

Link with sensitivity analysis

For every gamble f on $\mathscr{X}^n$:

$$\underline{E}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\}$$

$$= \sup\{\mu \in \mathbb{R} : \mathrm{Bn}^n(f) - \mu \in \mathcal{H}\}$$

Imprecise Bernoulli processes

Link with sensitivity analysis

For every gamble f on $\mathscr{X}^n$:

$$\underline{E}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\}$$

$$= \sup\{\mu \in \mathbb{R} : \mathrm{Bn}^n(f) - \mu \in \mathcal{H}\}$$

$$\Updownarrow$$

$$\mathrm{Bn}^n(f \mid \theta) - \mu \in \mathcal{H}$$

Imprecise Bernoulli processes

Link with sensitivity analysis

For every gamble f on $\mathscr{X}^n$:

$$\underline{E}(f) := \sup\{\mu \in \mathbb{R} : f - \mu \in \mathcal{D}_n\}$$

$$= \sup\{\mu \in \mathbb{R} : \mathrm{Bn}^n(f) - \mu \in \mathcal{H}\}$$

$$\Updownarrow$$

$$\mathrm{Bn}^n(f \mid \theta) - \mu \in \mathcal{H}$$

$$\Updownarrow$$

$$\mathrm{Bn}^n(f \mid \theta) - \mu > 0 \quad \forall \theta \in [\underline{\theta}, \overline{\theta}]$$

Imprecise Bernoulli processes

Link with sensitivity analysis

For every gamble f on $\mathscr{X}^n$:

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$$= \sup\{\mu \in \mathbb{R} : \mathrm{Bn}^n(f) - \mu \in \mathcal{H}\}$$

$$= \sup\{\mu \in \mathbb{R} : \mathrm{Bn}^n(f \mid \theta) > \mu \ \forall\theta \in [\underline{\theta}, \overline{\theta}]\}$$

Imprecise Bernoulli processes

Link with sensitivity analysis

For every gamble f on $\mathscr{X}^n$:

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$$= \sup\{\mu \in \mathbb{R} : \mathrm{Bn}^n(f) - \mu \in \mathcal{H}\}$$

$$= \sup\{\mu \in \mathbb{R} : \mathrm{Bn}^n(f \mid \theta) > \mu \ \forall\theta \in [\underline{\theta}, \overline{\theta}]\}$$

$$\boxed{= \min\{\mathrm{Bn}^n(f|\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}}$$

Link with sensitivity analysis

Sensitivity analysis:

$$\overline{E}(f) = \max\{\ \mathsf{Bn}^{\mathsf{n}}(f|\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$
$$\underline{E}(f) = \min\{\ \mathsf{Bn}^{\mathsf{n}}(f|\theta) : \theta \in [\underline{\theta}, \overline{\theta}]\}$$

Imprecise Bernoulli processes

EXCHANGEABILITY

+

EPISTEMIC INDEPENDENCE

↓

SENSITIVITY ANALYSIS