Do multiplication and division strategies rely on executive and phonological working-memory resources?

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Abstract

The role of executive and phonological working-memory resources in simple arithmetic was investigated in two experiments. Participants had to solve simple multiplication problems (e.g., 4 x 8; Experiment 1) or simple division problems (e.g., 42 : 7; Experiment 2) under no-load, phonological-load, and executive-load conditions. The choice/no-choice method was used to investigate strategy execution and strategy selection independently. Results on strategy execution showed that executive working-memory resources were involved in direct memory retrieval of both multiplication and division facts. Executive working-memory resources were also needed to execute non-retrieval strategies. Phonological working-memory resources, on the other hand, tended to be involved in non-retrieval strategies only. Results on strategy selection showed no effects of working-memory load. Finally, correlation analyses showed that both strategy execution and strategy selection correlated with individual-difference variables such as gender, math anxiety, associative strength, calculator use, arithmetic skill, and math experience.

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Working memory is a system devoted to short-term storage and processing, and is used in various cognitive tasks such as reading, reasoning, and mental arithmetic. The last decennia, research into the role of working memory in mental arithmetic has flourished (for review, see DeStefano & LeFevre, 2004) and showed that solving both simple-arithmetic problems (e.g., 8 + 5, 3 x 9) and complex-arithmetic problems (e.g., 23 + 98, 12 x 35) relies on working-memory resources. The present study further investigates the role of working memory in simple-arithmetic strategies, based on the multi-component working-memory model of Baddeley and Hitch (1974). In this model there is an attentional system (the central executive) that supervises a phonological subsystem and a visuo-spatial subsystem, which guarantee short-term maintenance of phonological and visuo-spatial information, respectively.

The role of executive working-memory resources in simple arithmetic has been shown extensively (e.g., Ashcraft, 1995; De Rammelaere, Stuyven, & Vandierendonck, 1999, 2001; De Rammelaere & Vandierendonck, 2001; Deschuyteneer & Vandierendonck, 2005a, 2005b; Deschuyteneer, Vandierendonck, & Coeman, 2006; Deschuyteneer, Vandierendonck, & Muyllaert, 2006b; Hecht, 2002; Lemaire, Abdi, & Fayol, 1996; Seitz & Schumann-Hengsteler, 2000, 2002). The role of phonological working-memory resources in simple arithmetic is less clear. In some studies an effect of phonological load on simple-arithmetic problem solving was observed (e.g., Lee & Kang, 2002; Lemaire et al., 1996; Seitz & Schumann-Hengsteler, 2002) whereas in other studies it was not (e.g., De Rammelaere et al., 1999, 2001; Seitz & Schumann-Hengsteler, 2000). The role of the visuo-spatial sketch pad in simple arithmetic has only scarcely been investigated (but see Lee & Kang, 2002; Seitz & Schumann-Hengsteler, 2000) and is equivocal.

A drawback of all the studies mentioned above, however, is that none of them made a separation between retrieval and non-retrieval trials. Yet, it has been shown that adults use several strategies to solve even the simplest arithmetic problems (e.g., Hecht, 1999; LeFevre, Bisanz, et al., 1996a; LeFevre, Sadesky, & Bisanz, 1996b). More specifically, although direct memory retrieval (i.e., 'knowing' that 3 x 4 = 12) is the most frequently used strategy, non-retrieval strategies (or 'procedural' strategies) are used as well. Examples of such procedural strategies are transformation (e.g., 9 x 6 = (10 x 6) – 6 = 60 – 6 = 54) and counting (e.g., 4 x 7 = 7… 14… 21… 28). Based on the studies mentioned above, it is impossible to know *in which* simple-arithmetic strategies executive and phonological working-memory resources are needed.

The role of executive and phonological working-memory across different simple-arithmetic strategies started to be investigated only very recently. Hecht (2002) conducted the first study on this topic. In his study, simple addition equations (e.g., 4 + 3 = 8) had to be verified under no load, phonological load, and executive load. After each trial, participants had to report which strategy they had used. Results showed that all strategies (i.e., retrieval, transformation, and counting) were slowed down under executive working-memory loads, whereas only the counting strategy was slowed down under phonological working-memory loads. Based on regression analyses however, Hecht concluded that retrieval does not rely on the central executive, whereas the counting strategy would rely on both executive and phonological working-memory resources.

Seyler, Kirk, and Ashcraft (2003) also studied the role of working memory in simple-arithmetic strategies. In a first experiment, simple subtraction problems had to be solved while a 2-, 4-, or 6-letter string had to be remembered. Results showed that solving subtraction problems with minuends of 11 or greater (e.g., 11 – 5) relied more heavily on working memory than problems with minuends smaller than 11 (e.g., 8 – 5). In another experiment, using strategy reports, Seyler et al. (2003) showed that subtraction problems with minuends of 11 or greater were more frequently solved with procedural strategies than problems with minuends smaller than 11. It was concluded that working memory is more involved when simple subtraction problems are solved via procedural strategies.

A drawback of both previous studies is that neither Hecht (2002) nor Seyler et al. (2003) controlled for strategy selection effects, since participants were always free to choose any strategy they wanted. Consequently, non-retrieval strategies will have been executed more frequently on large problems while retrieval will have been executed more frequently on small problems. Such strategy selection effects might have influenced strategy efficiency data and all resulting conclusions. In order to exclude such biasing effects of strategy selection on strategy efficiency, the choice/no-choice method (Siegler & Lemaire, 1997) should be used. Using the choice/no-choice method in combination with selective working-memory loads provides unbiased data about the role of working memory in strategy selection and strategy efficiency.

The combination of the choice/no-choice method and selective working-memory loads has first been used by Imbo and Vandierendonck (in press). They investigated the role of executive and phonological working-memory resources in simple-arithmetic strategies. In their study, simple addition and subtraction problems had to be solved under no-load, passive-phonological load, active-phonological load, or central-executive load conditions. Results showed that retrieval of addition and subtraction facts relied on executive working-memory resources. Solving addition or subtraction problems by means of a non-retrieval strategy on the other hand, required both executive and active-phonological working-memory resources. The passive phonological store was only involved when counting was used to solve subtraction problems. Obviously, the role of executive and phonological working-memory resources was significantly larger in non-retrieval strategies (i.e., transformation and counting) than in direct memory retrieval.

To summarize, the three studies described above showed that the role of working memory differs across strategies. Whether or not the central executive is needed in retrieval remains a debated topic: Hecht (2002) does not believe that this working-memory component is needed in retrieval whereas Imbo and Vandierendonck (in press) presented evidence that retrieval requires executive working-memory resources. Nevertheless, all three studies seem to agree that phonological working-memory resources are needed when non-retrieval strategies are used to solve simple addition and/or subtraction problems.

Our knowledge about the role of working memory in addition and subtraction strategies may be scarce; the knowledge about the role of working memory in multiplication and division strategies is non-existent. Despite that fact that solving simple multiplication and division problems requires working-memory resources (De Rammelaere & Vandierendonck, 2001; Deschuyteneer & Vandierendonck, 2005b; Deschuyteneer et al., 2006a, 2006b; Lee & Kang, 2002; Seitz & Schumann-Hengsteler, 2000, 2002), up until now, no study investigated the role of working memory across the different multiplication and division strategies.

As multiplication and division can certainly not be seen as the counterparts of addition and subtraction, studying the role of working memory in multiplication and division strategies is much more than merely an extension of previous research. Indeed, there exist many differences across operations; and especially between addition and subtraction on the one hand and multiplication and division on the other hand. Differences across arithmetic operations start from childhood on and continue to exist in adulthood. First, addition and subtraction problem-solving procedures are taught before multiplication and division problem-solving procedures. Furthermore, the acquisition of addition and subtraction is mainly based on counting procedures, whereas the acquisition of multiplication and division is based on the memorization of problem-answer pairs. In adults, the highest percentages retrieval use are observed in multiplication (98%) whereas the lowest percentages retrieval use are observed in division (69%), with addition and subtraction in between (88% and 72%, respectively; Campbell & Xue, 2001). Adults' strategy efficiencies differ also greatly across operations, with multiplication RTs (930 ms) being much faster than division RTs (1086 ms, Campbell & Xue, 2001).

These results seem to suggest that access to long-term memory and selecting the correct response is very difficult for division but rather easy for multiplication. As getting access to long-term memory and selecting the correct response are processes requiring *executive* working-memory resources, one might rather be sure that an executive load will affect division efficiency, but it might be questioned whether an executive load will affect the over-learned retrieval of multiplication facts. It might further be expected that *phonological* working-memory loads will affect non-retrieval strategy efficiencies but not retrieval strategy efficiencies. Indeed, when non-retrieval strategies are used, intermediate values have to be kept temporary in working memory, a function accomplished by the phonological working-memory component (Ashcraft, 1995). Effects of phonological working-memory loads on non-retrieval strategies have been observed in addition and subtraction, but as several authors (e.g., Campbell, 1994; Dehaene, 1997) suppose that multiplication is more heavily based on auditory-verbal number codes than other operations are, effects of phonological working-memory loads may be more heavily apparent in the present study.

In order to investigate the role of executive and phonological working-memory resources[1] in multiplication and division strategies, the present study used two frequently used and approved methods: the selective-interference paradigm and the choice/no-choice method. The selective-interference paradigm is the methodological approach most frequently chosen for studying the role of different working-memory resources in mental arithmetic. It entails using both a single-task condition in which the primary task (mental arithmetic) is executed without any working-memory load and a dual-task condition in which the primary task is combined with a secondary task loading a specific working-memory component. If both primary and secondary task demand the same working-memory resources, performance decrements should be observed in either task. In the present study, three secondary tasks were used to load three specific working-memory components, more specifically the passive-phonological component (the phonological store), the active-phonological component (the sub-vocal rehearsal process), and the central executive.

The choice/no-choice method (designed by Siegler & Lemaire, 1997) is used to collect data on strategy selection (which strategies are chosen?) and strategy efficiency (are strategies executed efficiently?) independently. In this method, each participant is tested under two types of conditions: a choice condition in which participants are free to choose any strategy they want and no-choice conditions in which participants are required to solve all the problems with one particular strategy. There are as many no-choice conditions as there are strategies available in the choice condition. Data obtained in no-choice conditions provide information about strategy efficiency, whereas data gathered in the choice condition provide information about strategy selection.

Besides investigating the role of working memory in multiplication and division strategies, the present study also wanted to test whether simple-arithmetic strategies are influenced by factors *not* imposed by the experimenter. To this end, several individual-difference measures were obtained for each participant, namely arithmetic skill, math experience, gender, calculator use, math anxiety, and associative strength. Effects of *arithmetic skill* have already been reported (e.g., Campbell & Xue, 2001; Gilles, Masse, & Lemaire, 2001; Kirk and Ashcraft, 2001; LeFevre & Bisanz, 1986; LeFevre et al., 1996a, 1996b). Generally, strategy use is more efficient (i.e., faster) in high-skill participants than in low-skill participants. Effects of *math experience*, in contrast, have been reported only rarely. However, Roussel, Fayol, and Barrouillet (2002) observed that experienced participants (primary school teachers) performed slower on arithmetic tasks than did inexperienced participants (undergraduate psychology students). In contrast, experienced and inexperienced participants did not differ in their strategy choices. In one of our own studies, arithmetic experience (based on the participants' high school curricula) was found to predict both strategy selection and strategy efficiency, albeit only for multiplication problems and not for addition problems (Imbo, Vandierendonck, & Rosseel, in press(b)). *Gender* effects have been investigated in children rather than in adults. Several child studies showed more frequent and more efficient retrieval use in boys than in girls (e.g., Carr & Jessup, 1997; Carr, Jessup, & Fuller, 1999; Royer, Tronsky, Chan, Jackson, & Marchant III, 1999). Whether or not these differences exist in adulthood, is a debated topic. Because some recent studies (e.g., Geary, Saults, Liu & Hoard, 2000; Imbo et al., in press(b)) observed significant gender differences in adults' arithmetic processing, with males outperforming females, gender was included in the present study as well. Only two studies investigated the possible effects of calculator use, one observing no effects (Campbell & Xue, 2001) and one observing effects of calculator use on strategy efficiency (Imbo et al., in press(b)); subjects who reported highly frequent calculator use were remarkably slower in both retrieval efficiency and procedural efficiency. The present study elaborated on this issue and included a short questionnaire about calculator use. Concerning *math anxiety*, it was expected that high-anxious participants would perform worse on the simple-arithmetic tasks than the low-anxious participants. Effects of math anxiety have previously been shown in complex-arithmetic tasks (e.g., Ashcraft & Kirk, 2001) but not yet in simple-arithmetic tasks. The *associative strength* variable, finally, is an estimate of how strong the participant's problem-answer associations are in long-term memory, and is operationalized as the participants' percentage retrieval use in choice conditions. It was hypothesized that participants with stronger problem-answer associations would be faster in retrieving arithmetic facts from long-term memory.

Experiment 1: Multiplication

Method

Participants. Sixty subjects participated in the present experiment (15 men and 45 women). Their mean age was 21 years and 0 months. Half of them were first-year psychology students at Ghent University who participated for course requirements and credits. The other half was paid €10 for participation.

Procedure. Each participant was tested individually in a quiet room for approximately 1 hour. The experiment started with short questions about the age of the participant, his/her math experience (i.e., the number of mathematics lessons per week during the last year of secondary school), calculator use (on a rating scale from 1 “never” to 5 “always”), and math anxiety (on a rating scale from 1 “low” to 5 “high”[2]). All participants solved the simple-arithmetic problems in four conditions: first the choice condition (in order to exclude influence of no-choice conditions on the choice condition), and then three no-choice conditions, the order of which was randomized across participants. In the choice condition, 6 practice problems and 42 experimental problems were presented. After the choice condition, participants needed no more practice; the no-choice conditions thus immediately started with 42 experimental problems. Each condition was further divided in two blocks: one in which no working-memory component was loaded, and one in which one working-memory component was loaded. The working-memory load differed across participants: for 20 participants the central executive was loaded, for 20 participants the active phonological rehearsal process was loaded, and for

20 participants the passive phonological store was loaded. For half of the participants, each condition started with the no-load block and was followed by the working-memory load block; the order was reversed for the other half of the participants.

Simple-arithmetic task. The multiplication problems presented in the simple-arithmetic task consisted of two one-digit numbers (e.g., 6 x 7). Problems involving 0, 1, or 2 as an operand (e.g., 5 x 0, 1 x 4, 2 x 3) and tie problems (e.g., 3 x 3) were excluded. Since commuted pairs (e.g., 9 x 4 and 4 x 9) were considered as two different problems, this resulted in 42 multiplication problems (ranging from 3 x 4 to 9 x 8). Small problems were defined as problems with a correct product smaller than 25 whereas large problems were defined as problems with a correct product larger than 25 (Campbell, 1997; Campbell & Xue, 2001). A trial started with a fixation point for 500 milliseconds. Then the multiplication problem was presented horizontally in the center of the screen, with the operation sign at the fixation point. The problem remained on screen until the subject responded. Timing began when the stimulus appeared and ended when the response triggered the sound-activated relay. To enable this sound-activated relay, participants wore a microphone that was activated when they spoke their answer aloud. This microphone was connected to a software clock accurate to 1ms. On each trial, feedback was presented to the participants, a green ‘Correct’ when their answer was correct, and a red ‘Incorrect’ when it was not.

Immediately after solving each problem, participants in the choice condition were presented four strategies on the screen (see e.g. Campbell & Gunter, 2002; Campbell & Xue, 2001; Kirk & Ashcraft, 2001; LeFevre et al., 1996b; Seyler et al., 2003): Retrieval, Counting, Transformation, and Other. These four choices had been extensively explained by the experimenter: *(1) Retrieval: You solve the problem by remembering or knowing the answer directly from memory. It means that you know the answer without any additional processing. For example: you know that 5 x 6 = 30 because 30 “pops into your head”. (2)*

Counting: You solve the problem by counting a certain number of times to get the answer. You recite the tables of multiplication. For example: 4 x 7 = 7… 14… 21… 28 or 5 x 3 = 5… 10… 15. (3) Transformation: You solve the problem by referring to related operations or by deriving the answer from known facts. You change the presented problem to take advantage of a known arithmetical fact. For example: 9 x 8 = (10 x 8) – 8 = 80 – 8 = 72 or 6 x 7 = (6 x 6) + 6 = 36 + 6 = 42. (4) Other: You solve the problem by a strategy unlisted here, or you do not know what strategy you used to solve the problem. For example: guessing. After each problem, participants were asked to report verbally which of these strategies they had used. The experimenter also emphasized that the presented strategies were not meant to encourage use of a particular strategy. If the participant felt like using only one of the presented strategies, he/she was completely free to do so; when the participant acknowledged generally using a mix of strategies; he/she was as free to do so.

In the no-choice conditions, participants were asked to use one particular strategy to solve all problems. In no-choice/retrieval, they were required to retrieve the answer. More specifically, participants were asked to pronounce the answer that first popped into their head. In no-choice/transformation, participants were required to transform the problem by making an intermediate step. The experimenter proposed several intermediate steps, and all participants recognized using at least a few of them. Examples were (a) going via 10, e.g., 9 x 6 = (10 x 6) - 6 = 60 - 6 = 54 and 5 x 7 = (10 x 7) : 2 = 70 : 2 = 35, (b) using the double, e.g., 4 x 6 = 2 x 2 x 6 = 2 x 12, and (c) using ties, e.g., 7 x 8 = (7 x 7) + 7 = 49 x 7 = 56. However, if participants normally used any transformation step not proposed by the experimenter, they were free to do so. In no-choice/counting, participants had to say (sub-vocally) the tables of multiplication until they reached the correct total (e.g., 4 x 7 = 7… 14… 21… 28). After having solved the problem, participants also had to answer with ‘yes’ or ‘no’ whether they had succeeded in using the required strategy. This enabled us to exclude non-compliant trials from analyses.

In choice and no-choice conditions, the answer of the participant, the strategy information, and the validity of the trial were recorded on-line by the experimenter. All invalid trials (e.g., failures of the voice-activated relay) were discarded and returned at the end of the block, which minimized data-loss due to unwanted failures.

Executive secondary task. A continuous choice reaction time task (CRT task) was used to load the executive working-memory component (Szmalec, Vandierendonck, & Kemps, 2005). Stimuli of this task consisted of low tones (262 Hz) and high tones (524 Hz) that were sequentially presented with an interval of 900 or 1500 ms. Participants had to press the 4 on the numerical keyboard when they heard a high tone and the 1 when a low tone was presented. The duration of each tone was 200 ms. The tones were presented continuously during the simple-arithmetic task. The CRT task was also performed alone (i.e., without the concurrent solving of arithmetic problems). In this single-task condition, the multiplication problems with their correct answer were presented, which the participants had to read off the screen. Doing so, the procedure and vocalization of the task remained very similar to the procedure and vocalization in the dual-task condition. Differences in the secondary-task performance could thus only be due to the mental-arithmetic process itself.

Active phonological secondary task. In this task, letter strings of 3 consonants (e.g., T K X) were read aloud by the experimenter. Known letter strings (e.g., BMW, LSD) were avoided. The participant had to retain these letters and repeat them after three simple-arithmetic problems. Following the response of the participant, the experimenter presented a new 3-letter string. This task was also tested individually (i.e., without the concurrent solving of arithmetic problems), using the same methodology as in the CRT single-task condition.

Passive phonological secondary task. In this task, irrelevant speech was presented to the participants. This speech consisted of dialogues between several people in the

Swedish language, which were taken from a compact disc used in language courses. The Swedish dialogues were presented with an agreeable loudness (i.e., around 70 dB) through the headphones. Because both Swedish and Dutch (i.e., the participants' native language) are German languages, phonemes strongly match between both languages. None of the participants had any knowledge of Swedish.

French Kit. After the simple-arithmetic experiment, all participants completed a paper-and-pencil test of complex arithmetic, the French Kit (French, Ekstrom, & Price, 1963). The test consisted of two subtests, one page with complex addition problems and one page with complex subtraction and multiplication problems. Participants were given 2 minutes per page, and were instructed to solve the problems as fast and accurately as possible. The correct answers on both subtests were summed to yield a total score of arithmetic skill.

Results

Of all trials 6.9% was spoiled due to failures of the sound-activated relay. Since all these invalid trials returned at the end of the block, most of them were recovered from data loss, which reduced the trials due to failures of the sound-activated relay to 1.8%. Further, all incorrect trials (4.4%), all choice trials on which participants reported having used a strategy 'Other' (0.1%), and all no-choice trials on which participants failed to use the required strategy (8.8%) were deleted. All data were analyzed on the basis of the multivariate general linear model; and all reported results are considered to be significant if $p < .05$, unless mentioned otherwise.

To test whether the three subject groups (i.e., loaded by the passive phonological task, the active phonological task, or the executive task) differed from each other, analyses of variance (ANOVAs) were conducted on the scores on the French Kit[3] ('arithmetic skill'), the scores of the calculator-use questionnaire, the amount of arithmetic lessons in the last year of secondary school (‘math experience’), and the scores of the math anxiety questionnaire. Results showed that the groups did not differ in any of these variables; all *F*s < 1.2 and all *p*s > .30.

Strategy efficiency. Only the RTs uncontaminated by strategy choices (i.e., no-choice RTs) will be considered, since only these RTs provide clear data concerning strategy efficiency. A 3 x 2 x 3 x 2 ANOVA was conducted on correct RTs with working-memory component (passive phonological, active phonological, executive) as between-subjects factor and load (no load vs. load), strategy (retrieval, transformation, counting), and size (small vs. large) as within-subjects factors (see Table 1).

The main effects of load, size, and strategy were significant, $F(1,57) = 10.24$, $MSE = 1326374$, $F(1,57) = 198.87$, $MSE = 1598084$, and $F(2,56) = 110.27$, $MSE = 5221560$, respectively. RTs were longer under load (3061ms) than under no-load (2786 ms), longer for large problems (3588 ms) than for small problems (2259). RTs were also longer for counting (4759 ms) than for transformation (2992 ms), $F(1,57) = 138.10$, $MSE = 3378924$, and longer for transformation than for retrieval (1020 ms), $F(1,57) = 82.98$, $MSE = 4514306$. The main effect of strategy was modified by a strategy x load interaction and a strategy x size interaction. The strategy x load interaction, $F(2,56) = 5.15$, $MSE = 683977$, indicated that the load effect (i.e. load RTs – no-load RTs) was larger for counting than for retrieval, $F(1,57) = 10.04$, $MSE = 750311$, and larger for counting than for transformation, $F(1,57) = 7.01$, $MSE = 807632$. Load effects did not differ between retrieval and transformation, $F(1,57) < 1$. The strategy x size interaction, $F(2,56) = 69.61$, $MSE = 1705536$, indicated that the problem-size effect (i.e., RTs on large problems – RTs on small problems) was larger in counting than in retrieval, $F(1,59) = 141.63$, $MSE = 2275821$, and larger in counting than in transformation, $F(1,59) = 132.01$, $MSE = 262806$, but as large in retrieval as in transformation, $F(1,59) = 2.13$, $MSE = 212582$ ($p = .15$).

Insert Table 1 about here

The working-memory component x load interaction did not reach significance, $F(2,57) = 1.91$, $MSE = 1326374$ ($p = .16$). However, as differential load effects were predicted for the different working-memory components, planned comparisons were conducted. These analyses showed that the effect of load (i.e., load RTs – no-load RTs) was significant for the executive component, $F(1,57) = 11.59$, $MSE = 1326374$, but did not reach significance for the active phonological component, $F(1,57) = 1.87$ ($p = .18$) or the passive phonological component, $F(1,57) < 1$. This interpretation was verified by separate ANOVAs that tested the effects of the different working-memory loads for each single strategy. Retrieval RTs were affected by an executive load, $F(1,57) = 35.69$, $MSE = 28055$, but not by an active phonological load, $F(1,57) = 2.38$ ($p = .13$) or a passive phonological load, $F(1,57) < 1$. Transformation RTs tended to be affected by an executive load, $F(1,57) = 2.88$, $MSE = 1054430$ ($p = .09$) but not by an active phonological load, $F(1,57) < 1$ or a passive phonological load, $F(1,57) < 1$. Counting RTs, finally, were affected by an executive load, $F(1,57) = 10.16$, $MSE = 1611840$, tended to be affected by an active phonological load, $F(1,57) = 2.75$, $MSE = 1611840$ ($p = .10$), and were not affected by a passive phonological load, $F(1,59) = 1.82$ ($p = .18$). High variance on the counting RTs might have prevented this effect to reach significance.

To consolidate the results described above, and to investigate the influence of individual differences, correlations[4] were calculated between strategy efficiency (i.e., retrieval RTs, transformation RTs, and counting RTs) strategy selection, working-memory load (i.e., executive, active phonological, and passive phonological), problem size, and individual-difference variables (i.e., math anxiety, arithmetic skill, calculator use, gender, and math experience.

When looking at the correlation measures presented in Table 2, we see that strategies were executed more slowly when problem size was higher and when the central executive was loaded, which confirms the ANOVA results. Moreover, the efficiency of the different strategies correlated with several individual-difference variables. The efficiency of all three strategies was higher in high-skill participants than in low-skill participants. Participants with stronger problem-answer associations were more efficient in retrieval but not in transformation and counting. Retrieval efficiency was higher in infrequent calculator users than in frequent calculator users, and higher in males than in females.

Insert Table 2 about here

Strategy selection. In order to investigate effects on strategy selection, a 3 x 2 x 2 ANOVA was conducted on percentages use of each single strategy (in the choice condition), with working-memory component (passive phonological, active phonological, executive) as between-subjects factor and load (no load vs. load) and size (small vs. large) as within-subjects factors (see Table 3).

Insert Table 3 about here

For retrieval, the main effect of size was significant, $F(1,57) = 71.47$, $MSE = 96$, indicating more frequent retrieval use on small problems (89%) than on large problems (72%). The main effects of load and working-memory component did not reach significance, and neither did any interaction (highest $F = 2.31$). For transformation, the main effect of size was significant as well, $F(1,57) = 50.22$, $MSE = 11395$, indicating more frequent transformation use on large problems (16%) than on small problems (3%). None of the other effects reached significance (highest F = 1.79). Finally, counting tended to be used more often on large problems (11%) than on small problems (9%), but this effect did not reach significance, $F(1,57) = 3.13$, $MSE = 403$ ($p = .08$). None of the other effects reached significance (highest F = 1.18).

In Table 2, the correlations between retrieval frequency, working-memory load, problem size, and individual differences are presented. Percentage retrieval use correlated with problem size but did not correlate with any of the working-memory loads, which confirms the ANOVA results. Percentage retrieval use correlated with all individual-difference variables, however. More specifically, retrieval was more frequently used by high-skill participants than by low-skill participants, by infrequent calculator users than by frequent retrieval users, by more-experienced participants than by less-experienced participants, by low-anxious participants than by high-anxious participants, and by males than by females.

Secondary task performance. An analysis of variance was conducted on CRT accuracy, CRT speed, and letter-task accuracy (see Table 4) with condition as within-subjects variable (single, choice, no-choice/retrieval, no-choice/transformation, and no-choice/counting). CRT speed tended to differ across conditions, $F(4,16) = 2.56$, MSE = 3862, ($p = .08$). Participants were faster to react to the tones in the CRT-only condition (626 ms) than in the other conditions (660 ms), but this difference did not reach significance, $F(1,19) = 2.21$, $MSE = 8516$ ($p = .15$). CRT accuracy differed across conditions as well, $F(4,16) = 6.51$, $MSE = 67$. More specifically, CRT accuracy was significantly higher in the CRT-only condition (87%) than in the other conditions (80%), $F(1,19) = 4.17$, $MSE = 167$. When few executive working-memory resources are left, performance was thus impaired not only on the primary task but also on the secondary task. CRT accuracy was also higher in the no-choice/retrieval condition than in the choice condition, $F(1,19) = 7.31$, $MSE = 32$ and than in the other no-choice conditions, $F(1,19)$ =

7.04, *MSE* = 40. Note that the slowest CRT performance was observed in the no-choice/transformation condition, i.e., where the effect of an executive load failed to reach significance (p = .09, see above). As such, a trade-off between efficient transformation use and efficient CRT performance may account for the insignificant effect of executive load on transformation RTs. Performance on the active phonological task (i.e. the letter task) differed across conditions as well, $F(4,16)$ = 12.56, *MSE* = 166. Accuracy was significantly higher in the single-task condition (84%) than in dual-task conditions (68%), $F(1,19)$ = 19.91, *MSE* = 210.

Insert Table 4 about here

Summary

Results concerning *strategy efficiency* showed that the role of the different working-memory resources differed across strategies. Executive working-memory resources were needed in all strategies, whereas phonological working-memory resources were especially needed in the counting strategy. Working-memory load did not have any effect on *strategy selection*. Both strategy efficiency and strategy selection correlated significantly with several individual-difference variables. The interpretation of the possible roles of these individual differences is postponed to the general discussion.

Experiment 2: Division

Participants. Sixty subjects (10 men and 50 women) participated in the present experiment. Their mean age was 21 years and 4 months. Half of them were first-year psychology students at Ghent University who participated for course requirements and credits. The other half was paid €10 for participation. None of them had participated in Experiment 1.

Stimuli and Procedure. The 43 division problems were the reverse of the multiplication problems used in Experiment 1. The procedure was identical to the one used in Experiment 1, with one exception. It has been shown that only two strategies are frequently used to solve simple division problems (Campbell & Xue, 2001; LeFevre & Morris, 1999; Robinson, Arbuthnott, & Gibbons, 2002): direct memory retrieval and solving the division problem via the related multiplication problem (e.g., solving 48 : 8 via ? x 8 = 48). Therefore, the choices in the choice condition of this experiment were restricted to three: *(1) Retrieval: You solve the problem by remembering or knowing the answer directly from memory. It means that you know the answer without any additional processing. For example: you know that 30 : 6 = 5 because 5 "pops into your head". (2) Via multiplication: You solve the division problem by using the related multiplication problem. For example: when you have to solve 42 : 6, you think about how many times 6 goes into 42, i.e., 6 x ? = 42. You might also check your answer by doing the multiplication 6 x 7 = ?. (3) Other: You solve the problem by a strategy unlisted here, or you do not know what strategy you used to solve the problem. For example: guessing*. Accordingly, there were only two no-choice conditions: no-choice/retrieval, in which participants were asked to retrieve the answer, and no-choice/via-multiplication, in which participants were asked to solve the division problem via the related multiplication problem.

Results

Of all trials, 5.6% were spoiled due to failures of the sound-activated relay. Since all these invalid trials returned at the end of the block, most of them were recovered from data loss, which reduced the trials due to failures of the sound-activated relay to 1.5%.

Further, all incorrect trials (10.0%), all choice trials on which participants reported having used a strategy ‘Other’ (0.7%), and all no-choice trials on which participants failed to use the required strategy (6.0%) were deleted. The low percentage of ‘Other’ strategy use confirms that the two strategies allowed in the choice condition (i.e., direct memory retrieval and the via-multiplication strategy) cover the choice pattern generally used by participants when solving simple division problems. All data were analyzed on the basis of the multivariate general linear model; and all reported results are considered to be significant if $p < .05$, unless mentioned otherwise.

To test whether the three subject groups (i.e., loaded by the passive phonological task, the active phonological task, or the executive task) differed from each other, four analyses of variance (ANOVAs) were conducted. Results showed no group differences in arithmetic skill, calculator use, math experience, or math anxiety; all Fs < 1.1 and all ps > .30.

Strategy efficiency. A 3 x 2 x 2 x 2 ANOVA was conducted on correct no-choice RTs with working-memory component (passive phonological, active phonological, executive) as between-subjects factor and load (no load vs. load), strategy (retrieval vs. via multiplication) and size (small vs. large) as within-subjects factors (see Table 1).

The main effects of load, strategy, and problem size were significant. RTs were longer under load (1505 ms) than under no-load (1304 ms), $F(1,57) = 29.08$, $MSE = 102768$; retrieving division facts (993 ms) was faster than solving them via multiplication (1860 ms), $F(1,57) = 52.84$, $MSE = 1400216$; and small problems (1261 ms) were solved faster than large problems (1591 ms), $F(1,57) = 59.60$, $MSE = 219528$.

Strategy further interacted with problem size and with load. The strategy x size interaction indicated a larger problem-size effect (i.e., RTs on large problems – RTs on small problems) when division problems were solved via multiplication than when they were retrieved from memory, $F(1,57) = 16.69$, $MSE = 157848$. The strategy x load interaction showed larger effects of working-memory load (i.e., load RTs – no-load RTs) when division problems were solved via multiplication than when they were retrieved from memory, $F(1,57) = 5.05$, $MSE = 99248$.

There was also a significant interaction between working-memory component and load, $F(2,57) = 11.30$, $MSE = 102769$, which showed that load effects were significant for the executive component, $F(1,57) = 48.72$, $MSE = 102769$, but not for the active phonological component, $F(1,57) < 1$, or the passive phonological component, $F(1,57) = 2.19$ ($p = .14$). This interpretation was verified by separate ANOVAs that tested the effects of the different working-memory loads for each single strategy. Retrieval RTs were affected by executive loads, $F(1,57) = 75.27$, $MSE = 27985$ but not by active phonological or passive phonological loads (each $F < 1$). Via-multiplication RTs were affected by executive loads, $F(1,57) = 16.87$, $MSE = 174031$ but not by active phonological loads, $F(1,57) = 1.33$ ($p = .25$). However, via-multiplication RTs tended to be affected by passive phonological loads, $F(1,57) = 3.59$, $MSE = 174032$ ($p = .06$).

To consolidate the results described above, and to investigate the influence of individual differences, correlations were calculated between strategy efficiency (i.e., retrieval RTs and via-multiplication RTs), strategy selection, working-memory load (i.e., executive, active phonological, and passive phonological), problem size, and individual-difference variables (i.e., math anxiety, arithmetic skill, calculator use, gender, and math experience).

Correlation measures are presented in Table 5 (see also Footnote 4). Strategy efficiencies were smaller when problem size was higher and when the central executive was loaded, which confirms the ANOVA results. Strategy efficiencies correlated with several individual-difference variables as well. More specifically, retrieval and via-multiplication efficiencies were higher in high-skill participants than in low-skill participants, and higher in low-anxious participants than in high-anxious participants. Associative strength correlated significantly with the efficiency of the via-multiplication strategy but not with retrieval efficiency. Finally, the efficiency of the via-multiplication strategy was higher in more-experienced participants than in less-experienced participants.

Insert Table 5 about here

Strategy selection. In order to investigate effects on strategy selection, a 3 x 2 x 2 ANOVA was conducted on percentages use of each single strategy (in the choice condition), with working-memory component (passive phonological, active phonological, executive) as between-subjects factor and load (no load vs. load) and size (small vs. large) as within-subjects factors (see Table 3).

For retrieval, the main effect of size was significant, $F(1,57) = 49.36$, $MSE = 10431$, indicating more frequent retrieval use on small problems (84%) than on large problems (71%). The main effects of load and working-memory component did not reach significance, and neither did any interaction (highest $F = 1.11$). The via-multiplciation strategy, in contrast, was used more frequently on large problems (29%) than on small problems (16%), $F(1,57) = 49.36$, $MSE = 10431$. None of the other effects reached significance (highest $F = 1.11$)

In Table 5, the correlations between retrieval frequency, working-memory load, problem size, and individual differences are presented. Percentage retrieval use correlated with problem size but did not correlate with any of the working-memory loads, which confirms the ANOVA results. None of the individual-difference variables correlated significantly with strategy selection.

Secondary task performance. An analysis of variance was conducted on CRT accuracy, CRT speed, and letter-task accuracy (Table 4) with condition as within-subjects variable (single, choice, no-choice/retrieval, no-choice/via-multiplication). CRT accuracy differed across conditions, $F(3,17) = 11.80$, $MSE = 56$. More specifically, CRT accuracy was higher in the CRT-only condition (88%) than in the other conditions (75%), $F(1,19) = 33.86$, $MSE = 78$. CRT speed did not differ across conditions, $F(3,17) = 1.06$ ($p = .39$). Performance on the active phonological task (i.e., the letter task) differed across conditions, $F(3,17) = 15.06$, $MSE = 180$. Accuracy was higher in the single-task condition (90%) than in dual-task condition (72%), $F(1,19) = 13.26$, $MSE = 350$.

Summary

Concerning *strategy efficiency*, it was shown that, as in Experiment 1, the role of the different working-memory resources differed across strategies. The retrieval strategy was affected by an executive load only, whereas the multiplication strategy was affected by an executive load and by a passive phonological load. Strategy efficiency further correlated significantly with several individual-difference variables; the interpretation of which is postponed to the general discussion. Also as in Experiment 1, *strategy selection* was not influenced by working-memory load.

General Discussion

In the present study, the choice/no-choice method and the selective-interference paradigm were combined in order to investigate the role of working memory in simple-arithmetic strategy selection and strategy efficiency. Results showed that the executive working-memory component was involved in all strategies (i.e., retrieval, transformation and counting in the multiplication experiment and retrieval and via-multiplication in the division experiment). Phonological working-memory components played a much smaller role, and tended to be needed in some non-retrieval strategies (i.e., counting in the multiplication experiment and via-multiplication in the division experiment).

The role of executive working-memory resources

Executive working-memory resources were needed in direct *retrieval* of multiplication and division facts. Getting access to information stored in long-term memory is indeed one of the main executive (or attentional) functions (e.g., Baddeley, 1996; Baddeley & Logie, 1999; Cowan, 1995; Engle, Kane, & Tuholski, 1999; Ericsson & Kintsch, 1995). Consequently, executive (or attentional) working-memory resources have for long been hypothesized to play a significant role in retrieving arithmetic facts from long-term memory (e.g., Ashcraft, 1992, 1995; Ashcraft, Donley, Halas, & Vakali, 1992; Barrouillet, Bernardin, & Camos, 2004; Geary & Widaman, 1992; Kaufmann, 2002; Kaufmann, Lochy, Drexler, & Semenza, 2003; Lemaire et al., 1996; Seitz & Schumann-Hengsteler, 2000, 2002; Zbrodoff & Logan, 1986) and the present study succeeded to show this by using a rigorous method (i.e., solving simple-arithmetic problems in a no-choice/retrieval condition under an executive working-memory load).

We suppose that executive working-memory resources are needed to select the correct response. Indeed, the presentation of a simple multiplication or division problem does automatically activate several candidate answers in long-term memory (e.g., Campbell, 1997; De Brauwer & Fias, 2006; Galfano, Rusconi, & Umiltà, 2003; Rusconi, Galfano, Speriani, & Umiltà, 2004; Rusconi, Galfano, Rebonato, & Umiltà, 2006; Thibodeau, LeFevre, & Bisanz, 1996). After this automatic activation of several associated responses, a deliberate choice of the correct response has to be executed in order to complete the retrieval.

Executive working-memory resources did also play a role when *non-retrieval* strategies were used to solve multiplication or division problems. Of course, executing non-retrieval strategies does also require retrieval of known responses, which relies on executive resources. Moreover, executing non-retrieval strategies requires other demanding processes as well, such as performing calculations (e.g., Ashcraft, 1995; Imbo, Vandierendonck, & De Rammelaere, in press(a); Imbo, Vandierendonck, & Vergauwe, in press(c); Logie, Gilhooly, & Wynn, 1994), manipulating interim results (Fürst & Hitch, 2000), and monitoring counting sequences (e.g., Ashcraft, 1995; Case, 1985; Hecht, 2002; Logie & Baddeley, 1987).

The central executive did not play a role in *strategy selection*: percentages of strategy use did not change under an executive working-memory load. This is in agreement with previous studies (e.g., Hecht, 2002; Imbo & Vandierendonck, in press) and suggests that selecting simple-arithmetic strategies does not rely on executive working-memory resources. The absence of load effects on the strategy selection process is in agreement with the adaptive strategy choice model of Siegler and Shipley (1995). In this model, strategy selection is based solely on problem-answer association strengths (i.e., the answer that is most strongly associated with the presented problem is retrieved) and not on meta-cognitive processes such as executive (or attentional) processes.

The role of phonological working-memory resources

Phonological working-memory resources tended to be needed in non-retrieval strategies. More specifically, an active phonological load tended to affect the counting strategy in Experiment 1 (p = .10) and a passive phonological load tended to affect the via-multiplication strategy in Experiment 2 (p = .06). These results are in agreement with previous studies (Hecht, 2002; Imbo & Vandierendonck, in press; Seyler et al., 2003) that also observed a significant role for the phonological loop in non-retrieval strategies.

The main function of the active phonological rehearsal process is storing intermediate and partial results (Ashcraft, 1995; Logie et al., 1994; Hitch, 1978), a function which is needed in non-retrieval strategies only. Without doubt, using the counting strategy to solve multiplication facts (e.g., 4 x 7 = 7… 14… 21… 28) requires storing intermediate results and thus relies on active phonological resources. The passive phonological store would come into play when more than one number needs to be maintained at any one time (Logie & Baddeley, 1987). This may explain why passive phonological resources were needed when the via-multiplication strategy was used to solve division problems. In order to transform a division problem into a multiplication problem (e.g., transforming 56 : 8 into 8 x ? = 56), participants have to maintain the dividend and the devisor while they are (sub-vocally) reciting their multiplication tables.

The present study also sheds further light on the equivocal results observed in previous studies investigating the role of the phonological loop in simple arithmetic. Whereas some studies did observe an effect of phonological load (e.g., Lee & Kang, 2002; Lemaire et al., 1996; Seitz & Schumann-Hengsteler, 2002), others did not (e.g., De Rammelaere et al., 1999, 2001; Seitz & Schumann-Hengsteler, 2000). Present results suggest that strategy choices might have played a role. Studies in which participants relied more heavily on non-retrieval strategies might have observed larger effects of phonological working-memory loads than studies in which participants relied mainly on direct memory retrieval.

The impact of individual differences

Besides investigating the role of working memory in people's arithmetic strategy use, we also explored whether individual differences might influence strategy efficiency and/or strategy selection processes. In the following, the possible roles of these individual difference variables are discussed.

Arithmetic skill correlated significantly with all strategy efficiencies. More specifically, high-skill participants were more efficient (i.e., faster) in executing both retrieval and non-retrieval strategies to solve multiplication and division problems. This observation is not very surprising, however, as both our primary task (solving simple arithmetic problems) and the French Kit are speeded performance tests. Hence, correlations between arithmetic skill and strategy efficiency have been observed previously (e.g., Campbell & Xue, 2001; Imbo et al., in press(b); Kirk & Ashcraft, 2001; LeFevre & Bisanz, 1986). Arithmetic skill correlated with strategy selection only in the multiplication experiment: high-skill participants used retrieval more frequently than did low-skill participants, an observation that is in agreement with previous studies as well (e.g., Imbo et al., in press(b); LeFevre et al., 1996a, 1996b).

Associative strength (i.e., percentages retrieval use) correlated with retrieval efficiency in Experiment 1 but not in Experiment 2 (in which the correlation was quite high and in the correct direction, but not significant). Indeed, it has been asserted that problems with higher associative strengths are retrieved more efficiently from long-term memory (e.g., Ashcraft et al., 1992; Hecht, 2002). The correlation between associative strength and the via-multiplication strategy efficiency in Experiment 2 may be due to the fact that fast retrieval of multiplication facts is a critical component of this strategy.

Concerning *math anxiety*, the results of Experiment 1 indicated effects on strategy selection; retrieval use was significantly less frequent in high-anxious participants than in low-anxious participants. Anxious participants might set higher confidence criteria, which entails that they will only retrieve an answer when they are very sure about its correctness.

No effects of math anxiety on strategy efficiency were found in Experiment 1, probably because solving simple multiplication problems is rather easy. Indeed, math anxiety would affect arithmetic performance only when the task is resource-demanding (Ashcraft, 1995; Faust, Ashcraft, & Fleck,, 1996). This reasoning also explains why math anxiety affected strategy efficiency in Experiment 2. In this experiment, in which division problems had to be solved, both retrieval and non-retrieval strategy use was less efficient in high-anxious participants than in low-anxious participants. Math-anxious participants are often occupied by worries and intrusive thoughts when performing arithmetic tasks. Because such thoughts load on working-memory resources, high-anxious participants have less working-memory resources left to solve the arithmetic task efficiently (Ashcraft & Kirk, 2001; Faust et al., 1996). It is reasonable that solving division problems is more resource-demanding than solving multiplication problems, which explains why math anxiety affected strategy efficiency in Experiment 2 but not in Experiment 1.

The frequency of *calculator use* correlated with strategy selection and strategy efficiency in Experiment 1 (multiplication) but not in Experiment 2 (division). More frequent calculator use was related to less efficient and less frequent retrieval use. Effects of calculator use on strategy efficiency have been observed earlier (Imbo et al., in press(b)), but no previous study observed a reliable effect of calculator use on simple-arithmetic strategy selection.

Math experience correlated with strategy selection and strategy efficiency. More-experienced participants used the retrieval strategy more frequently (Experiment 1) and were more efficient in the execution of the via-multiplication strategy (Experiment 2). Comparable effects have been observed previously (e.g., Imbo et al., in press(b)) and indicate that daily arithmetic practice has great effects on strategy selection and strategy efficiency.

Gender, finally, correlated with strategy selection and strategy efficiency in Experiment 1 but not in Experiment 2. When solving multiplication problems, men more frequently used retrieval than did women, an effect observed earlier (e.g., Carr & Jessup, 1997; Carr et al., 1999; Fennema et al., 1998; Geary et al., 2000). We also observed more efficient retrieval use in men than in women, which confirms the hypothesis that gender differences in mental arithmetic are due to that fact that retrieval use is faster in men than in women (Royer et al., 1999). However, gender might correlate with many other individual-difference variables such as calculator use, math experience, math anxiety and arithmetic skill. Hence, further research is needed to disentangle gender effects from other confounding variables.

Based on these exploratory correlations, it might be concluded that individual differences influence people's strategy efficiency and strategy selection processes. However, the effects were not always significant and differed across operations (multiplication vs. division) and across strategic performance measures (efficiency vs. selection). This was especially the case for the individual-difference variables which were based on one single question (e.g., calculator use, math anxiety). We acknowledge that the reliability of such measures can be questioned. Hence, future studies, in which individual differences are tested more thoroughly, are needed to confirm or disconfirm the exploratory results found here. For example, one might think to use the full Mathematics Anxiety Rating Scale (MARS, Richardson & Suinn, 1972) in order to test participants' math anxiety. Further research might also investigate the impact of individual differences in a more experimental way, e.g., by training participants, by manipulating their anxiety level, or by augmenting /reducing their calculator use.

Conclusion

The present study used a combination of two frequently used and approved methods, the selective-interference paradigm and the choice/no-choice method. The selective-interference paradigm enabled us to investigate the role of three different working-memory components; the choice/no-choice method enabled us to study strategy selection and strategy efficiency independently. Another novelty of the present study is that multiplication and division strategies were investigated. These operations differ greatly from addition and subtraction; already from childhood on up until adulthood. Moreover, the role of working memory in multiplication and division strategies has never been investigated before. A final novelty of the present study was that several individual-difference variables were included.

Concerning strategy *efficiency*, results showed that executive working-memory resources were involved in both retrieval and non-retrieval strategies. Active and passive phonological working-memory resources played a much smaller role and tended to be involved in non-retrieval strategies only. Strategy *selection*, on the other hand, was not affected by executive or phonological working-memory loads. It was further shown that individual differences had a large impact as well. Arithmetic skill, calculator use, math experience, gender, and math anxiety influenced strategy efficiency and/or strategy selection. Individual differences should thus not be ignored when the cognitive systems underlying simple-arithmetic performance are investigated. Indeed, many effects caused by individual differences can be explained by cognitive variables. Effects of math anxiety for example, can be explained by working-memory limits (Ashcraft & Kirk, 2001; Faust et al., 1996) and effects of math experience can be explained by differential problem-answer strengths in long-term memory (Imbo et al., in press(b)). Arithmetic models and theories might be challenged to incorporate these individual differences and their respective cognitive processes.

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Foot notes

1. Given the poorer elaboration of the role of the visuo-spatial sketchpad in simple arithmetic (on theoretical, methodological, and empirical level), this working-memory component was not included in the present study.
2. The correlation between rating math anxiety on a scale from 1 to 5 and rating math anxiety with the Mathematics Anxiety Rating Scale (MARS, Richardson & Suinn, 1972) ranges from .45 to .85 (Mark Ashcraft, personal communication).
3. Both subtests of the French kit correlated significantly with each other ($p < .01$); r = .675 in Experiment 1 and r = .531 in Experiment 2, indicating high reliability. Correlations are not 100% because both subtests test other operations (addition vs. multiplication-subtraction).
4. Gender was coded as a dummy variable: girls were coded as -1 and boys were coded as 1. Each working-memory load was coded a dummy variable as well. This variable was -1 for no-load conditions and 1 for load conditions.

Table 1

Mean correct RTs (in milliseconds) of Experiment 1 (multiplication) and Experiment 2 (division) as a function of load, working-memory component, size, and strategy. Standard errors between brackets.

Multiplication		PL passive		PL active		Executive	
		No load	Load	No load	Load	No load	Load
Retrieval	Small	854	843	922	977	736	957
		(52)	(58)	(52)	(58)	(52)	(58)
	Large	1129	1089	1259	1319	964	1191
		(80)	(78)	(80)	(78)	(80)	(78)
Transformation	Small	2874	2954	3280	3240	2379	2761
		(357)	(380)	(357)	(380)	(357)	(380)
	Large	3235	3126	3110	3312	2616	3013
		(304)	(334)	(304)	(334)	(304)	(334)
Counting	Small	2881	3162	2980	3342	2556	2964
		(269)	(292)	(269)	(292)	(269)	(292)
	Large	6261	6704	6284	6863	5874	7275
		(661)	(761)	(661)	(761)	(661)	(761)

Table 1 (continued)

Division		PL passive		PL active		Executive	
		No load	Load	No load	Load	No load	Load
Retrieval	Small	745	725	917	908	906	1210
		(59)	(75)	(59)	(75)	(59)	(75)
	Large	893	860	1159	1131	1057	1402
		(78)	(96)	(78)	(96)	(77)	(96)
Via multiplication	Small	1593	1696	1590	1671	1410	1764
		(195)	(193)	(195)	(193)	(195)	(193)
	Large	1996	2246	1972	2107	1930	2342
		(281)	(321)	(281)	(321)	(281)	(321)

Table 2

Correlation table for Experiment 1 (multiplication).

	Transform RT	Count RT	Retrieval use[1]	Problem size	Arithmetic skill	Calculator use	Math experience	Math anxiety	Gender	Phon. passive	Phon. active	Exec.
Retrieval RT	.424*	.393*	-.370*	.311*	-.415*	.294*	-.006	.009	-.210*	-.021	.045	.193*
Transform RT		.509*	-.113	.539*	-.208*	.093	.002	-.051	-.047	.037	.046	.096
Count RT			-.002	.063	-.284*	.109	.006	-.112	.012	-.012	.045	.080
Retrieval use[1]		.		-.349*	.190*	-.205*	.256*	-.202*	.270*	.016	.007	.048
Arithmetic skill						-.440*	.014	.012	.410*	--	--	--
Calculator use							.127	.096	-.332*	--	--	--
Math experience								-.455*	.159	--	--	--
Math anxiety									-.186	--	--	--
Gender										--	--	--

[1] Associative strength is operationalized by the participants' percentage retrieval use
* $p < .0038$ (the Bonferroni-corrected *alpha* level of .05 when correlating 13 variables)
df = 238

Table 3

Mean percentages strategy use of Experiment 1 (multiplication) and Experiment 2 (division) as a function of load, working memory component, and size. Standard errors between brackets.

Multiplication		PL passive		PL active		Executive	
		No load	Load	No load	Load	No load	Load
Retrieval	Small	88 (4)	90 (4)	87 (4)	86 (4)	88 (4)	91 (4)
	Large	70 (6)	71 (6)	68 (6)	70 (6)	76 (6)	79 (6)
Transformation	Small	2 (2)	2 (4)	4 (2)	3 (1)	2 (2)	2 (1)
	Large	17 (5)	17 (4)	20 (5)	16 (4)	15 (5)	13 (4)
Counting	Small	9 (3)	8 (4)	10 (3)	11 (4)	9 (3)	7 (4)
	Large	13 (3)	12 (3)	12 (3)	14 (3)	10 (3)	8 (3)

Division		PL passive		PL active		Executive	
		No load	Load	No load	Load	No load	Load
Retrieval	Small	82 (5)	88 (5)	80 (5)	81 (5)	86 (5)	86 (5)
	Large	68 (6)	69 (5)	68 (6)	72 (5)	74 (6)	74 (5)
Via multiplication	Small	18 (5)	12 (5)	20 (5)	19 (5)	14 (5)	14 (5)
	Large	32 (6)	31 (5)	32 (6)	28 (5)	26 (6)	26 (5)

Table 4

Performance on the secondary tasks in Experiment 1 (multiplication) and Experiment 2 (division). Standard errors between brackets.

Experiment 1	Single	Choice	Retrieval	Transform	Count
CRT accuracy (%)	87 (5)	79 (3)	84 (3)	79 (4)	79 (3)
CRT speed (ms)	626 (26)	656 (17)	646 (20)	672 (18)	666 (16)
Letter-task accuracy (%)	84 (3)	56 (3)	75 (4)	68 (5)	74 (4)
Experiment 2	**Single**	**Choice**	**Retrieval**	**Via multiplication**	
CRT accuracy (%)	88 (2)	73 (3)	75 (3)	75 (3)	
CRT speed (ms)	647 (23)	661 (8)	664 (15)	646 (13)	
Letter-task accuracy (%)	90 (2)	62 (5)	78 (4)	77 (4)	

Table 5

Correlation table for Experiment 2 (division).

	Multiplication RT	Retrieval use[1] (%)	Problem size	Arithmetic skill	Calculator use	Math experience	Math anxiety	Gender	Phon. passive	Phon. active	Exec.
Retrieval RT	.494*	-.149	.233*	-.264*	.019	-.047	.195*	-.130	-.020	-.014	.240*
Multiplication RT		-.206*	.210*	-.328*	.083	-.230*	.233*	-.105	.045	.027	.097
Retrieval use[1] (%)			-.274*	.003	-.063	.150	-.006	.062	.041	.031	.000
Arithmetic skill					-.241*	.299*	-.321*	.002	--	--	--
Calculator use						.208*	.241*	-.030	--	--	--
Math experience							-.207*	.040	--	--	--
Math anxiety								-.128*	--	--	--
Gender									--	--	--

[1] Associative strength is operationalized by the participants' percentage retrieval use.
* $p < .0042$ (the Bonferroni-corrected *alpha* level of .05 when correlating 12 variables)
$df = 238$