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game & train, track & teach

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Problem difficulties of simple and complex addition and subtraction problems

Marthe Straatemeier, Brenda R. J. Jansen,
Sharon Klinkenberg, & Han L. J. van der Maas

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Which problem is more difficult?

8 + 9
2 + 2
44 + 3
1 + 70
3 + 6
9 + 66
80 + 10
19 + 58

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Which problem is more difficult?

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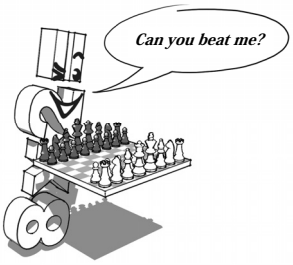
Item or person perspective?



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What can we learn from studying from the item perspective?

- Insight into processes underlying arithmetic performance
  - Memory processes
  - Strategy use
  - Learning processes
- Statistical approach
  - Simple versus complex problems
  - Addition versus subtraction

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Ideas behind Math garden

- Use daily practicing as input for monitoring
  - detailed data on development
- Computer adaptive testing
  - adapted to the ability level of the child
- Testing dependent on accuracy and response time
- Results can be used to inform teachers
  - progress, strategies, errors
- Results can be used for scientific research

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New computer-adaptive method

- Based on Elo rating system for paired comparisons from chess
  Chess players gain or lose rating based on the outcome of chess games and the rating of their opponent.
- Children and items play against each other
  If a child responds incorrectly, the item wins rating points, dependent on the rating difference between the item and the child.
- Pretesting is not necessary
  less time and money

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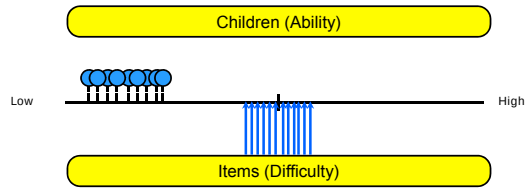
Elo system



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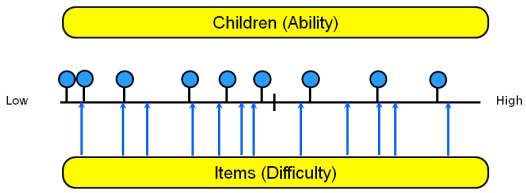
Elo system



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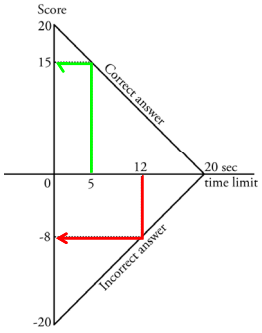
High Speed High Stakes rule

- Speed and accuracy integrated
- HSHS punishes quick guesses
- Easy to visualize



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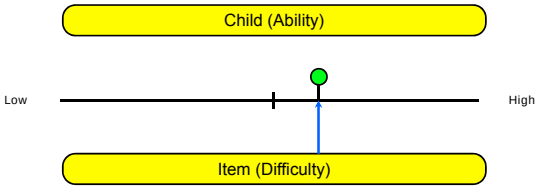
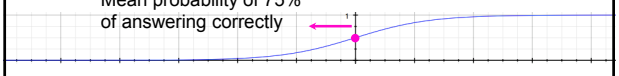
Motivation



Mean probability of 75% of answering correctly



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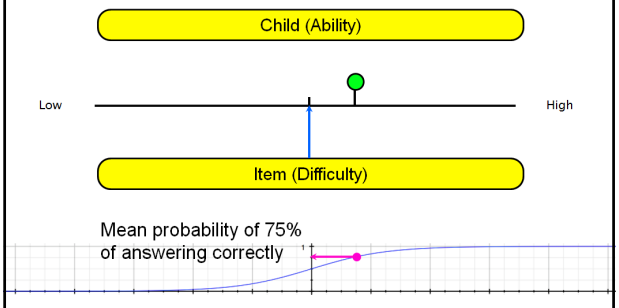
Motivation



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Validity & reliability

Klinkenberg, Straatemeier, & van der Maas (in press)

Validity
- High correlations with Cito scores (> .78)
- Rating increases significantly with age
- High correlations between domains (> .85)

Reliability
- Parallel test correlation:
  n + m, m + n .88
  n x m, m x n .98

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Item sets
r + s =m & m - s = r

Simple
- r > 0 & s > 0
- r < 10 & s < 10

Complex
- r > 0 & s > 0
- m < 100

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Sample

- Data from school year 2008-2009
- Number of participants (4 – 14 years):
  Addition: 2561 children
  Subtraction: 2392 children
- Number of items answered in games
  Addition: 747,711 items
  Subtraction: 540,263 items

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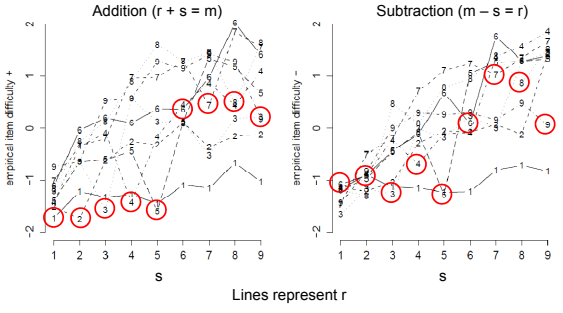
Simple problems



Lines represent r

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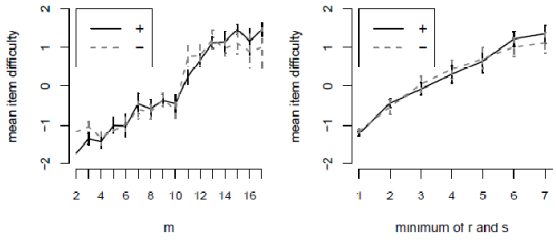
Problem size effect



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Simple Arithmetic

Predictor	Addition (r + s = m)			Subtraction (m − s = r)		
	B	*SE*	*t-value*	*B*	*SE*	*t-value*
Intercept	-.86	.18	-4.73***	-.43	.12	-3.48***
m	.04	.03	1.41			
Minimum of r & s	.15	.05	3.43**	.10	.03	2.91**
Tie (r = s)	-1.32	.13	-10.05***	-.98	.14	-6.82***
Order (r > s vs r < s)	-.10	.07	-1.38	-.63	.08	-7.60***
Cross 10	.86	.17	5.06***	1.33	.15	8.76***
Units r|s = 1	-.58	.11	-5.29***	-.47	.12	-3.78***
Units r|s = 5	-.19	.09	-2.22*			
Units r|s = 9	-.30	.11	-2.79**	-.29	.11	-2.56*
m is decade	-.87	.15	-5.74***	-1.22	.16	-7.82***
Units m = 1	-.58	.14	-4.03***			
Units m = 5						
Units m = 9				.27	.15	1.87

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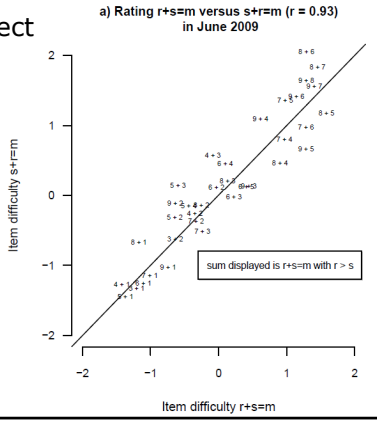
Order effect



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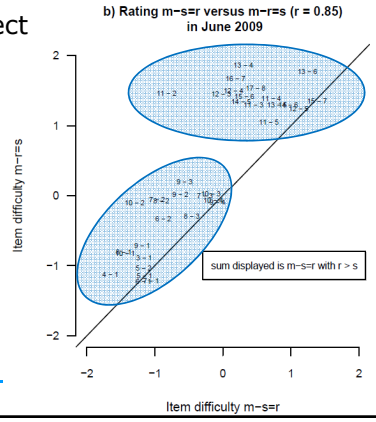
Order effect



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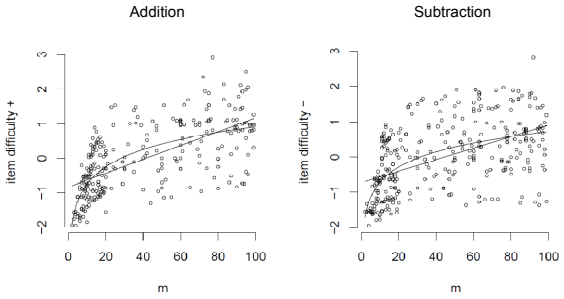
Complex problems



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Predictor	**Simple +**	**Simple -**	**Complex +**	**Complex -**
R²	***91%***	***87%***	***91%***	***86%***
Intercept	***	***	***	***
m	O			
Minimum of r & s	**	**	***	***
Order: r = s (tie)	***	***	***	***
Order: r > s	O	***	O	***
Cross 10	***	***	***	***
Units r|s = 1	***	***	***	***
Units r|s = 5	*		**	
Units r|s = 9	**	*	*	*
m is decade	***	***	***	***
Units m = 1	***		***	
Units m = 5				
Units m = 9		O		O
Logarithm of m			***	
Tie units			***	**
Tie decades				
r|s is decade			***	***
# digits			***	***

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Conclusions problem size

- Influence of m is weak when other effects are accounted for.
  - minimum of r and s
  - cross 10 (borrowing and carrying)
- Complex addition & subtraction:
  - Number of digits in problem
  - Addition: logarithm of m

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Conclusions

- Large amount of explained variance
  - **Addition** Simple: 91% Complex: 91%
  - **Subtraction** Simple: 87% Complex: 86%
- Comparable models for simple and complex problems.
  - Do similar processes underlie both?
- Addition and subtraction are strongly related.
  - Salient difference: order-effect
    - What does this tell us about memory processes and/or strategy use?

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Future/possibilities

- Comparing all operations (+, - , x, /)
- Studying more complex problems
- Error analysis

- Data today:
  > 28,000,000 arithmetic problems solved
  > 18,000 children

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Thank you for your attention!

Contact details:
m.straatemeier@uva.nl
www.rekentuin.nl
www.mathsgarden.com