Coupling Max-Ent principle and \( \mu \)CT imaging to model uncertain bone microstructure

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Abstract

Mechanical properties of cortical bone depend on its micro-structural features such as Haversian porosity (HP – the amount of large pores in bone) and Tissue mineral density (TMD – the mineral content of bone). HP and TMD can be measured using micro-computed tomography (\( \mu \)CT) and used to deduce bone composition in terms of volume fractions: \( \Phi_{\text{mineral}}, \Phi_{\text{water}}, \Phi_{\text{bone}} \) (the relative amount of mineral, collagen and water-filled pores, respectively) and, in turn, estimate its elastic tensor \( \mathbf{C}_\text{Hertz} \) (or moduli, ex.: \( Y \), axial modulus in the axial direction of bone). However, resolution of \( \mu \)CT images can produce blurry data which may affect these estimates. In order to account for the uncertainty on the experimental data, modeling variables can be treated as random variables and their probability laws computed using the Maximum Entropy (Max-Ent) Principle (ref.[1,2]). In this work we compared two approaches, considering as modeling variables either [I] the volume fractions \( \Phi_{\text{mineral}}, \Phi_{\text{water}}, \Phi_{\text{bone}} \) or [II] the tensor \( \mathbf{C}_\text{Hertz} \) or \( \Phi_{\text{mineral}}, \Phi_{\text{water}}, \Phi_{\text{bone}} \) (the relative amount of mineral, collagen and water-filled pores, respectively) and, in turn, estimate its elastic tensor \( \mathbf{C}_\text{Hertz} \) (or moduli, ex.: \( Y \), axial modulus in the axial direction of bone).

Biological context

- **Hierarchical structure**
  - (i) Organ
  - (ii) Cortical Tissue (50-100 \( \mu \)m)
  - (iii) Ultrastructure (5-100 \( \mu \)m)
  - (iv) Mineral Foam (300-500 \( \mu \)m)

- **Hierarchical Micromechanical model**
  - At the organ scale, bone is a complex multiphase, anisotropic medium with intrinsic variability.
  - At the scale of several hundreds of micrometers, cortical bone tissue can be represented as a porous medium made of a solid matrix (the ultrastructure) crossed by the cylindrical pores known as Haversian canals.
  - At the scale of several micrometers, ultrastructure is made up of collagen fibers embedded in the mineral matrix.
  - At the scale of few hundred nanometers, mineral foam is made up of highly disordered mineral (hydroxyapatite, HA), and water-filled spaces.

Micromechanical homogenization

- Microscopic properties of the bone can be computed using Micromechanical models that allow computing the effective mechanical properties of bone.
- **General implicit equation**
  \[
  \Phi_{\text{mineral}} \varepsilon_{\text{mineral}} + \Phi_{\text{water}} \varepsilon_{\text{water}} + \Phi_{\text{bone}} \varepsilon_{\text{bone}} = 0
  \]
- Hypothesis of the study:
  \[
  \Phi_{\text{mineral}} \varepsilon_{\text{mineral}} + \Phi_{\text{water}} \varepsilon_{\text{water}} + \Phi_{\text{bone}} \varepsilon_{\text{bone}} = 0 \]

Experimental data and imaging

- Acquisition of the dataset (ref.[3]):
  - 50 \( \mu \)CT of femoral neck (79 p.a. patient)
- Volume reconstruction 5x5x5 mm\(^3\)
- Sampling cortical bone 12 cubic RVEs (5x5x5 mm\(^3\))
- Identification of volume fractions (ref.[2]):
  - \( X = VFs \): mean value of the stochastic model

Stochastic (\( \mu \)Mech. homogenization) model by Max-Ent

- **MaxEnt** is a stochastic modeling approach that provides reliable estimates of bone-elastic properties.
- **Experimental information at mm-scale (stress value and dispersion)** is known to be sufficient for obtaining a reliable stochastic MaxEnt based model accounting for the variability of the elasticity of cortical bone.
- **\( \Phi \)-based and TMD-based approaches produce quite similar results**. TMD-based approach should be preferred since it scales on a directly measurable quantity.

Conclusions and Perspectives

- In the present work, the set of realizations are independent and thus spatially uncorrelated. In future work, the natural spatial correlation of the bone would be also accounted for in terms of both TMD and HP measures through random fields.
- In this work the bone variability has been induced indirectly from TMD and HP. In further study we will compare this approach with a direct random representation of the random elastic tensor of cortical bone.

References