The Taylor Rule: A Useful Monetary Policy Benchmark for the Euro Area?*

Gert Peersman and †Frank Smets


Abstract

This paper explores the Taylor rule – defined as an instrument rule linking the central bank’s policy rate to the current inflation rate and the output gap – as a benchmark for analysing monetary policy in the euro area. First, it analyses the stabilization properties of the Taylor rule in a closed economy model of the euro area, estimated using aggregate data from five EU countries. An optimized Taylor rule performs quite well compared to the unconstrained optimal feedback rule. Second, the robustness of these results to estimation error in the output gap and model uncertainty is examined.

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I. Introduction

In this paper we explore the usefulness of the Taylor rule – defined loosely as an instrument rule linking the central bank’s instrument (the short-term interest rate) to the current inflation rate and the output gap – as a benchmark for analysing monetary policy in the euro area. Ever since Taylor (1993) proposed the rule as a good description of the behaviour of the Federal Reserve System in the USA, economists have increasingly used the rule to analyse monetary policy decisions. The increasing popularity of the Taylor rule rests on two features.

First, it is simple and clear, as it explicitly links the current policy rate to current economic conditions, as captured by the inflation rate and the output gap. In contrast, this link is only implicit in targeting rules where the central bank tries to minimize deviations of an intermediate variable (be it an inflation forecast or monetary aggregates) from a target. In addition, compared to inflation forecast targeting, one does not need a forecasting model. It suffices to observe current inflation and estimate the current output gap.

Second, the Taylor rule has been shown to be a good description of monetary policy behaviour not only in the USA, but also in many other countries. For example, Clarida, Gali and Gertler (1998) show that a version of the Taylor rule with interest rate smoothing can explain short-term interest rates in the G-3 countries. Similarly, Gerlach and Schnabel (1998) find that in the 1990s average short-term interest rates in the euro area can be described by a simple Taylor rule with a coefficient of 0.5 on the output gap and 1.5 on inflation. In sum, simplicity and success in tracking short-term interest rates explain why many private-sector economists use a Taylor rule to analyse monetary policy decisions.

However, as a guide for monetary authorities the Taylor rule has two big disadvantages. First, it is too restrictive, as the number of variables in the feedback list is very limited. In general, there is no reason why central banks in the pursuit of price stability would not want to respond to other information, such as the exchange rate, other asset prices, money and credit aggregates, and so on. Second, instrument rules may not be robust to changes in the structure of the economy. Generally speaking, the efficient feedback coefficients will be complicated functions of the structural parameters of the model economy and the central bank’s preferences. For example, changes in the transmission mechanism of monetary policy will typically lead to changes in the

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1 For an application to the euro area, see Gerlach and Schnabel (1998). Many investment banks use Taylor rules to analyse and predict monetary policy decisions.

2 For a discussion of instrument and targeting rules, see Svensson (1998).
efficient feedback coefficients. For these two reasons, central banks, including the European Central Bank (ECB), would never want to commit to such simple instrument rules. The need to be able to change policies flexibly in response to new information and/or structural changes in the economy puts a premium on central bank discretion.3

Nevertheless, simple policy guidelines like the Taylor rule can be useful in two respects. First, they can be used internally as a benchmark to assess policy decisions which are based on the widest information set available. The availability of a benchmark puts some discipline on the central bank’s staff to explain why its analysis deviates from what the benchmark suggests. Second, and more controversially, a Taylor rule could also be used as a communication device to explain policy decisions to the general public.4

As mentioned before, the benefits of using a simple benchmark rule will depend on how robust the ability of the rule to stabilize inflation and output is to changes in the structure of the economy. If optimal policy deviates frequently and persistently from the benchmark and/or the rule needs to be revised frequently, the advantages of having such a benchmark will quickly disappear.5 Recent research has therefore focused on analysing the stabilization properties of such policy rules in different models of the economy.6

This paper adds to the existing literature in two ways. First, it analyses the stabilisation properties of the Taylor rule in a model of the euro area economy (Section II). Obviously it is difficult to come up with a convincing aggregate model of the euro area economy when the single currency has just been created. Nevertheless, in Section IIA we estimate a version of the closed-economy model presented in Rudebusch and Svensson (1999) using a weighted average of output and inflation in five euro countries as a measure of aggregate output and inflation and the real German policy rate as a measure of the common monetary policy.

3The stability-oriented monetary policy strategy announced by the ECB Council in October 1998 was partly designed to communicate the long-run commitment to price stability, while allowing for enough short-run flexibility to face the many uncertainties related to the establishment of the new currency. It includes a quantitative definition of the price stability objective as ‘an increase of the area-wide harmonised index of consumer prices of below 2%’ and two pillars – a reference value for the growth of a broad money aggregate and a broad-based assessment of the outlook for inflation – to explain monetary policy decisions. In the context of inflation targeting, Bernanke et al. (1999) describe such strategies as involving ‘constrained discretion’.

4For a more extensive description of the advantages of instrument rules, see Taylor (1996).

5The importance of robustness in the design of monetary policy rules has often been stressed by Ben McCallum (McCallum 1997).

6Taylor (1999) and the references therein. Taylor (1998b) surveys the literature on the Taylor rule.
We argue that this model may approximate the working of monetary policy in the euro area. Part of this justification is given in Appendix 1. There we show that once one controls for changes in bilateral exchange rates and interest rate differentials, a rise in the German real interest rate has similar effects on output in each of the five countries. Moreover, the external transmission channel through the Deutschmark–dollar exchange rate does not appear to be significant. While these results need to be taken with more than the usual degree of caution, we consider them as supporting the view that, overall, the euro area will function as a relatively closed economy. Moreover, to the extent that differences in the impact of the common monetary policy on the other countries are mitigated by the cross-border spill-overs, the effects will be relatively uniform across the whole euro area.

We then use the EU5 model to compare the performance of a simple Taylor rule with various other instrument rules and the optimal feedback rule in Section IIB. Our measure of comparison is a standard loss function which captures the fact that the central bank dislikes output, inflation and interest rate variability. Our results are similar to the ones obtained by Rudebusch and Svensson (1999) and others for the USA. We find that a Taylor rule performs quite well compared to the optimal feedback rule, although the feedback on the output gap is larger than suggested by Taylor (1993).

Our second contribution is to analyse the robustness of the results to various forms of uncertainty (Section III). Given the importance of the output gap in the Taylor rule, and the fact that typically the confidence band around estimates of the output gap is quite large, we first analyse the impact of estimation error in the output gap on the Taylor rule’s stabilization properties (Section IIIA). Consistent with recent research by Aoki (1998), Orphanides (1998), Rudebusch (1998) and Smets (1998), we find that estimation error reduces the optimal feedback coefficient on output in a simple Taylor rule and makes these coefficients more consistent with actual monetary policy behaviour as described by estimated reaction functions. However, estimation error in the output gap does not affect the Taylor rule’s relative performance.

In Section IIIB we then ask how sensitive the Taylor rule is to model parameter uncertainty. As in Estrella and Mishkin (1999) and Rudebusch (1998), we find that the estimated parameter uncertainty has only negligible effects on the efficient feedback parameters. Moreover, the stabilization properties of a simple Taylor rule with coefficients of 1.5 on inflation and 1.0 on output as recently proposed by Taylor (1998a) appear quite robust to changes in the parameters of the estimated economy as long as the basic closed economy structure is maintained. The final section contains some conclusions and suggestions for future research.
II. The Taylor Rule in an Aggregate Model for the EU5

One of the obvious problems with analysing optimal monetary policy in the euro area is that it is difficult to predict how the economy and the transmission mechanism will work under the new monetary regime. Nevertheless, the establishment of the European System of Central Banks (ESCB) is not a completely new policy environment as a gradual process of monetary convergence has preceded it. In particular, France, Germany and some of their smaller neighbours, have had fixed exchange rates with occasional parity adjustments since the end of the Bretton Woods system. In this section we use a simple model of the transmission process in these countries to analyse more formally the performance of a simple Taylor rule. The model is similar to that estimated by Rudebusch and Svensson (1998) for the USA. As our measures of output and inflation we take a weighted average of real GDP and the consumer price index (CPI) in Germany, France, Austria, Belgium and the Netherlands. The monetary policy indicator used to estimate the effects of a change in the common monetary policy stance is the real German day-to-day rate.7

This aggregate EU5 model may be a useful approximation of the working of the euro economy as a whole in a number of respects. First, while two large euro countries, Italy and Spain, are excluded from the aggregate model, the five countries included still account for almost two thirds of GDP in the euro area and will thus to a large extent determine the characteristics of the aggregate euro economy. Second, the countries included have had a history of fixed bilateral exchange rates, with the German Bundesbank de facto playing the anchor role.8 As a result, the transmission of the German interest rate through to aggregate output and inflation under a fixed exchange-rate regime may be as close as one can get to a historical description of the effects of a common monetary policy in EMU.

Although Italy has been a long-standing member of the ERM, we decided against including Italy in the estimation of the aggregate model because its inflation persistence was much higher than in the EU5. Until recently, this difference in inflation persistence was based to a large extent on differences in the credibility of the monetary authorities in maintaining the exchange-rate peg, partly related to political instability and the unsustainability of the fiscal situation. As these factors should no longer play a dominant role in stage III of EMU, it is more appropriate to analyse the EU5 countries as a model for the current euro area. Of course, this decision is somewhat arbitrary, and in Section IIIB we examine to what extent our conclusions could depend on

7See also Taylor (1998b).

8See, for example, the references in Gros and Thygesen (1992) and De Grauwe (1994).
differences in inflation persistence. Other countries participating in EMU are excluded either because of problems with data availability (Ireland) or because they started participating in the ERM only recently (Spain, Portugal and Finland).

Third, the model takes into account that, in terms of openness, the euro area as a whole will be more like the USA than like any of its individual members. The ratio of exports of goods to euro area-wide GDP is about 14% and by and large comparable to that of the USA and Japan. The disaggregated analysis of the transmission mechanism in Appendix 1 confirms this hypothesis. Two results from this analysis need to be highlighted. First, we find that once one controls for changes in bilateral exchange rates and interest rate differentials, the output effects of a rise in the German real rate are similar in the five countries comprising the EU5 (with the possible exception of Belgium). Second, the external exchange rate approximated by the Deutschmark–dollar exchange rate has only negligible effects on aggregate output. Thus, in contrast to recent estimation results in Dornbusch, Favero and Giavazzi (1998) we find that the coefficient on the external exchange rate in an implicit Monetary Conditions Index (MCI) for the ECB would be close to zero.

It is nevertheless obvious that the aggregate EU5 model can only be a rough approximation of the transmission process in the euro area. First, while we argue in Appendix 1 that the output effects of monetary policy in these five countries are similar, this may not be the case for the other euro area countries. Indeed, in Appendix 1 we find some evidence that the impact of a common monetary policy shock on Italian output may be significantly larger than in these countries. In Section IIIB we therefore analyse the implications of a higher interest elasticity of aggregate demand than estimated in the EU5 model. Second, the bilateral exchange rates were not completely fixed during the estimation period. The omission of changes in bilateral exchange rates or interest rate differentials may bias the estimation of the aggregate model. Third, it is hard to predict how inflation will respond to the output gap under the new policy regime. Implicitly we assume that the euro area-wide Phillips curve will resemble the one in the EU5 countries over the last two decades. Finally, not only is the monetary regime changing but at the same time many other structural changes are taking place which may have an impact on the transmission process. For all these reasons, the results of this section need to be treated very cautiously.

The rest of this section is structured as follows. In Section IIA we estimate a simple aggregate model for the EU5 based on Rudebusch and Svensson (1999). In Section IIB we analyse the performance of various optimal instrument rules in the estimated model.
A. An Estimated Aggregate Model for the EU5

In this section we estimate a simple aggregate model for the EU5 along the lines of Rudebusch and Svensson (1999). The main methodological difference with the latter paper is that we simultaneously estimate the model and the output gap using unobservable component techniques.9

The estimated model has the following form:

\[ \pi_{t+1} = \alpha(L)\pi_t + \beta z_t + \epsilon_{\pi, t+1} \]  
\[ z_{t+1} = \varphi_1 z_t + \varphi_2 z_{t-1} + \lambda(i_t - \bar{\pi}) + \epsilon_{z, t+1} \]  
\[ y_{p, t+1} = \mu + y_{p, t} + \epsilon_{y, t+1} \]  
\[ y_t = y_{p, t} + z_t \]

where:
- \( \pi_t \) is an EU5 weighted average of quarterly inflation in percentage points at an annual rate;
- \( \bar{\pi}_t \) is four-quarter inflation in Germany;
- \( i_t \) is the quarterly average German day-to-day rate in percentage points at an annual rate;
- \( y_{p, t} \) is a weighted average of the log of unobserved potential GDP in percentage points;
- \( z_t \) is the unobserved output gap, that is, the log difference between actual real GDP (\( y_t \)) and potential GDP in percentage points.

Equation (1) can be interpreted as a Phillips curve which relates inflation to the lagged output gap and to lags in inflation. The second equation is the reduced form of an aggregate demand equation which relates the output gap to its own lags and to a lagged real interest rate, which is approximated by the difference between the nominal day-to-day rate and average inflation over the previous four quarters. Equation (3) assumes that potential output follows a random walk process with constant drift. Finally, equation (4) is an identity that defines the output gap.

In Appendix 2 we show how this model can be written in state space form and estimated using the Kalman filter and maximum likelihood methods. Table 1 reports the estimation results with quarterly data over the period 1975 Q1–1997 Q4. For comparison we also add the estimation results for the same model estimated for the United States over the same period. As can be seen, all

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the parameters have the expected sign and are significant. It is useful to compare the EU5 estimates with the US ones. While the effect of the real policy rate on the output gap is almost the same in both cases (\(\lambda = -0.10\)), we estimate the slope of the Phillips curve to be steeper in EU5 than in the USA (\(\beta = 0.33\) instead of 0.11).

The EU5 output gap is somewhat more persistent than the US one, but does not exhibit its hump-shaped pattern. In contrast, the inflation process is much less persistent in the EU5 than in the USA. The sum of the \(\alpha\)-parameters is 0.74 in the EU5 case versus 0.92 in the US case. One interpretation for the fact that we can easily reject a unit root in the inflation process in the EU5 is that during this period agents in the EU5 put a positive weight on the constant inflation target (which equals the average inflation rate over the sample) in forming their inflation expectations. An important issue for the analysis of efficient Taylor rules is whether this weight will be different in the euro area. This will in part depend on the reputation of the new central bank. Everything else equal, lower anti-inflationary credibility will result in a higher persistence of inflation.\(^{10}\) Implicitly we assume that the ECB will inherit the credibility of the EU5 central banks. If this turns out not to be the case and — for example, the weight on the ECB’s inflation target is less than implicit in the EU5 model — then one implication for the optimal Taylor rule would be that the central bank will have to lean more against inflation and output (see the results of Section IIIB).

Figure 1 compares the effects of a temporary one-percentage point rise in the real policy rate during eight quarters on the output gap and inflation in the

\[^{10}\text{See, for example, the discussion in McLean (1998).}\]

### Table 1: Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>EU5 (1975 Q1–1997 Q4)</th>
<th>USA (1975 Q1–1997 Q4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varphi_1)</td>
<td>0.84 (0.22)</td>
<td>1.41 (0.15)</td>
</tr>
<tr>
<td>(\varphi_2)</td>
<td>0.10 (0.22)</td>
<td>–0.52 (0.13)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.33 (0.13)</td>
<td>0.11 (0.05)</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>–0.10 (0.04)</td>
<td>–0.12 (0.03)</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>0.45 (0.09)</td>
<td>0.48 (0.09)</td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>0.17 (0.11)</td>
<td>0.19 (0.08)</td>
</tr>
<tr>
<td>(\alpha_3)</td>
<td>0.06 (0.10)</td>
<td>0.13 (0.09)</td>
</tr>
<tr>
<td>(\alpha_4)</td>
<td>0.06 (0.09)</td>
<td>0.12 (0.10)</td>
</tr>
<tr>
<td>(\sigma_y)</td>
<td>0.19</td>
<td>0.39</td>
</tr>
<tr>
<td>(\sigma_z)</td>
<td>0.22</td>
<td>0.14</td>
</tr>
<tr>
<td>(\sigma_a)</td>
<td>0.98</td>
<td>0.74</td>
</tr>
<tr>
<td>likelihood</td>
<td>129.3</td>
<td>129.5</td>
</tr>
</tbody>
</table>

Note: Standard errors in parenthesis.
EU5 and the USA. Consistent with the discussion above, one can see that the effects on EU5 output are less in magnitude, but more persistent than in the USA, while the effects on inflation are stronger. Comparing these results with the results of the disaggregated analysis in Figures A.1 and A.2 of Appendix 1, it is likely that the output effects would be larger if the aggregate data had included Italy.¹¹

Turning to the estimates of the variances of the shocks, we find that the variance of the inflation shocks is very similar to that in the USA. However, estimated supply shocks are relatively less important, while demand shocks are more important in the EU5 compared to the USA. Figure 2 plots

¹¹In Section IIIB we analyse the robustness of the stabilization properties of the Taylor rule to larger output effects of a monetary policy shock.
the estimate of the EU5 output gap together with a two standard-deviations confidence band. Consistent with the findings of Gerlach and Smets (1997) the confidence band around the estimates of the output gap is quite wide, but somewhat less so than for the USA (Smets 1998). Typically, the standard deviation of the output gap is a bit less than 1%.

B. How well does the Taylor Rule Perform?

i. Instrument rules and the loss function
In order to analyse how well an optimized Taylor rule performs in the EU5 model estimated in the previous section, we consider the following loss function,\textsuperscript{12}

\[ E(L_t) = \gamma \text{Var}(\bar{\pi}_t) + (1 - \gamma) \text{Var}(z_t) + \nu \text{Var}(i_t - i_{t-1}) \] (5)

\textsuperscript{12}This discussion follows Rudebusch and Svensson (1999). They show how this loss function is equivalent to a more standard intertemporal loss function with a discount rate equal to one.
The central bank cares about variability in the deviations of annual inflation from a constant inflation target, variations in the output gap and changes in the short-term interest rate. As all variables are demeaned before the analysis, equation (5) implies that the inflation target equals the mean inflation rate over the sample.

In this section we assume that the central bank takes the model estimated in Section IIA as given and observes the current state of the economy, including not only current and past inflation and interest rates, but also the current and past output gap. The central bank’s task is then to set its policy instrument, \( i_t \), in such a way as to minimize the loss function (5), subject to the dynamics of the economy described by equations (1) and (2).

We consider seven instrument rules. The benchmark rule is the unrestricted optimal feedback rule. Given the linear-quadratic nature of the optimal control problem, the optimal rule is linear in each of the seven state variables. In addition, we consider six restricted instrument rules. The first four of these are all variants of the popular Taylor rule. The first restricted rule is the simple Taylor rule (T), and constrains the feedback of the policy rate to the current annual inflation rate and the current output gap:

\[
i_t = g_p \overline{\pi}_t + g_z z_t
\]

The second restricted rule is a forward-looking Taylor rule (FT). In such a rule the central bank responds to an inflation forecast rather than to current inflation. Following Rudebusch and Svensson (1989), we assume the central bank responds to a constant-interest-rate inflation forecast, that is, the inflation forecast is calculated under the assumption of a constant interest rate. The forecast horizon is assumed to be eight quarters.

\[
i_t = g_p \overline{\pi}_t^e + g_z z_t
\]

The third and fourth restricted rules (TS) and (FTS) correspond to the previous two rules, but allow for interest rate smoothing by including the lagged interest rate in the feedback list:

\[
i_t = g_p \overline{\pi}_t + g_z z_t + g_i i_{t-1}
\]

\[
i_t = g_p \overline{\pi}_t^e + g_z z_t + g_i i_{t-1}
\]

Finally, the last two restricted rules (F) and (FS) are pure inflation-forecast rules with and without smoothing:

\[
i_t = g_p \overline{\pi}_t^e
\]

\[
i_t = g_p \overline{\pi}_t^e + g_i i_{t-1}
\]
For each of these rules the feedback parameters are optimized so as to minimize the unconditional variance of the period loss function in equation (5) (see Appendix 2). In addition, we also report the performance of the original Taylor rule (OT) and a modified Taylor rule with a somewhat larger response to output (MT):

\[ i_t = 1.5\pi_t + 0.5\pi_t \]  
(OT)

\[ i_t = 1.5\pi_t + 1.0\pi_t \]  
(MT)

### ii. Results

The upper panel of Table 2 gives the feedback parameters for each of the nine instrument rules, the corresponding standard deviations of the goal variables, the value of the loss function and the ranking among the rules considered. Following Rudebusch and Svensson (1999), we assume for the benchmark case that the central bank puts equal weight on inflation and output deviations \((\gamma = 0.5)\) and a weight of 0.25 \((\nu = 0.25)\) on the interest rate smoothing component.

The optimal feedback rule in the estimated EU5 model is given by:

\[ i_t = 0.34\pi_t + 0.17\pi_{t-1} + 0.09\pi_{t-2} + 0.05\pi_{t-3} + 1.17\pi_t + 0.12\pi_{t-1} + 0.56i_{t-1} \]  
(6)
This rule implies a quite strong response to the current output gap with policy rates increasing more than one for one with increases in the output gap. Not surprisingly given the considerable weight on interest-rate smoothing in the objective function, the optimal feedback rule also implies a significant feedback on the lagged interest rate.

The importance of the output gap is also obvious in the restricted instrument rules. We find that the weight on the output gap in the simple Taylor rule is as large as the weight on inflation and equals about 1.5. In other words, while the weight on inflation is close to the weight proposed by Taylor (1993), the optimal weight on the output gap is three times as large (1.5 instead of 0.5). This result is consistent with the findings of Ball (1997) who – using a small calibrated model of the US economy – argued that an efficient weight on the output gap should be much larger than the 0.5 proposed by Taylor (1993). The third and fourth rows in Table 2 give an indication of the cost of following a Taylor rule with lower weights on output. Using the original Taylor rule (OT) increases a typical deviation of the output gap by almost 30 basis points and a typical deviation of inflation from target by about 20 basis points. Using a weight of 1.0 reduces these losses considerably.\(^\text{13}\)

Obviously, the optimal feedback coefficients in the Taylor rule will also depend on the weights in the objective function. Figure 3 plots the efficient Taylor rule parameters as a function of the weights on output relative to inflation and the weight on the interest rate smoothing component. In this graph the symbol O on the left side of the solid curves stands for strict output targeting \(\gamma = 0\), while the symbol I on the right side stands for strict inflation targeting \(\gamma = 1\).\(^\text{14}\) The middle curve corresponds to a weight on interest-rate smoothing, \(v\), equal to 0.25 as in the benchmark case. The upper and lower curves correspond to respectively a smaller and greater weight on interest-rate smoothing in the loss function.

A couple of observations are worth making. For a given weight on interest-rate smoothing it appears that the optimal feedback coefficient on the output gap is not much affected by the relative weight on output versus inflation stabilization. A higher weight on inflation does increase the response to inflation considerably. Again, this reflects the crucial role of the output gap in attempts at inflation stabilization in this model. In contrast, the weight on interest-rate smoothing does affect the optimal response to output quite significantly. As interest-rate smoothing becomes more important, the coefficient on the output gap falls quite considerably. Over the sample period the standard deviation of changes in the German policy rate was 0.68. A

\(^{13}\)Recently John Taylor seems to acknowledge that his original proposal may be inefficient and considers rules with a weight on output of 1.0 (Taylor 1998a).

\(^{14}\)A weight in between is what Svensson (1997a) calls flexible inflation targeting.
comparison with the standard deviations of interest-rate changes reported in Table 2 suggests that one needs to assume a weight on interest-rate smoothing that lies between $\nu = 0.25$ and $\nu = 0.5$ to replicate the historical standard deviation.

In view of the considerable weight on interest-rate smoothing in the objective function, allowing for a response to the lagged interest rate in the restricted instrument rule improves the performance of the Taylor rule quite considerably. While the long-run feedback on inflation is not much affected, the long-run response to the output gap doubles compared to the simple Taylor rule (T). As is clear from Table 2, a Taylor rule with interest-rate smoothing (TS) comes very close to the optimal feedback rule in this EU5 model.

Allowing the central bank to respond to a constant interest-rate inflation forecast rather than current inflation does not particularly improve the performance of the Taylor rule. While the optimal feedback coefficient on the output gap falls somewhat and that on the inflation forecast rises, the losses are comparable. The crucial role of the output gap in the transmission mechanism of monetary policy is most obvious when comparing the Taylor rules with the simple inflation forecast rules. The latter clearly perform much worse than the simple Taylor rule. As there are only two shocks in the economy, it is not very surprising that in this economy, one cannot improve very much upon

Figure 3: Efficient Taylor Rule Coefficients
a simple Taylor rule by using inflation forecasts rather than current inflation. Obviously, in a more realistic setting, filtering out temporary shocks from more permanent ones by using an inflation forecast will be optimal, given the considerable lags in the transmission process. In addition, in the simulation results presented in Table 2, we fixed the forecast horizon to eight quarters. Optimizing over the forecast horizon may improve the performance of inflation forecast rules.\(^{15}\)

To summarize, if the model estimated for the EU5 in Section IIA is a reasonably good approximation of the way the euro-area economy works, then the results in this section suggest that a simple Taylor rule with a relatively strong feedback on the output gap would perform quite well in stabilizing the economy in the face of macroeconomic shocks. How do these results relate to the existing literature on optimal monetary policy rules? They are quite similar to the findings in Rudebusch and Svensson (1999). One difference is that Rudebusch and Svensson find a much stronger feedback coefficient on inflation, which is due to the higher persistence of inflation in their estimated US model. Lower inflation persistence, which may be interpreted as higher credibility of the inflation target, implies that the central bank will need to lean relatively less against changes in inflation. Rudebusch (1998) shows that this is indeed the case.

There is more evidence that a simple Taylor rule with a relatively strong feedback on the output gap performs quite well in the US economy. Earlier work includes studies by Henderson and McKibbin (1993) and Levin (1996). More recently Levin, Wieland and Williams (1999) examined the performance of a Taylor-like rule in a range of models for the US economy.\(^{16}\) Similarly to our findings, they find that such a rule outperforms simple inflation-forecast rules. In addition, they find that not much can be gained from including other information (such as lagged variables, foreign variables or the exchange rate) in the feedback rule. In a simple optimizing model of the US economy, McCallum and Nelson (1999) find that increasing the response to both inflation and the output gap in a Taylor rule considerably improves the stabilization of the economy.

The overall positive results that have been found for the US economy contrast with the less favourable results researchers have found for smaller, more open economies. Black, Macklem and Rose (1997), Battini and Haldane (1999) and Drew, Hunt and Scott (1998), for example, find that inflation-forecast

\(^{15}\)See Battini and Haldane (1999) for a lucid discussion of the encompassing features of forward-looking rules.

\(^{16}\)One difference with the findings in this paper is that they find that a strong persistence in the policy rate is optimal. This result is in part due to the fact that it is the long-term interest rate that matters in the aggregate demand equations.
rules have the ability to perform much better than simple Taylor rules in, respectively, the Bank of Canada’s QPM-model, a calibrated model of the UK economy and the Reserve Bank of New Zealand’s FPS-model. Similarly, de Brouwer and O’Regan (1997) find that including the exchange rate and foreign variables improves the performance of the Taylor rule in a model for the Australian economy.

These results are not very surprising. While in closed economies the current output gap and inflation may be close to sufficient statistics to describe the state of the economy, this is unlikely to be the case for more open economies. The most important difference is probably the importance of the exchange-rate channel, which is, for example, emphasized in Svensson (forthcoming) and Battini and Haldane (1999). Indeed, the importance of this channel in relatively open economies is reflected in a significant response to the exchange rate in estimated reaction functions (Clarida, Gali and Gertler 1998). However, as long as the underlying paradigm of a relatively closed economy with the main transmission channel working through the output gap is reasonable for the euro-area economy, a simple Taylor rule would appear to be a useful benchmark.

III. Uncertainty and the Robustness of Simple Taylor Rules

A. The Effect of Estimation Error in the Output Gap

In light of the crucial role of the output gap in the efficient Taylor rules of the previous section, an important question that needs to be addressed concerns the impact of estimation error in the output gap on the efficient feedback parameters and the performance of the Taylor rule. Several authors, including Kuttner (1994), Staiger, Stock and Watson (1996) and Gerlach and Smets (1997) have shown that indicators of capacity utilization such as the output gap or the NAIRU are estimated with a considerable margin of uncertainty. Given the initial aggregation problems and the lack of reliable historical data, this is likely to be even more true for the measurement of a euro area-wide output gap. One counterargument is that the variability of the euro area-wide output gap will be less than that of the individual countries because some of the idiosyncrasies will be averaged out. In this section, we analyse the effect of estimation error in the output gap on the efficient instrument rules and their performance in the estimated model of Section IIA.

To address this question we follow Smets (1998), who argues that estimation error in the output gap may in part explain why the actual central bank response to movements in the output gap is less than optimal control exercises suggest. The loss function and the dynamics of the economy are again given
by equations (1) to (5). However, now we assume, consistent with the estimated model, that output gaps are not directly observed, but need to be estimated. In other words, two of the state variables, $z_t$ and $z_{t-1}$, are unobserved. Current and past growth rates of real GDP, $\Delta y_t$, are observed, but movements could be due to either a change in the growth of potential output or a change in the output gap, so that the central bank faces a signal extraction problem. The Kalman filter which was used in Section IIA to estimate the model gives the optimal estimate of the output gap given the observed data and the structure of the economy.

The middle panel of Table 2 gives the results of the optimal control exercise when we take the estimation error in the output gap due to the unobservable potential output shocks into account. As emphasized in Estrella and Mishkin (1999) and shown by Chow (1970) estimation error in the state variables does not affect the optimal unconstrained feedback rule in a linear-quadratic framework. As a result of this certainty equivalence theorem, the only difference between the optimal linear feedback rule in panel 1 and 2 of Table 2 is that in the latter case the feedback is on the estimated state variables rather than on the actual state variables. However, the loss function is affected as the policy feedback on measurement error in the output gap will filter through into the economy and increase the variability of the goal variables. Indeed, the loss under the optimal feedback rule increases from 1.03 to 1.60.

The results concerning the restricted feedback rules are more interesting. The relative ranking of the different rules is not affected. However, comparing panel 1 and 2 of Table 2, it is obvious that in the simple Taylor rule the weight on output falls from 1.58 to 1.41 and the weight on inflation increases from 1.53 to 1.65. The effect of estimation error is to put less weight on the variable that is measured with error and more on the variable that is perfectly observed. Figure 4 plots the Taylor rule coefficients as a function of the estimation error in the output gap. The optimal response to the output gap in a simple Taylor rule falls at an increasing rate as the standard deviation of the estimation error in the output gap increases, while the optimal response to inflation rises. In both cases the optimal feedback parameter is still relatively high at the estimated standard deviation (around 0.8%). However, increasing the standard deviation beyond its estimate results in a rapid drop of the feedback parameter. A standard deviation of 1.13 would in this model be consistent with a coefficient of 0.5 on output, as suggested by Taylor (1993). It becomes

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*See Appendix 2 for some of the technical details.*

*This result is sometimes called a separation theorem, as one can separate the estimation of the state variables from the optimal control problem. See Chow (1970) for a discussion.*

*See Staiger, Stock and Watson (1996).*
optimal not to respond to the output gap when its standard deviation is larger than 1.15%.

Figure 3 shows that the negative effect of higher estimation error on the Taylor rule coefficients is robust to various weights in the objective function. The dashed lines give the optimal Taylor rule coefficients for various weights taking the estimated output gap uncertainty into account. In almost all cases, the efficient Taylor rule coefficient on output falls, but how much depends on the weights in the objective function. It is worth noting that, although the effect of the estimated output gap uncertainty is to move the efficient Taylor rule parameters in the direction of the values suggested by Taylor (1993), it is clear that one needs either larger than estimated output gap uncertainty or a strong interest-rate smoothing objective to explain why estimated feedback parameters on output are around 0.5. However, from the third and fourth row in the middle panel of Table 2 it is obvious that the loss of having a lower feedback parameter on output (0.5 or 1.0) is much less when one takes into account the measurement error in the output gap.

In sum, estimation error in the output gap can partly explain why central banks in practice respond less to the output gap than suggested by optimal control exercises which do not take into account this uncertainty. In the extreme, high uncertainty may result in a zero response to the output gap.
With the estimated standard deviation of the EU5 output gap, a quite strong feedback is still optimal. It is an empirical question whether estimates of the area-wide output gap are subject to much larger confidence bands. These results are consistent with other recent research that has analysed the effects of measurement error in both output gaps and inflation on the optimal feedback coefficients in a Taylor rule. Rudebusch (1998) and Orphanides (1998) document that there are significant revisions in US estimates of inflation and the output gap, and show that the presence of measurement error reduces the efficient feedback parameters and brings them more in line with the Taylor (1993) ones using a very similar model for the US economy. Aoki (1998) performs a theoretical analysis in a simple, but optimizing model of the US economy. Consistent with the previous results, he shows that noise contained in the data offers a reason for policy conservatism.

B. The Effect of Uncertainty about the Transmission Mechanism

In Section II we argued that a simple model of the transmission mechanism for a relatively closed economy may be a useful starting point for analysing monetary policy in the euro area. However, the uncertainties remain large. In part, this is a generic problem facing central banks. In spite of decades of economic research on this issue, there is still a considerable degree of uncertainty about the precise effects on output and inflation of changes in the monetary policy stance.20 In the case of the euro area, the fact that monetary policy may impact the economy of the different nations differently, the associated aggregation problem, the absence of aggregate historical data and the potential for a structural break under the new regime make an analysis of the euro area-wide transmission mechanism even more complicated. In this section, we make a preliminary, and necessarily limited attempt at assessing the impact of parameter uncertainty on the optimal policy rules considered in this paper.

Following the original work of Brainard (1967), a number of authors, including Svensson (forthcoming), Clarida, Gali and Gertler (1997), Cecchetti (1997) and Estrella and Mishkin (1999), have recently analysed the effect of parameter uncertainty on optimal monetary policy rules using simple, mostly theoretical models of the transmission mechanism.21 Focusing on uncertainty regarding the policy multiplier, these papers show that monetary policy-makers

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20 For an overview of some of the empirical research on the transmission mechanism in the European context, see Kieler and Saarenheimo (1998).

21 A particularly innovative paper is Wieland (1997). He analyses the trade-off between caution and experimentation faced by the central bank.
will be more cautious in the presence of such uncertainty. Until recently, there was, however, little attempt to quantify the effects of parameter uncertainty in an empirical model. Such quantification is important because only in the special case where none of the parameters is correlated can one unambiguously show that higher uncertainty about the transmission mechanism will result in a more cautious response of the central bank to the economy’s state variables.

In this section we use two admittedly limited ways of assessing the impact of model parameter uncertainty on optimal policy behaviour and the performance of the Taylor rule in particular. First, following the literature discussed above, we analyse the optimal Taylor rule taking into account the estimated variance-covariance matrix of the parameter estimates as a measure of model uncertainty. The lower panel of Table 2 presents the results. We basically confirm the results of Estrella and Mishkin (1999) and Rudebusch (1998) who show that parameter uncertainty only marginally reduces the efficient feedback parameters in the instrument rules. This is true for both the optimal linear feedback rule and the simple Taylor rule. Moreover, even doubling the estimated standard deviations of the parameters does not significantly change this result. In sum, conventional parameter uncertainty does not seem to matter very much for the efficient instrument rules.

However, the estimated parameter uncertainty of the EU5 model may not take into account the model uncertainty that arises from the fact that the transmission in the other euro countries may be different, or from structural breaks due to the establishment of the new monetary regime. While a full analysis of the robustness of simple Taylor rules to such model uncertainty deserves a separate paper, Figure 5 presents some suggestive preliminary evidence. In this figure, we plot the efficiency frontier of both the optimal linear feedback rule and the efficient simple Taylor rule (T) for four different versions of the EU5 model. In each case the symbol MT corresponds to the outcome of the Modified Taylor rule with a feedback coefficient of 1.5 on inflation and 1.0 on the output gap as recently suggested by Taylor (1998a). The solid lines correspond to the estimated model.

The lines indicated by Model 1 correspond to a similar model with a reduced slope of the Phillips curve (β is 0.15 instead of 0.33). In other words, compared to the estimated model, the sacrifice ratio is higher and similar to the one estimated for the USA. As also noted in Rudebusch (1998), a steeper slope of the Phillips curve will reduce the feedback coefficient on the output

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23Some of the technical details can be found in Appendix 2.
gap while increasing the one on inflation. The lines indicated by Model 3 correspond to a model in which the output effects of an interest rate rise are much higher (\(\lambda\) is \(-0.15\) instead of \(-0.10\)). A higher interest rate sensitivity will generally reduce the feedback coefficients on both output and inflation. Finally, the lines with short dashes (Model 2) correspond to a model in which the persistence of inflation is greater (\(\alpha(1)\) is 0.85 instead of 0.74).

Except in the latter case, the results of Figure 5 seem to indicate that the modified Taylor rule does relatively well in stabilizing output and inflation compared to the efficiency frontier. The gains from moving to the frontier are typically less than 10 basis points in terms of a reduced standard deviation of inflation and the output gap. More substantial gains can be achieved when inflation is much more persistent than estimated in the EU5 model. In this case an efficient Taylor rule would do much better and could potentially reduce the standard deviation of inflation by more than 20 basis points. The reason for this is that when shocks to inflation are highly persistent it pays for the central bank to be much more aggressive. In this particular case the efficient Taylor rule parameters are 1.9 on inflation and 1.8 on the output gap if the central bank cares equally about output and inflation. Most evidence seems, however, to suggest that inflation persistence has fallen as inflation has come down. This would tend to reduce the efficiency loss due to the use of a modified Taylor rule.
Overall, the evidence presented here suggests that the relatively good performance of a simple Taylor rule with coefficients of 1.5 on inflation and 1.0 on output is robust to small variations in the parameters of the estimated model. This is consistent with tests of the robustness of simple Taylor rules in US models. Levin et al. (1999), for example, find that the Taylor rules they consider are quite robust across the different models of the US economy they analyse. One possible exception that we identified is when the inflation process turns out to be much more persistent than estimated in the EU5 model. In that case optimal Taylor rules are still performing quite well, but the feedback parameters need to be much higher than the ones typically suggested. Of course, the significance of these results is somewhat reduced by the fact that we did not consider radically different models of the euro area economy.

IV. Conclusions

In this paper we have examined whether a simple policy guideline like the one suggested by Taylor (1993) could be used as a benchmark for analysing monetary policy in the euro area. The main attraction of the Taylor rule is its simplicity and clarity. Whether it is a useful benchmark also depends, however, on how good an approximation the guideline is for optimal policy behaviour and how robust its stabilization properties are to changes in the structure of the economy. The latter is particularly important in the euro area because there is considerable uncertainty about the working of the area-wide economy due to both a scarcity of reliable aggregate data and the potential effects of the regime change.

Using a simple estimated model for the euro area based on Rudebusch and Svensson (1999), we show in Section II that simple Taylor rules do a rather good job in stabilizing output and inflation. In addition, we find that estimation error in the output gap does not significantly affect the performance of the Taylor rule, although it does reduce the optimal feedback coefficient on the output gap. We also find that the performance of the Taylor rule is robust to small changes in the parameters of the model. Overall, these rather favourable results are consistent with research on Taylor rules using a variety of models for the US economy.

However, a number of issues remain to be resolved. First, there is still a discrepancy between the actual central bank response to the output gap as estimated in policy reaction functions and the much higher feedback coefficient suggested by the optimal control exercises used in this and other papers. Second, most empirical studies of central bank behaviour reveal that central banks smooth interest rates and only gradually move towards the policy suggested by a Taylor rule. The reasons for interest rate smoothing need
to be better understood. In an innovative study, Sack (1998b) finds that in a situation where the central bank learns about the policy multiplier by observing the reaction of the economy to recent interest-rate changes, it may be optimal to move gradually over time. Third, implementing the Taylor rule requires an estimate of the equilibrium real interest rate. The implications of considerable uncertainty about its level need to be examined.

Frank Smets  
European Central Bank  
Kaiserstrasse 29  
D-60311 Frankfurt-am-Main  
Germany

References


Appendix 1: Some More Evidence on the Transmission Mechanism in Europe

In this appendix we provide some additional evidence on the monetary policy transmission mechanism in six European countries – Austria, Belgium, France, Germany, Italy and the Netherlands. The emphasis is on the output effects of a common monetary policy shock, taking into account the interaction effects among the EU countries due to their trade links. The effects of such a shock will most closely replicate the effects of a common monetary policy under EMU. The immediate goal of the exercise is to provide some suggestive evidence that, first, a common monetary policy shock has quite similar effects on output in the five EU countries that we use in the aggregate model and, second, that the external (dollar) exchange rate has negligible effects on output.

To do so, we estimate for each country \( i \) the following output equation:\(^{24}\)

\[
y^i_t = \sum_{j=1}^{k} A^{i}_{j} y^{i}_{t-j} + \sum_{j=1}^{k} B^{i}_{j} y^{i}_{t-j} + \sum_{j=1}^{k} C^{i}_{j} r^{DM}_{t-j} + \sum_{j=1}^{k} D^{i}_{j} \left( r^{j}_{t-j} f^{DM}_{t-j} \right)
+ \sum_{j=1}^{k} E^{i}_{j} x^{US/DM}_{t-j} + \sum_{j=1}^{k} F^{i}_{j} x^{i/DM}_{t-j} + \epsilon^{i}_t
\]

(A1.1)

where \( y^i \) is output growth in country \( i \), \( r^{DM} \) is the real German interest rate, \( r^i \) the real rate of country \( i \), \( x^{US/DM} \) the dollar–Deutschmark exchange rate and the exchange rate of country \( i \) in Deutschmark.\(^{25}\) We include the German real rate and \( x^{i/DM} \) the Deutschmark–dollar exchange rate as measures of the common monetary policy stance in Europe. In addition, we add the real interest-rate differential and the Deutschmark exchange rate of each European currency to control for deviations of the domestic policy stance from the common monetary policy stance.

Equation (A1.1) can be seen as the output equation of a Vector Auto Regression, which also includes real interest rates and exchange rates. We estimate this system for the six EU-countries mentioned above using SUR estimation and quarterly data over the period 1978 Q1–1995 Q4.\(^{26}\) The

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\(^{24}\)This partially follows the methodology used in Dornbusch, Favero and Giavazzi (1998). In contrast to that paper, we include the real German rate and deviations of real short-term interest rate from the German rate. Dornbusch et al. (1998) include expected and unexpected interest rates in their equation.

\(^{25}\)Real rates are defined as the corresponding nominal rate minus current annual inflation.

\(^{26}\)We also estimated the system excluding Italy over the period (1975 Q1–1995 Q4) with very similar results. The six EU countries represent about 84% of PPP-adjusted real GDP of the euro area in 1990. Without Italy this share falls to about two-thirds of euro area-wide GDP. While it would be interesting to include Spain an the other smaller EMU countries – Ireland, Finland, Portugal and Luxembourg – data limitations prevented us from doing so.
common lag length of two quarters was derived using a sequence of (likelihood ratio) exclusion tests. With this system, we simulate the effect of a 1 percentage point increase in the German real interest rate and the effect of a 1% appreciation of the Deutschmark–dollar exchange rate during eight quarters on GDP in each of the six countries, as well as on a weighted average.

Figures A.1 and A.2 highlight the main results of the analysis. Figure A.1 shows the effect of a one percentage point increase in the interest rate and a 1% appreciation of the dollar exchange rate on a weighted average of GDP in the six countries, together with a two standard deviations confidence band. It is immediately clear that while there is a strong interest rate channel, in contrast to Dornbusch et al. (1998), we do not find a systematic negative effect of an appreciation of the dollar on output in these six countries. Only in France did we find a significant negative effect (See lower panel of Figure A.2). However, even there the implicit weight on the exchange rate in a Monetary Conditions Index is small. While we find these estimates more plausible than the ones suggested by Dornbusch et al. (1998), our results need to be taken with more than the usual degree of caution. In estimating the system, we assumed the real interest rate and the exchange rate to be predetermined. While this may be a reasonable assumption for the real interest rate, it is much less clear for the real exchange rate. In particular, if the exchange rate appreciates in response to forecasts of stronger growth, a simultaneous equations

Figure A.1: Transmission channels in EU6
Note: estimation period = 1978 Q1–1995 Q4
**Figure A2:** Transmission channels in six EU countries: Differences from the average output effect on EU6

Note: estimation period = 1978 Q1–1995 Q4
bias may reduce the estimated output effects of such an appreciation. In future research we intend to address this problem by using instrumental variables in the estimation.

Figure A.2 suggests that there are significant differences in the strength of the interest-rate channel across the six countries. The graph shows the difference in output effects between each country and the EU6 (a weighted average of the six countries) together with a 95% confidence band. An interest rate shock has very similar effects on output in Austria, France, Germany and the Netherlands, but considerably larger effects in Belgium and Italy. In view of the high government debt in both countries, one possible explanation is that higher interest rates increase the government debt burden and lead to a procyclical tightening of fiscal policy. The fact that monetary policy may have larger effects in Italy than in the rest of the EMU area has also been noted in other studies (BIS 1995, or Dornbusch et al. 1998). One explanation emphasized in these studies is that the share of the private debt incurred at adjustable interest rates is larger in Italy than in the other EMU countries. This implies that a rise in interest rates has a more direct effect on the interest-rate burden of the private sector.

**Appendix 2: Estimation and optimal control of the EU5 model**

**A. Estimation of the EU5 Model**

In order to estimate models (1) to (4) using the Kalman filter and maximum likelihood methods, we write the model in state space form. The measurement equations are given by:

\[
\begin{bmatrix}
\Delta y_t \\
\pi_t 
\end{bmatrix} = 
\begin{bmatrix}
1 & -1 \\
0 & \beta 
\end{bmatrix} 
\begin{bmatrix}
z_t \\
z_{t-1}
\end{bmatrix} + 
\begin{bmatrix}
\mu \\
\alpha(L)\pi_{t-1}
\end{bmatrix} + 
\begin{bmatrix}
\epsilon_t^y \\
\epsilon_t^\pi
\end{bmatrix} \tag{A2.1}
\]

The corresponding state equation is:

\[
\begin{bmatrix}
z_{t+1} \\
z_t 
\end{bmatrix} = 
\begin{bmatrix}
\varphi_1 & \varphi_2 \\
1 & 0 
\end{bmatrix} 
\begin{bmatrix}
z_t \\
z_{t-1}
\end{bmatrix} + 
\begin{bmatrix}
\lambda(i_{t-1} - \bar{\pi}_{t-1}) \\
0
\end{bmatrix} + 
\begin{bmatrix}
\epsilon_t^z \\
0
\end{bmatrix} \tag{A2.2}
\]

Assuming that each of the three shocks are independently normally distributed with the following variance-covariance matrix,

\[
\Sigma_\epsilon = 
\begin{bmatrix}
\sigma_y^2 & 0 & 0 \\
0 & \sigma_z^2 & 0 \\
0 & 0 & \sigma_\pi^2
\end{bmatrix} \tag{A2.3}
\]
one can form the likelihood function using the Kalman filter and derive the estimates of the model using maximum likelihood estimation. The results are presented in Table 1.

B. Optimal Control of the EU5 Model

In order to derive the optimal feedback parameters of the different instrument rules, we write the EU5 model in its companion form. The state-space representation of the economy is then given by,

$$X_{t+1} = AX_t + B_i + v_t,$$  \hspace{1cm} (A2.4)

where the vector $X_t$ of state variables, the matrix $A$, the column vector $B$, the disturbance vector $v_t$ are given by

$$X_t = \begin{bmatrix} z_t \\ \pi_t \\ z_{t-1} \\ \pi_{t-1} \\ \pi_{t-2} \\ \pi_{t-3} \\ i_{t-1} \end{bmatrix}, \quad A = \begin{bmatrix} \varphi_1 & -\lambda/4 & \varphi_2 & -\lambda/4 & -\lambda/4 & -\lambda/4 & 0 \\ \beta & \alpha_1 & 0 & \alpha_{21} & \alpha_3 & \alpha_4 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \lambda \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad v_t = \begin{bmatrix} \epsilon_t^z \\ \epsilon_t^\pi \\ 0 \\ 0 \end{bmatrix}.$$

The vector $Y_t$ of goal variables fulfils:

$$Y_t = C_X X_t + C_i i_t,$$  \hspace{1cm} (A2.5)

where:

$$Y_t = \begin{bmatrix} \bar{\pi} \\ z_t \\ i_t - i_{t-1} \end{bmatrix}, \quad C_X = \begin{bmatrix} 0 & 1/4 & 0 & 1/4 & 1/4 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad C_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The period loss function can then be written as

$$E[L_t] = E[Y_t'KY_t],$$  \hspace{1cm} (A2.6)

where $K$ is a $3 \times 3$ diagonal matrix with $(\gamma, 1-\gamma, \nu)$ on the diagonal.

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i. The loss function when the model and the state variables are known

If one assumes that both the parameters of the model in (A2.1) and the state variables are known, then each of the restricted rules can be written as a linear function of the vector of state variables (say \( i_t = gX_t \)). The dynamics of the model and the goal variables for a given rule are then given by:

\[
X_{t+1} = MX_t + v_{t+1} \quad (A2.7)
\]
\[
Y_t = CX_t \quad (A2.8)
\]

where \( M = A + Bg \) and \( C = C_X + C_i g \).

The optimal feedback parameters in each of the restricted instrument rules can then be calculated by minimizing the unconditional loss:

\[
E[L_t] = E[Y'_t KY_t] = \text{trace}(K\Sigma_Y) \quad (A2.9)
\]

where \( \Sigma_Y \) is the unconditional covariance matrix of the goal variables and is given by:

\[
\Sigma_Y = C\Sigma_X C' \quad (A2.10)
\]

and \( \Sigma_X \) is the covariance matrix of the state variables and is in turn related to the covariance matrix of the disturbances by the following equation,

\[
\text{vec}(\Sigma_X) = [I - (M \otimes M)^{-1}] \text{vec}(\Sigma_v) \quad (A2.11)
\]

ii. The loss function with measurement error in the output gap

In matrix notation the observation equation equivalent to equation (A2.1) is given by:

\[
W_t = DX_t + \eta_t \quad (A2.12)
\]

where the vector of observables \( W_t \), the matrix \( D \), and the vector \( \eta_t \) are given by

\[
W_t = \begin{bmatrix} \Delta y_t \\ \pi_t \\ i_{t-1} \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \eta_t = \begin{bmatrix} \varepsilon_i^t \\ 0 \\ 0 \end{bmatrix}
\]

27See Dornbusch and Svensson (1999) for a discussion of how the constant interest-rate forecast can be calculated.
The central bank’s estimate of the current state of the economy is then given by \( E_X = E[X_t | W_t] \).

In this case the objective function is still given by equation (A2.9). However, with measurement errors in the output gap the covariance matrix of the goal variables and the state variables need to be modified to include the effect of measurement error. This results into the following expressions:

\[
\Sigma_Y = C \Sigma_X C' + C \Sigma_S C' \quad (A2.13)
\]

\[
vec(\Sigma_Y) = [I - (M \otimes M)]^{-1} [vec(\Sigma_u) + [(A \otimes A) - I] \vec{vec(\Sigma_S)}] \quad (A2.14)
\]

where \( \Sigma_S \) is the covariance matrix of the measurement errors in the vector of state variables and can be derived separately from the Kalman filter used in (A2.1). (See Chow 1970.)

iii. The loss function under model uncertainty

If the state variables are observed, but the parameters of the estimated model in equation (A2.4) are subject to uncertainty, then equations (A2.9) and (A2.10) again describe the loss function and the covariance matrix of the goal variables, but in this case the covariance matrix of the state variables is given by the following equation:

\[
vec(\Sigma_Y) = [I - E[(A + Bf) \otimes (A + Bf)]]^{-1} vec(\Sigma_u) \quad (A2.15)
\]

In order to calculate the expectation in (A2.15) we use the estimated covariance matrix of the parameters (Chow 1970).