A Cautionary Note on the Effects of Range Restriction on Predictor Intercorrelations

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The purpose of this research report is to highlight a unique set of issues that arise when considering the effects of range restriction in the context of estimating predictor intercorrelations. Three approaches are used to illustrate the issue: simulation, a concrete applied example, and a reanalysis of a meta-analysis of ability–interview correlations. The general conclusion is that a predictor intercorrelation can differ dramatically from the population value when both predictors are used in a composite that is used operationally for selection. The compensatory nature of a composite means that low scorers on one predictor can only obtain high scores on the composite if they obtain very high scores on the other predictor; this phenomenon distorts the correlation between the predictors.

Keywords: range restriction, predictor intercorrelations, ability, interview, meta-analysis

The effects of range restriction on correlations are well documented in the psychometric literature. Pearson published correction formulas in 1903, Aitken (1934) and Lawley (1943) developed general formulas applicable to multivariate cases, and Thorndike’s (1949) treatment of direct versus indirect restriction has been widely cited. Sackett and Yang (2000) offered a review of this area, as well as an expanded typology of ways in which restriction can occur and of techniques for estimating population correlations based on restricted sample correlation.

In applied psychology, the effects of range restriction are most commonly considered in one specific context, namely, the use of the correlation as an index of criterion-related validity, although it is recognized that the same issues apply to any use of the correlation coefficient. For example, Sackett, Laczo, and Arvey (2002) examined the effects of range restriction on the reliability estimates used in correcting validity estimates for attenuation. The purpose of the present research report is to highlight a unique set of issues that arise when considering the effects of range restriction in the context of estimating interpredictor correlations.

We have seen a growing interest in interpredictor correlations. There have been numerous recent meta-analyses of intercorrelations among specific predictors, including interview–cognitive ability relationships (Huffcutt, Roth, & McDaniel, 1996), interview–personality relationships (Cortina, Goldstein, Payne, Davison, & Gilliland, 2000; Huffcutt, Weekley, Wiesner, Degroot, & Jones, 2001), cognitive ability–situational judgment test relationships (McDaniel, Morgeson, Finnegan, Campion, & Braverman, 2001), and personality–situational judgment test relationships (McDaniel & Nguyen, 2001). There have been yet broader studies, combining meta-analytic values to estimate a correlation matrix between an array of predictors and one or more criterion dimensions (e.g., Bobko, Roth, & Potosky, 1999; Cortina et al., 2000). Also, a range of studies have examined the incremental contribution to validity of one or more newer predictors over one or more established predictors (e.g., Clevenger, Pereira, Wiechmann, Schmitt, & Schmidt-Harvey, 2001; Mount, Witt, & Barrick, 2000). All of these efforts were aimed at a better understanding of the nomological network of relationships among predictors and dimensions of job performance.

Perhaps the most basic distinction in conceptualizing range restriction is between direct restriction (e.g., the effects of direct truncation on x or y on the correlation between x and y) and indirect restriction (e.g., the effects of truncation on a third variable z on the correlation between x and y). In the context of predictor–criterion relationships, it is generally the case that direct restriction has a larger effect than indirect restriction. Consider the case in which all population correlations among x, y, and z are .50. Truncation at the mean on x reduces r_{xy} to .33; truncation at the mean on z reduces r_{zy} to .41. In the predictor–criterion relationship context, it is also the case that applying the correction for direct restriction in situations in which restriction is actually indirect results in an undercorrection (Linn, Harnish, & Dunbar, 1981). Although the undercorrection has generally been thought to be small, recent work by Hunter, Schmidt, and Le (2006) showed that the population correlation can be underestimated by as much as 25%.

In this article, we show that a very different pattern of findings can result in some cases when the focus shifts from predictor–criterion relationships to interpredictor relationships. We show that when examining interpredictor relationships, indirect restriction
can have a much larger effect than direct restriction. And we show that applying the correction for direct restriction to scenarios in which restriction is actually indirect can underestimate the population correlation much more dramatically than is the case when examining predictor–criterion relationships (by over 700% in one of our examples). We build an argument for much more careful attention to the process by which range restriction occurs than has been the case in the past when examining interpredictor correlations. The effects of range restriction when examining interpredictor correlations has been noted by others (e.g., Dawes, 1971); we go well beyond this prior work by exploring issues in correcting interpredictor correlations for range restriction and developing implications for meta-analysis.

The key distinction between range restriction in the context of interpredictor correlations (i.e., $r_{xy}$) and in the context of predictor–criterion relationships (i.e., $r_{z}$) lies in the fact that a form of range restriction is possible in the interpredictor context that is not possible in the predictor–criterion context. Consider truncation on a third variable $z$ (i.e., indirect range restriction due to selection on $z$). In the predictor–criterion context, it is possible that $x$ is a part of $z$ (as in the case where $x$ is one of several predictors going into a composite $z$ that is used for selection). It is not possible that $x$ and $y$ are both simultaneously part of $z$, as the predictor domain and the criterion are conceptually and operationally distinct. In the interpredictor context it is also the case that $x_1$ can be part of a composite $z$ used for selection. But it is also possible that the two predictors $x_1$ and $x_2$ are both simultaneously part of $z$. In fact, $z$ may be the composite $x_1 + x_2$. As a simple example, consider estimating the correlation between Scholastic Assessment Test (SAT) scores and high school grade point average (GPA) among students in a college that operationally used a composite of SAT scores and GPA to select students. Here the correlation between SAT scores and GPA is indirectly restricted due to selection on the SAT–GPA composite. In sum, unlike the examination of predictor–criterion relationships in which predictor and criterion cannot both be simultaneously used as part of the operational selection process, the study of interpredictor relationships includes situations in which both predictors contribute to the operational selection composite. We show below that this form of range restriction can produce dramatic departures from the population interpredictor correlation and that failure to apply the proper correction approach can have severe consequences.

Illustrating the Effects of Various Restriction Processes on Interpredictor Correlations

To show how different range restriction processes affect interpredictor correlations, we generated a data set with known properties and then restricted this population in a variety of ways. Specifically, we generated a data set with 1,000 cases, containing four predictors, $A$, $B$, $C$, and $D$, and a criterion measure. Each of the four predictors correlated .50 with the criterion in the population. $A$, $B$, and $C$ each had interpredictor correlations of .50 among themselves, and each of them correlated .00 with predictor $D$. Our focus was on the correlation between $A$ and $B$. These values were chosen for illustrative purposes; we turn later to concrete examples from empirical research.

Table 1 shows six different ways in which range restriction might occur in this data set. The six are illustrative of various ways restriction can occur; the set is not exhaustive. In all cases, a selection ratio of 50% was used. Our first goal was to show how different restriction processes affect $r_{AB}$. In Table 1 we restate the population $r_{AB}$ of .50, then show that direct truncation on $A$ (i.e., eliminating those cases in the bottom 50% on $A$) reduces $r_{AB}$ to .33 and that truncation on a correlated third variable $C$ reduces $r_{AB}$ to .42. We then illustrate truncation on a composite that includes one of the variables of interest. Truncation on $A + C$ (i.e., eliminating those cases in the bottom 50% on the composite $A + C$; recall that $A$ and $C$ correlate .50) reduces $r_{AB}$ to .28; truncation on $A + D$ (recall that $A$ and $D$ correlate .00) reduces $r_{AB}$ to .41. We then turn to the situation of most interest, namely, truncation on a composite that includes both of the variables of interest. Truncation on $A + B$ produces our most dramatic finding: It reduces $r_{AB}$ to .05.

The dramatic reduction of $r_{AB}$ from the population value of .50 to .05 when truncating on the composite $A + B$ merits discussion. Conceptually, a person with a low score on $A$ can only get a high score on the composite $A + B$ if he or she has a very high score on $B$, and vice versa for a person with a low score on $A$. This results in the substantial lowering of $r_{AB}$. The process is perhaps seen most clearly if the scenario changes: We repeated our simulation, setting population $r_{AB} = .00$. Truncating at the mean on the composite $A + B$ reduced $r_{AB}$ to -.47. The compensatory nature of the composite created a negative correlation between the two predictors. This is the phenomenon observed by Dawes (1971): Compensatory selection can cause the sign of the correlation between two predictors to change (e.g., two predictors that are positively correlated in an applicant sample can be negatively correlated in the selected sample). See the Appendix for a more detailed explanation of the process by which truncation on a composite of two variables affects the correlation between the two variables.

Imagine now that one has obtained six sample estimates of the $r_{AB}$ relationship. Each was based on a different one of the six illustrative range restriction processes included above. The result would be a wide range of estimates of $r_{AB}$: .42, .41, .33, .28, .20, and .05. All are large-sample estimates, so sampling error is not a significant issue. The researcher confronting these six estimates would appropriately note the need to take range restriction into account.

Table 1

<table>
<thead>
<tr>
<th>Range restriction scenario</th>
<th>$r_{AB}$</th>
<th>$SD_A$</th>
<th>$r_{AB}$ corrected for direct range restriction on $A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population: no restriction</td>
<td>.50</td>
<td>1.0</td>
<td>.50</td>
</tr>
<tr>
<td>Truncation at mean on $A$</td>
<td>.33</td>
<td>.60</td>
<td>.50</td>
</tr>
<tr>
<td>Truncation at mean on $C$</td>
<td>.42</td>
<td>.90</td>
<td>.46</td>
</tr>
<tr>
<td>Truncation at mean on $A + C$</td>
<td>.28</td>
<td>.74</td>
<td>.37</td>
</tr>
<tr>
<td>Truncation at mean on $A + D$</td>
<td>.41</td>
<td>.86</td>
<td>.46</td>
</tr>
<tr>
<td>Truncation at mean on $A + B + D$</td>
<td>.05</td>
<td>.75</td>
<td>.07</td>
</tr>
<tr>
<td>Truncation at mean on $A + B$</td>
<td>.20</td>
<td>.82</td>
<td>.24</td>
</tr>
</tbody>
</table>

Note. $r_{AC} = r_{BC} = .50$; $r_{AD} = r_{BD} = .00$. As variables are normally distributed, truncation at the mean results in a 50% selection ratio.
account before drawing conclusions as to whether or not population $r_{AB}$ varies substantially from setting to setting.

But let us assume that while we know the specific process producing the restriction, the researcher does not. This is, in fact, typical of practice in the domain of meta-analysis. If meta-analysts make range restriction corrections, they generally do so assuming direct restriction, even if restriction in fact takes place by other processes (Hunter et al., 2006). To simulate this practice in our data, we examined the restricted standard deviations of $A$ in each of our scenarios and report these in the third column of Table 1. Let us assume that the researcher does know the population standard deviation (based on published or local norms). The researcher can use the ratio of restricted to unrestricted standard deviation to apply the formula for correction for direct range restriction; we report these corrected $r_{AB}$ values in Table 1. As shown, in some instances this correction produces a reasonable approximation to the population value; in others, though, the corrected values remain quite far from the population value (e.g., .07, .24).

Thus, in situations in which the predictors of interest are both used in the restriction process, applying the direct range restriction formula does not even approximate the population value. This highlights the importance of understanding the mechanism producing the range restriction in order to accurately estimate the population interpredictor correlation. For example, the restricted $r_{AB}$ of .05 produced by truncating at the mean on the composite $A + B$ corrects up to the population value of .50 if appropriately corrected for indirect restriction. The same finding applies to all other scenarios: If the restriction process is modeled correctly, the corrected correlation differs from the population value only by sampling error.

At this point it is important to differentiate between range restriction due to selecting on the composite $A + B$ and restriction due to sequential selection: selection on $A$ followed by selection on $B$. In the composite case, the process is compensatory: A high score on $A$ can compensate for a low score on $B$ and vice versa. In the sequential selection case, the process is noncompensatory: A low score on $A$ at Stage 1 eliminates a candidate from consideration. In both cases, both $A$ and $B$ are used operationally in the selection process, but the two cases are very different in terms of their implications for attempting to make range restriction corrections. To illustrate this, we used the same simulation data as above to document the effects of sequential selection. As above, we used a selection ratio of 50%: At Stage 1 we used scores on $A$ to eliminate the bottom 25% of the pool, and at Stage 2 we used scores on $B$ to eliminate the bottom 33% of the remaining pool (which corresponds to 25% of the initial pool, thus giving $A$ and $B$ equal weight). This sequential selection on $A$ then $B$ reduced the population $r_{AB}$ from .50 to .35. Recall that selecting on the composite $A + B$ reduced the correlation from .50 to .05. Thus, selecting on the composite $A + B$ has a much more dramatic effect than sequential selection on $A$ then $B$. We also examined the consequences of correcting the sequentially range-restricted $r_{AB}$ using the direct range restriction formula. This resulted in a corrected $r_{AB}$ of .48. Thus, in the case of sequential selection, using the “wrong” correction formula results in only a slight underestimate (.48 vs. .50), whereas using the wrong formula in the composite selection case results in a radical underestimate (.07 vs. .50).

In short, when selection is done on the basis of a composite of two predictors there is great potential for misestimating the correlation between the two.

One additional issue meriting attention is the weight given to each predictor in creating the composite. The illustrative examples used thus far all give equal weight to both predictors. Note that the degree to which an observed interpredictor correlation differs from the population value due to selection on a composite that includes both predictors is maximized when both predictors are equally weighted in the composite. To illustrate, we showed in our examples above that truncating at the mean on $A + B$ reduced $r_{ab}$ from .50 to .05. We performed additional analyses assigning differing weights to $A$ and $B$. Truncation at the mean on $3A + B$ reduced $r_{ab}$ from .50 to .13; using $10A + B$ reduced $r_{ab}$ from .50 to .25. Thus, to accurately correct for range restriction one must take the weights applied to each predictor into account; simply knowing that both variables played a role in the selection process is not sufficient for accurately modeling the range restriction process. In some settings, the weights are known; in others they can be inferred from the pattern of correlations between the predictors and the composite. One must consider both explicit weighting (e.g., assignment of differing weights, as in our examples above) and implicit weighting that result from combining predictors with differing standard deviations.

An Empirical Example

We now offer a concrete example of the importance of attending to this issue. It is drawn from work we have done on selecting students for admission to medical and dental school in Belgium (see Lievens, Buyse, & Sackett, 2005, for more detail). Our primary interest was the introduction of a situational judgment test into the admission process, which also included other components such as a test of general cognitive ability and a test of science knowledge. For purposes of our discussion here, though, we focus on the cognitive ability and science knowledge tests as predictors of GPA in science-oriented classes, and we analyze data from individuals ($N = 1,787$) who passed the admissions test battery and undertook medical or dental studies. The passing rate was about 30%.

Table 2 shows correlations between the science knowledge test, the cognitive ability test, and GPA. It also includes correlations with the overall composite used to determine entry decisions. Both the science knowledge test and the cognitive ability test were part of this composite, as was a situational judgment test and a writing-oriented work sample. We focus on the uncorrected correlations below the diagonal. Surprisingly, a relatively low correlation of .13 is observed between the science knowledge test and the cognitive ability test. Given the long history of finding strong relationships between cognitive tests and measures of knowledge in specific domains (Borman, Hanson, Oppler, & Pulakos, 1993; Schmidt, Hunter, & Outerbridge, 1986) this finding is unexpected (unless one is aware of Dawes’ 1971 work on the effects of compensatory selection on interpredictor correlations).

In this case, however, we know that selection took place on a composite variable that included both the science knowledge test and the cognitive ability test. Because we had scores on this composite variable on which selection took place, we could make use of the multivariate range restriction formula to correct for this indirect range restriction; the resulting values are above the diag-
The observed science–cognitive test correlation of .13 corrects to a value of .46. Thus, the unexpected value in the observed data is due to the specific form of range restriction present here. It is important to note that if we did not have scores on the composite actually used for selection and attempted to rely instead on a direct range restriction correction (e.g., using the restricted vs. unrestricted standard deviation for the cognitive test as the basis for correction), we would have obtained values of .16 for the science–cognitive test correlation. This corrected correlation is dramatically different from the correlations corrected for indirect range restriction.

Implications for Research on Interpredictor Correlations

We have shown that different range restriction processes can have dramatically differing effects on interpredictor correlations. Thus, varying restriction processes can be an important source of variance in meta-analytic investigations of interpredictor correlations. We note, though, that these varying processes have not been consistently taken into account in meta-analyses. For example, McDaniel and Nguyen (2001) reported a meta-analysis of relationships between situational judgment tests and the Big Five personality dimensions. They reported a “bare bones” analysis, which did not consider range restriction. Huffcutt, Roth, and McDaniel (1996) reported a meta-analysis of relationships between cognitive ability and employment interviews. They corrected for direct range restriction, using an artifact distribution obtained from another study, as they found that there was not enough information available in the studies available for their meta-analysis.

We argue that it is a lack of information in primary studies, rather than lack of awareness of range restriction, that results in an inability to accurately model range restriction processes when conducting meta-analyses. But we do believe that the arguments and examples offered in this article offer some useful insights for meta-analysts. The most significant is based on the demonstration here that the greatest departures from population intercorrelation values occur when both predictors in question contribute to a composite used for selection. Thus, even if details about restriction processes are not reported or not available (as in the case in which the variable used for selection is a composite including the predictors in question as well as other unscored information), it may be possible to differentiate between settings in which the predictors in question were or were not part of the selection process. To shed light on range restriction effects, one could code information about predictor use as a moderator, for example, with (a) both predictors definitely part of the selection process; (b) both predictors potentially part of the selection process but information missing as to whether they were actually used; (c) only one predictor used as part of the selection process; (d) only one predictor potentially part of the selection process; (e) neither predictor part of the selection process, but the sample is screened on some other predictors; or (f) no screening occurs, as data are from an unscreened applicant sample. For example, in studies using a concurrent design in which both predictors are administered to a current employee sample, one can conclude with confidence that neither predictor was part of the selection process, and thus that the form of range restriction of greatest concern in terms of departures from population values is not possible. A comparison of values from concurrent designs versus other designs may serve as a useful quick indicator of whether differential range restriction processes might affect interpredictor correlations.

We illustrate the above ideas by a meta-analytic reanalysis of ability–interview relationships. This domain has been examined in meta-analyses by Huffcutt et al. (1996) and Cortina et al. (2000). We emphasize that the intent is not to be critical of these prior meta-analyses. We are introducing what we believe is an important new issue here; we do not expect prior researchers to have been aware of this. We located the studies listed in their reference sections and supplemented them with a search for more recent studies. Adding 13 new studies to the database from prior meta-analyses resulted in 71 ability–interview correlations. We set aside 9 studies in which the interviewer had access to the cognitive ability scores, reasoning that such access contaminated the ability–interview relationship. Two of the authors independently coded each study, and the few disagreements were resolved by discussion. Each study was coded into one of four categories: (a) neither variable (ability or interview) used in the selection process (27 studies, $N = 7,225$, mean ability–interview $r = .18$); (b) one of the two variables used in the selection process (13 studies, $N = 2,947$, mean $r = .22$); (c) both variables used in the selection process (15 studies, $N = 2,077$, mean $r = .12$); and (d) both variables possibly used in the selection process but insufficient information to be certain (7 studies, $N = 3,115$, mean $r = .13$).

In the first two categories (neither variables used in the selection process or only one used in the selection process), the form of range restriction of concern here (i.e., selection on the composite of ability and interview) is not possible; the mean ability–interview

<table>
<thead>
<tr>
<th>Variable</th>
<th>$M_U$</th>
<th>$SD_U$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>$M_R$</th>
<th>$SD_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Selection composite</td>
<td>20.63</td>
<td>4.76</td>
<td>—</td>
<td>.88</td>
<td>.61</td>
<td>.49</td>
<td>26.06</td>
<td>2.51</td>
</tr>
<tr>
<td>2. Science knowledge test</td>
<td>20.36</td>
<td>6.22</td>
<td>.69</td>
<td>.84</td>
<td>.46</td>
<td>.47</td>
<td>26.50</td>
<td>4.18</td>
</tr>
<tr>
<td>3. Cognitive ability test</td>
<td>27.58</td>
<td>5.99</td>
<td>.38</td>
<td>.13</td>
<td>.84</td>
<td>.16</td>
<td>31.73</td>
<td>4.67</td>
</tr>
<tr>
<td>4. GPA</td>
<td>—</td>
<td>—</td>
<td>.28</td>
<td>.25</td>
<td>.07</td>
<td>—</td>
<td>0.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Note. $N = 1,787$. Reliabilities can be found on the diagonal. Dashes indicate that no reliability estimates were available. Uncorrected correlations are below the diagonal. Correlations corrected for indirect range restriction of greatest concern here (i.e., selection on the composite of ability and interview) is not possible; the mean ability–interview...
$r$ across these two categories is .192. In the second two categories, it is possible that the ability–interview composite was used for selection; the mean ability–interview $r$ across these two categories is .126.

These findings illustrate the argument developed in this article, namely, the importance of understanding the mechanism producing range restriction. The findings show that studies in which both predictors are used in compensatory fashion, as well as studies with limited information such that the compensatory use of the predictors cannot be ruled out, produce lower mean correlations than studies in which compensatory selection is not present. Thus, this analysis documents that the ability–interview correlation is lowered as a result of the simultaneous selection on both variables, relative to studies in which simultaneous selection does not take place. Note that standard meta-analytic procedure would be to apply a correction for direct range restriction to all of these studies; both Huffcutt et al. (1996) and Cortina et al. (2000) did so in their meta-analyses of this domain. Doing so will result in an underestimation of the correlation between the two predictors.

We want to reiterate that the ability–interview domain was examined in detail simply to illustrate the issue at hand. The issue is relevant to any domain in which interpredictor correlations are examined. For example, Cortina et al. (2000) also examined the ability–conscientiousness relationship. Included in their analysis is a study by Hansen (1989), in which ability and conscientiousness had both been used operationally to select production workers. In the personality–assessment center domain, Goffin, Rothstein, and Johnston (1996) reported a study in which both personality score and assessment center ratings were used in selection decisions. Careful attention to range restriction processes is warranted in all domains.

In light of our findings we suggest that future meta-analyses of interpredictor correlations sort studies into those in which compensatory selection on the two predictors in question can be ruled out and those in which it cannot be ruled out. As we did above, mean correlations can be computed between the predictors in the two conditions. If they differ, the options are to (a) set aside the studies in which compensatory selection cannot be ruled out and base one’s conclusions on the studies in which compensatory selection can be ruled out or (b) appropriately correct for the specific form of range restriction in those studies with compensatory selection, should the information needed to do so be available. We call attention to a number of useful resources for making corrections, should the needed data be available. If the composite of predictors is reported directly, that composite can be treated as the third variable on which selection occurs in the standard formula for incidental range restriction. If the components of the selection composite are reported, but the composite itself is not reported, it may be possible to use the theory of composites to compute the missing correlations. Ghiselli, Campbell, and Zedeck (1981) provided an extensive treatment of relationships between composites and the variables making up the composites.

We also note that failure to account for difference in the type of range restriction processes will result in more unexplained variance in correlations across studies, thus falsely suggesting a greater impact of substantive moderator variables. Careful attention to differences in range restriction processes may result in more accurate estimates of the true variability of the relationship in question across settings.

Implications for Incremental Validity

The incremental validity of one predictor over another is influenced by the intercorrelation between the predictors. Underestimating the predictor intercorrelation results in overestimation of incremental validity. Thus, the underestimation of interpredictor correlations due to compensatory selection that is the focus of this article has implications for incremental validity.

To demonstrate this we integrate our findings—that ability–interview correlations are higher when compensatory selection can be ruled out than when compensatory selection is possible—with prior work by Cortina et al. (2000) on the incremental validity of interviews over ability and conscientiousness measures. Cortina et al. assembled a correlation matrix involving ability, conscientiousness, interview, and job performance measures, in which each value in the matrix was a meta-analytic estimate. They estimated the incremental validity for three levels of interview structure; for demonstration purposes here we pool the data across the three levels of structure. We used our ability–interview correlation estimates discussed earlier to document the effects of differing values of ability–interview correlations on the estimate of the incremental validity of the interview. When using the ability–interview correlation estimated from studies in which compensatory selection on an ability–interview composite could be ruled out, the increment in $R^2$ for the interview was .058; when estimated on studies in which compensatory selection could not be ruled out, the increment in $R^2$ for the interview was .042. Although the difference is not particularly large in this instance, it could certainly be far larger in other domains. The interest here is not in the incremental validity of the interview per se but in calling attention to the need for care in estimating interpredictor correlations when examining incremental validity.

The summary message of this brief note is that certain forms of range restriction processes can distort observed interpredictor correlations to a much greater degree than we suspect would be expected by many researchers used to thinking about range restriction in the context of predictor–criterion relationships. Attention to range restriction processes can prevent misinterpretation of primary studies (as in the medical school admissions example offered above) and may help gain a better understanding of variability in findings across studies in meta-analytic contexts.

References

three principles of human decision making. American Psychologist, 26, 180–188.


Appendix

In the text we note that selection on a composite of two variables can radically alter the correlation between the two variables. Here we elaborate on how and why this happens.

First, it is necessary to determine the correlation between the two variables (A and B) and the equally weighted composite A + B. For simplicity, call that composite C. The correlation between each variable (A and B) and the composite of the two (C) is:

\[ r_{AC} = r_{BC} = \frac{1 + r_{AB}}{\sqrt{2 + 2r_{AB}}} \]  

(1)

(see Ghiselli, Campbell, & Zedeck, 1981, p. 180, for a generalization of this formula to any number of variables).

Note that if A and B are uncorrelated, the formula reduces to 1/\sqrt{2}, or .707. Thus, even with uncorrelated variables, each correlates highly with the composite; if \( r_{AB} \) is .50, A and B each correlate .866 with the composite C.

These three correlations (\( r_{AB}, r_{AC}, \) and \( r_{BC} \), with the latter two correlations necessarily identical) are needed to use the standard formula for indirect range restriction to examine the effects of truncation on C on \( r_{AB} \). The formula for showing how indirect range restriction results in a different correlation in the restricted sample than in the unrestricted sample is

\[ r'_{AB} = \frac{r_{AB} - r_{AC}r_{BC} + r_{AC}r_{BC}sc_{C}^{2}/s_{C}^{2}}{\sqrt{1 - r_{AC}^{2} + r_{AC}^{2}sc_{C}^{2}/s_{C}^{2}}} \]  

(2)

where \( r'_{AB} \) is the restricted correlation between A and B, \( s_{C}^{2} \) is the restricted variance of the composite C, and \( s_{C}^{2} \) is the unrestricted variance of the composite C. This formula requires the three correlations discussed above, plus the ratio of restricted variance to unrestricted variance on C, the composite used for selection. This is a direct function of the selection ratio, and tables exist showing the restricted variance corresponding to various selection ratios (e.g., Schmidt, Hunter, & Urry, 1973). Here we use a selection ratio of .30 for illustrative purposes; that selection ratio produces a restricted standard deviation of .515, relative to an unrestricted standard deviation of 1.000. Expressed as a variance, the ratio of restricted variance to unrestricted variance is .265.

Now consider the impact of these values when using the indirect range restriction equation to show the effects of truncation on the composite on \( r_{AB} \). We first focus on the numerator of the equation. The numerator has three terms. It starts with the unrestricted correlation of interest (\( r_{AB} \)), and subtracts the product of the.
correlations of each variable with the composite \( r_{AC} \) and \( r_{BC} \). Here we see one key aspect of range restriction: the higher the correlation between the two variables of interest \((A \text{ and } B)\) and the third variable on which selection takes place \((C)\), the larger the value that is subtracted from the population correlation. It is useful to consider the question “under what circumstances can truncation on \( C \) change the sign of \( r_{AB} \).” The answer is straightforward: Only if the product of \( r_{AC} \) and \( r_{BC} \) is greater than \( r_{AB} \) itself is such an outcome possible. A key insight is that when the truncation variable \( C \) is the composite of \( A \) and \( B \), the values of \( r_{AC} \) and \( r_{BC} \) must be large (unless \( A \) and \( B \) correlate negatively with each other). As noted above, if \( r_{AB} \) is .50, \( r_{AC} \) and \( r_{BC} \) are both .866. The product of \( r_{AC} \) and \( r_{BC} \) is .75; thus, the restriction process begins by subtracting .75 from the unrestricted \( r_{AB} \) of .50. In contrast, in many cases of indirect range restriction the truncation variable is operationally independent of \( A \) and \( B \) (as in a predictor-criterion context where \( A \) is the predictor, \( B \) is the criterion, and \( C \) is the indirect truncation variable), and \( r_{AC} \) and \( r_{BC} \) are considerably lower. If \( A \) and \( B \) both correlate .50 with \( C \), the product of \( r_{AC} \) and \( r_{BC} \) is .25, and thus this value of .25 is subtracted from the unrestricted value of .50. Thus, this is the critical mechanism: Selecting on the composite of \( A \) and \( B \) means that \( A \) and \( B \) will each correlate very highly with the composite; at any given selection ratio this term drives the range restriction process.

Moving to the third term in the numerator, we see that the product of \( r_{AC} \) and \( r_{BC} \), which was subtracted from unrestricted \( r_{AB} \) at Step 2, is now multiplied by the term reflecting the selection ratio (e.g., the ratio of restricted to unrestricted variance on the truncation variable \( C \)). This third term thus captures the effects of the selection ratio: it is maximized when the selection ratio is 1.00 and minimized as the selection ratio approaches zero. Whereas the second term adjusts unrestricted \( r_{AB} \) down as a function of the correlation between \( A \) and \( B \) and the truncation variable \( C \), the third term scales this downward reduction as a function of the selection ratio. Thus, in conjunction, the second and third terms of the numerator indicate that unrestricted \( r_{AB} \) is reduced (a) the greater that \( A \) and \( B \) correlate with \( C \) and (b) the lower the selection ratio.

The denominator has two sets of terms, which reflect the degree to which truncation on \( C \) reduces the variance in \( A \) and \( B \), respectively. They parallel the numerator: The variance (which is 1.00 in the no-restriction case) is reduced as a function of (a) the correlation of each variable with the truncation variable and (b) the selection ratio.

Thus, if we begin knowing nothing more than the fact that \( A \) and \( B \) correlate .50, we can use the theory of composites to determine that \( A \) and \( B \) each must correlate .866 with an equally weighted composite \((C)\) of the two. Selecting a selection ratio (30% in this example), we can use the formula for the effects of indirect range restriction to determine the effects of truncation on \( C \) on \( r_{AB} \). Inserting these values into the formula produces a restricted \( r_{AB} \) of -148. In sum, the key to the dramatic departure from the unrestricted value of \( r_{AB} \) when selection is done on the basis of \( C \) is that (a) \( A \) and \( B \) both correlate highly with \( C \) as they are part-whole correlations and (b) at any given selection ratio, the product of \( r_{AC} \) and \( r_{BC} \) drives the degree to which selection on \( C \) reduces \( r_{AB} \). High values of this product result in restricted values of \( r_{AB} \) far lower than unrestricted \( r_{AB} \).

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