***Completely monotone outer* approximations**

of

***lower* probabilities**

on

finite possibility spaces

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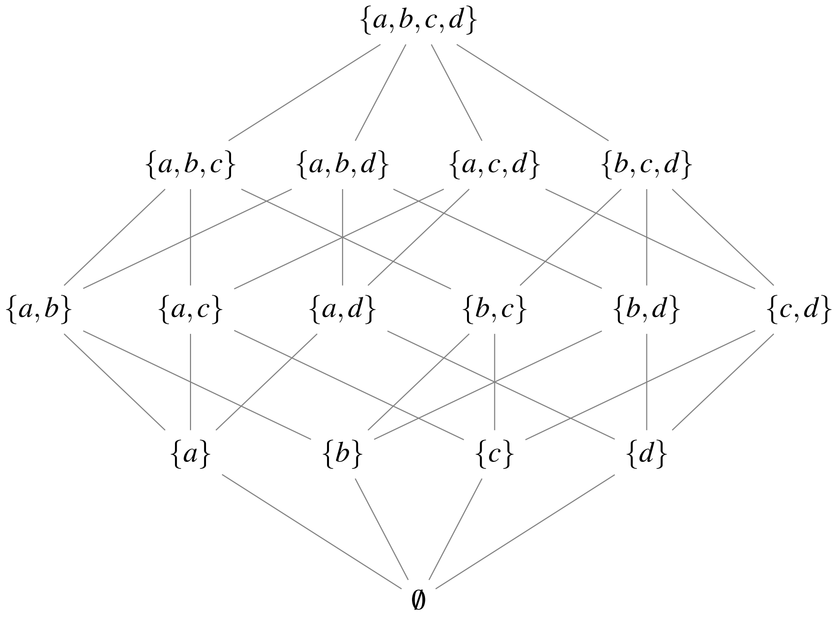
Belgium



$\{a,b,c,d\}$
$\{a,b,c\}$
$\{a,b,d\}$
$\{a,c,d\}$
$\{b,c,d\}$
$\{a,b\}$
$\{a,c\}$
$\{a,d\}$
$\{b,c\}$
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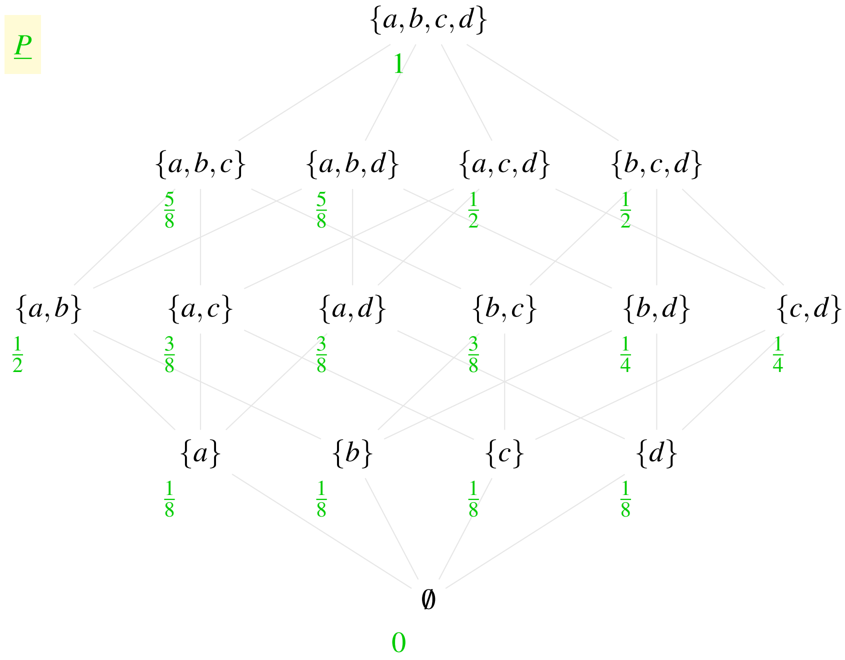
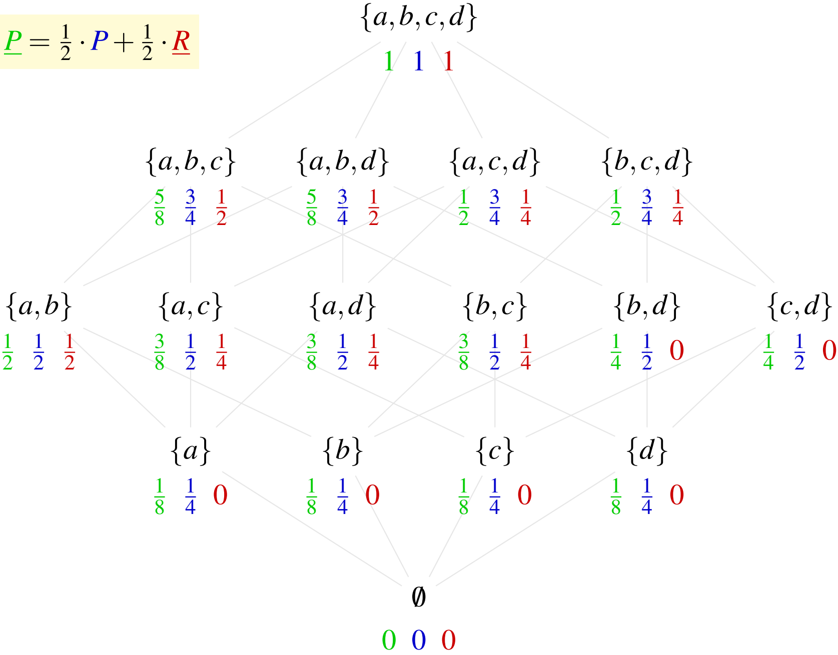
$\{c,d\}$
$\{a\}$
$\{b\}$
$\{c\}$
$\{d\}$
$\emptyset$



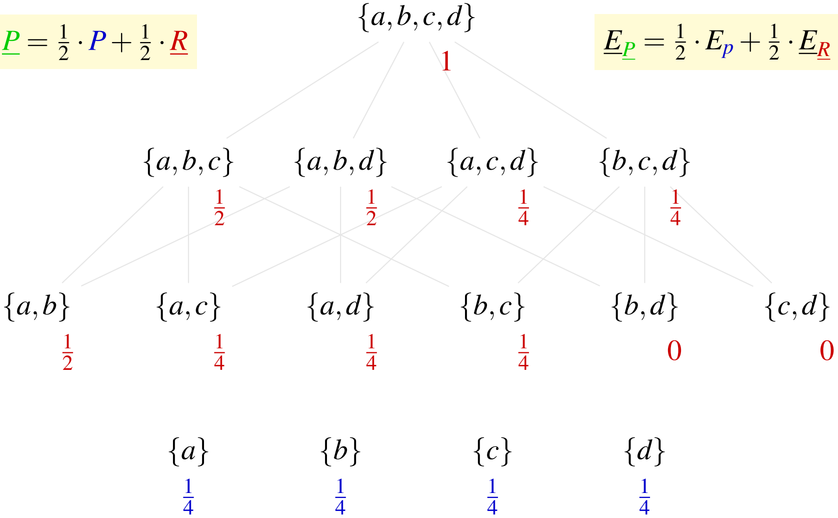
$\underline{P}$
$\{a,b,c,d\}$ 1
$\{a,b,c\}$ $\frac{5}{8}$
$\{a,b,d\}$ $\frac{5}{8}$
$\{a,c,d\}$ $\frac{1}{2}$
$\{b,c,d\}$ $\frac{1}{2}$
$\{a,b\}$ $\frac{1}{2}$
$\{a,c\}$ $\frac{3}{8}$
$\{a,d\}$ $\frac{3}{8}$
$\{b,c\}$ $\frac{3}{8}$
$\{b,d\}$ $\frac{1}{4}$
$\{c,d\}$ $\frac{1}{4}$
$\{a\}$ $\frac{1}{8}$
$\{b\}$ $\frac{1}{8}$
$\{c\}$ $\frac{1}{8}$
$\{d\}$ $\frac{1}{8}$
$\emptyset$ 0

$\underline{P} = \frac{1}{2} \cdot P + \frac{1}{2} \cdot \underline{R}$

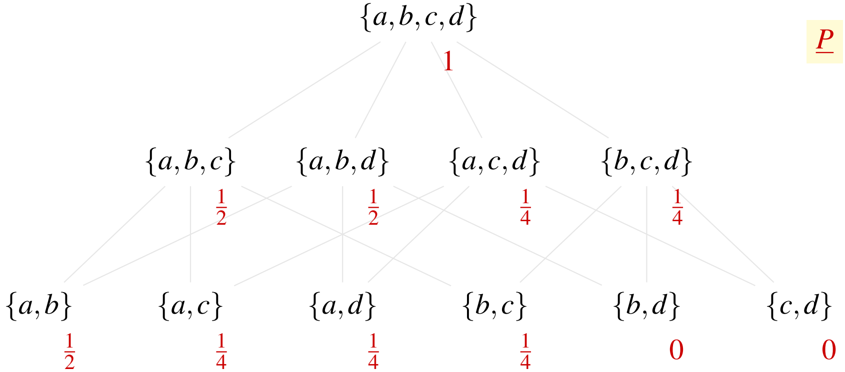


$\underline{P} = \frac{1}{2} \cdot P + \frac{1}{2} \cdot \underline{R}$

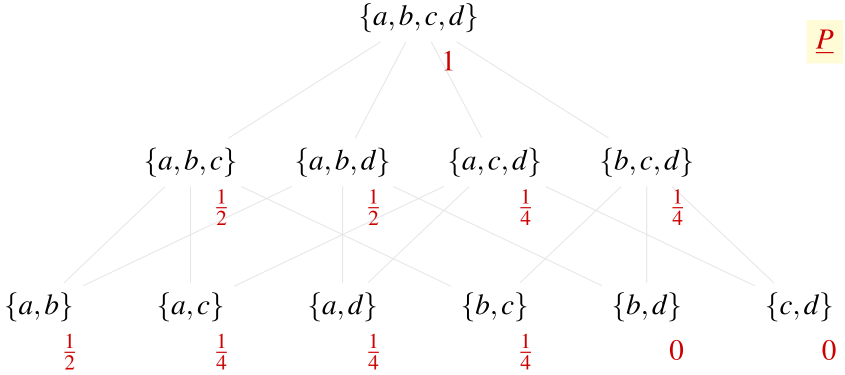
$\underline{E}_{\underline{P}} = \frac{1}{2} \cdot E_p + \frac{1}{2} \cdot \underline{E}_{\underline{R}}$



linear–imprecise decomposition

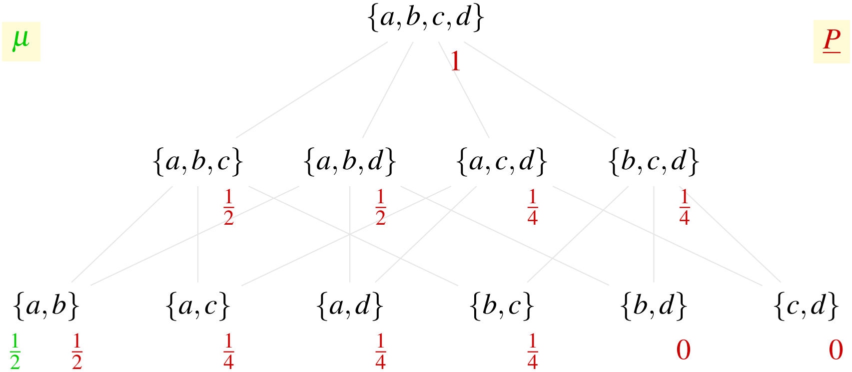


$\underline{P}$
$\{a,b,c,d\}$
1
$\{a,b,c\}$
$\frac{1}{2}$
$\{a,b,d\}$
$\frac{1}{2}$
$\{a,c,d\}$
$\frac{1}{4}$
$\{b,c,d\}$
$\frac{1}{4}$
$\{a,b\}$
$\frac{1}{2}$
$\{a,c\}$
$\frac{1}{4}$
$\{a,d\}$
$\frac{1}{4}$
$\{b,c\}$
$\frac{1}{4}$
$\{b,d\}$
0
$\{c,d\}$
0



Recursive
Möbius transform

$$\mu A = \underline{P} A - \sum_{B \subset A} \mu B$$



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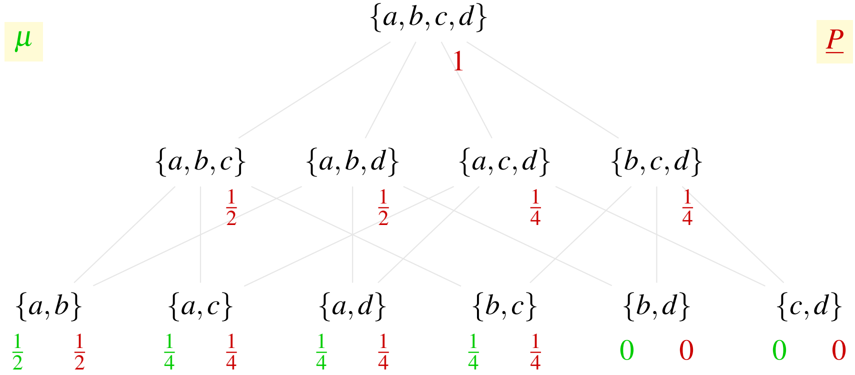

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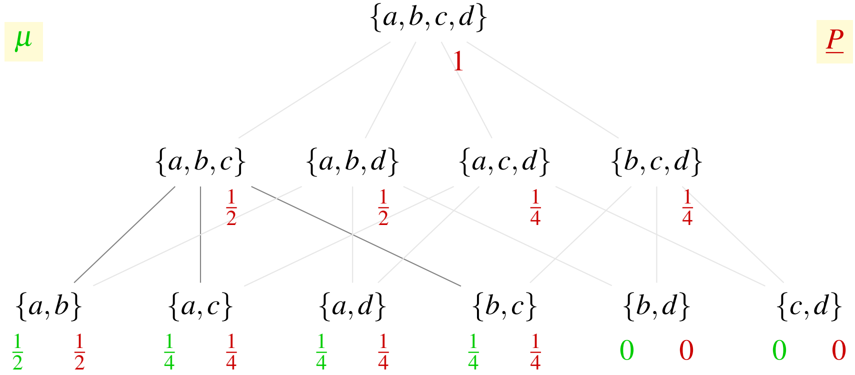

Recursive
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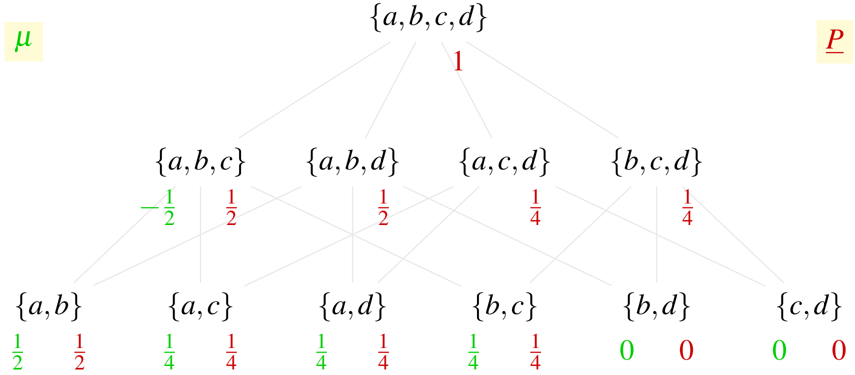

Recursive
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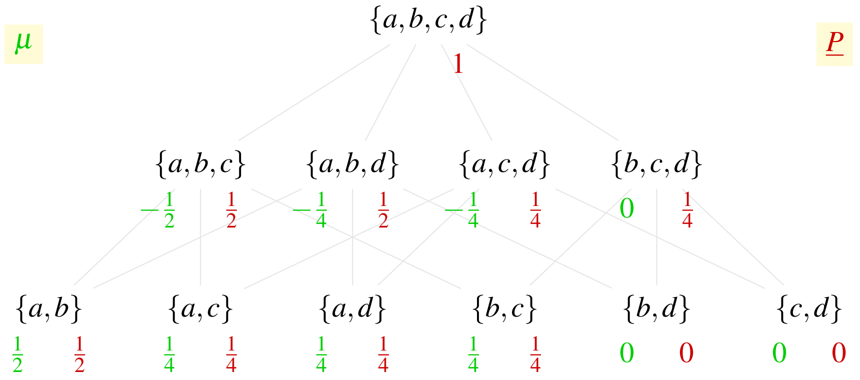
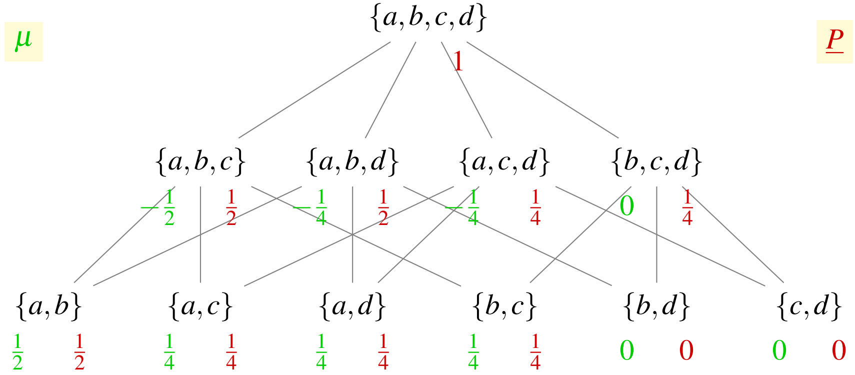
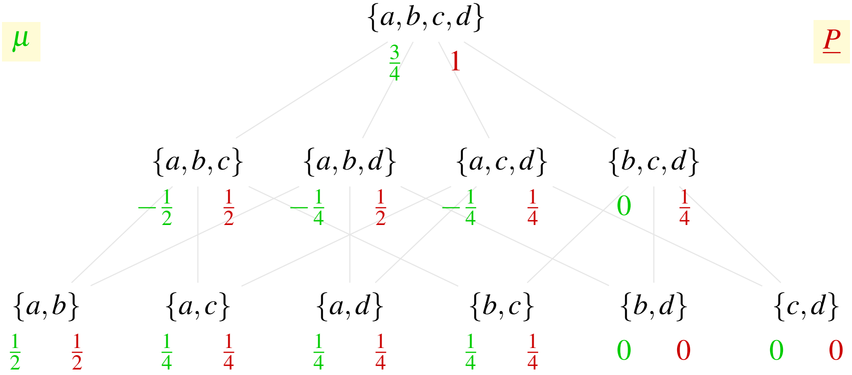


Recursive
Möbius transform

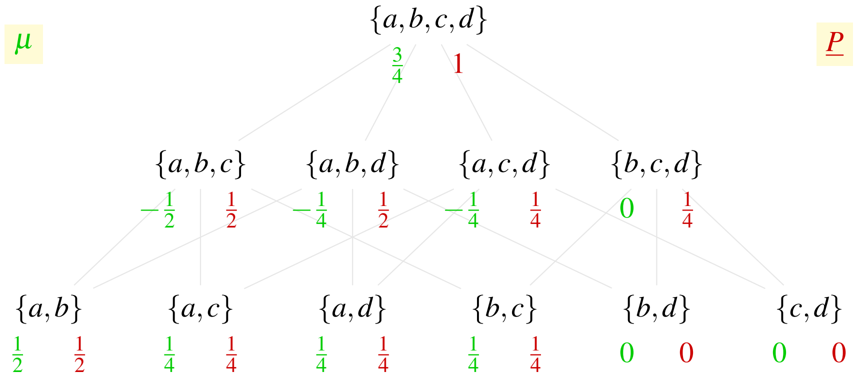
$$\mu A = \underline{P}A - \sum_{B \subset A} \mu B$$



Recursive
Möbius transform $\mu A = \underline{P}A - \sum_{B\subset A} \mu B$



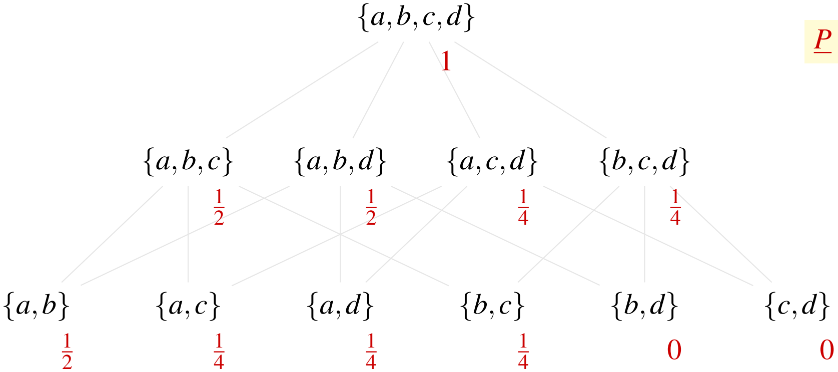
Recursive Möbius transform $\mu A = \underline{P} A - \sum_{B \subset A} \mu B$



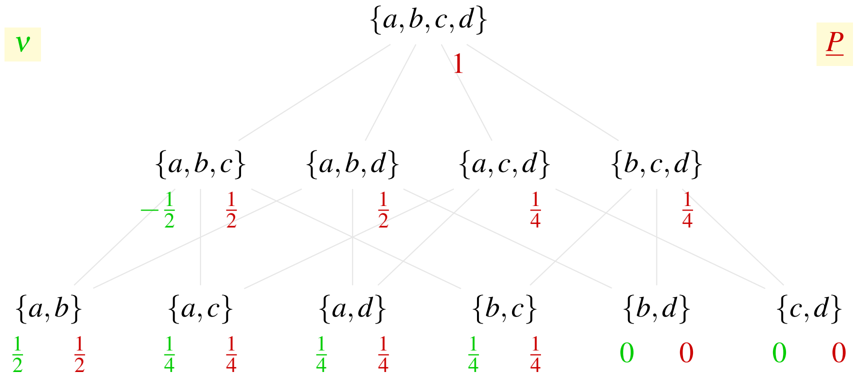
Recursive Möbius transform $\mu A = \underline{P} A - \sum_{B \subset A} \mu B$

$\underline{P} A = \sum_{B \subseteq A} \mu B$ Möbius inverse

$\underline{P}$

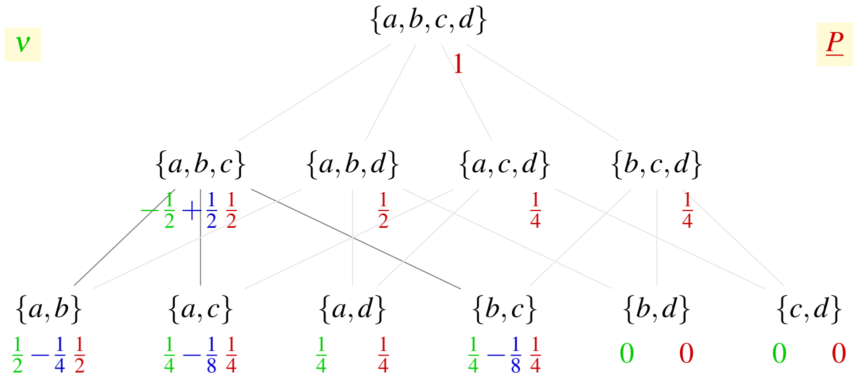


Iterative Rescaling Method



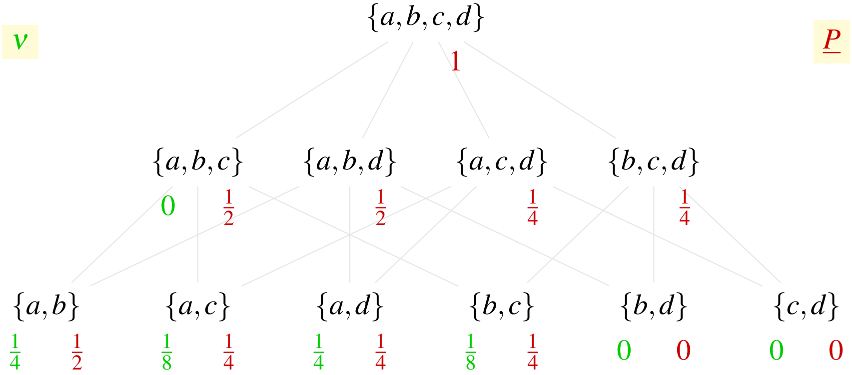
Recursive
Möbius transform

Iterative Rescaling Method

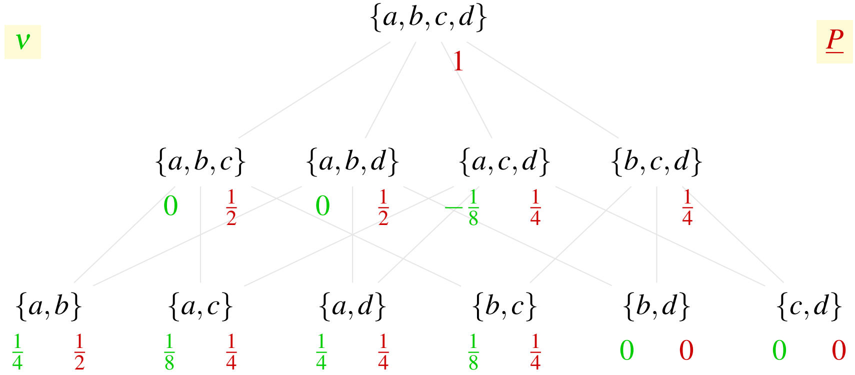


Iterative Rescaling Method

Rescale
when negative

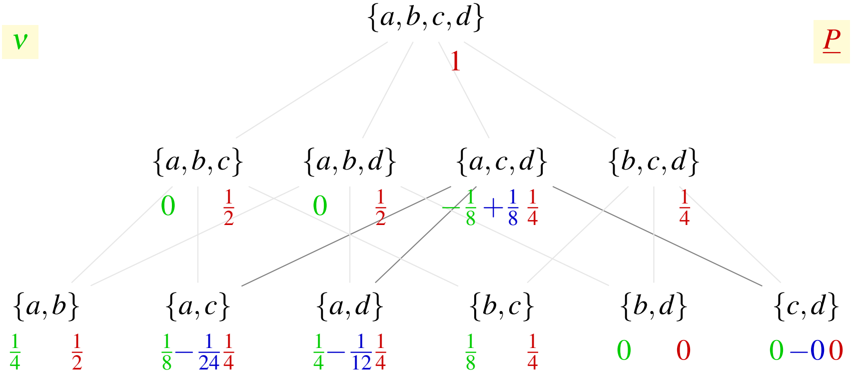


Iterative Rescaling Method



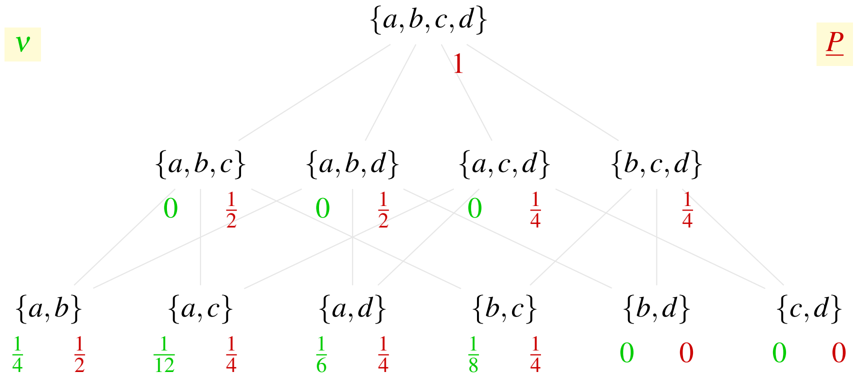
Recursive Möbius transform

Iterative Rescaling Method

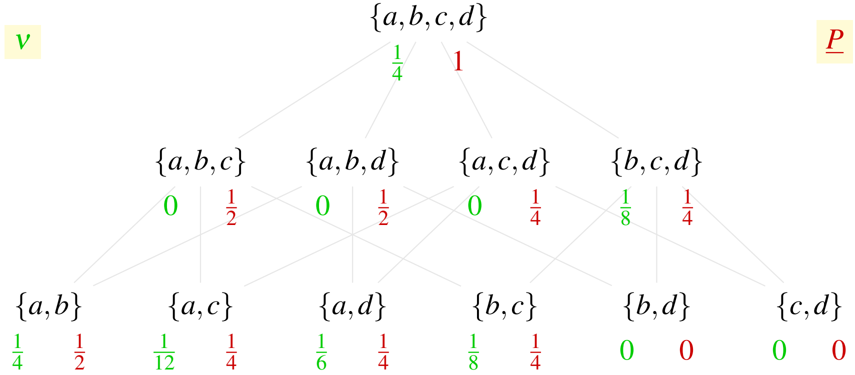


Iterative Rescaling Method

Rescale
when negative

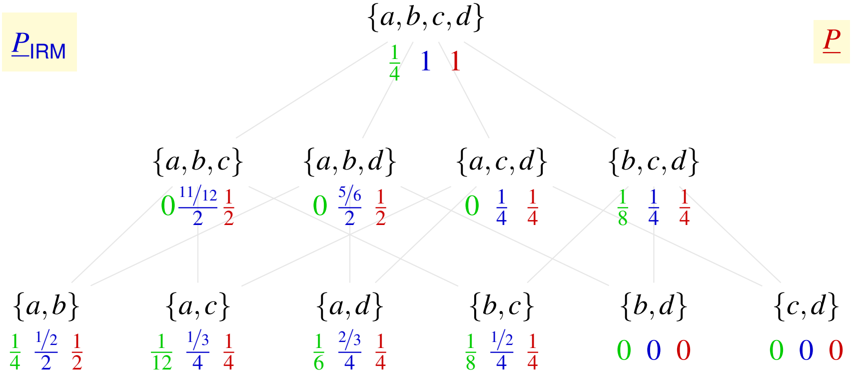


Iterative Rescaling Method



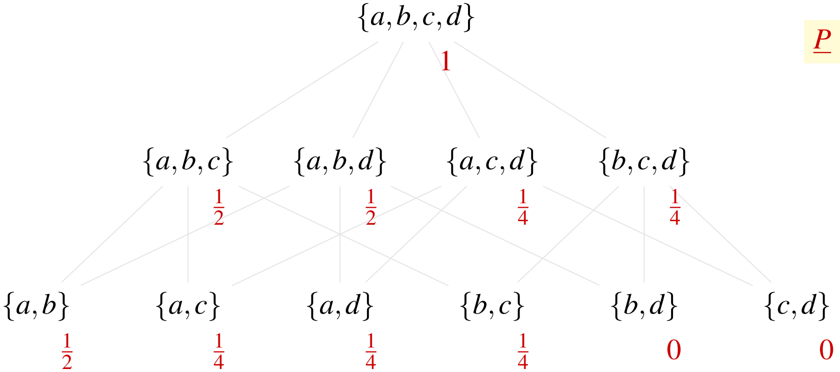
Recursive Möbius transform

Iterative Rescaling Method

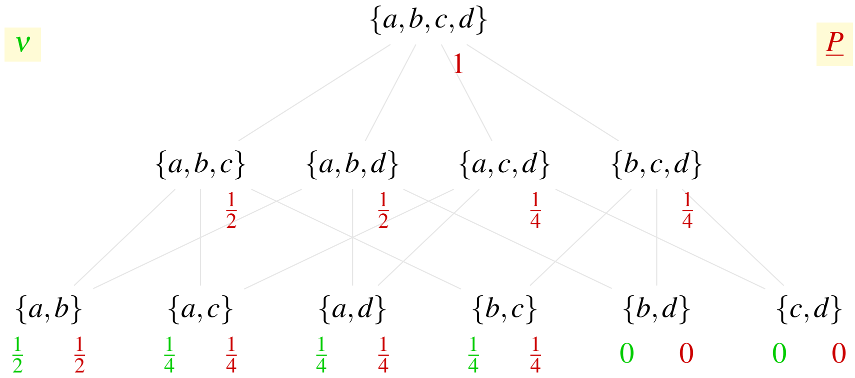


Iterative Rescaling Method

$\underline{P}_{\text{IRM}} A = \sum_{B \subseteq A} \nu B$ Möbius inverse

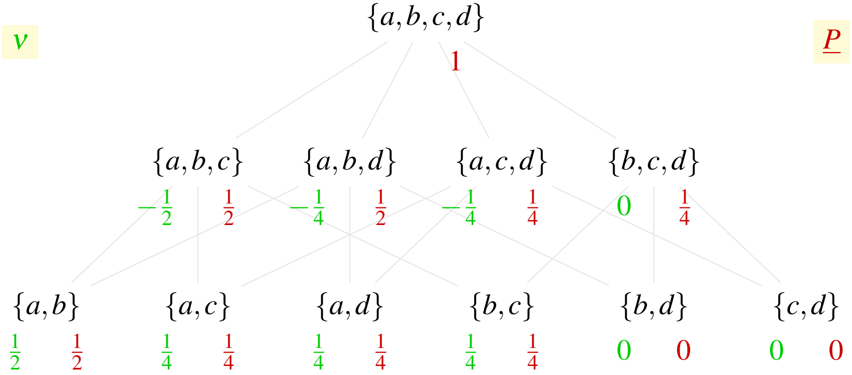


Iterative ***Minimal*** Rescaling Method



Möbius transform
per cardinality

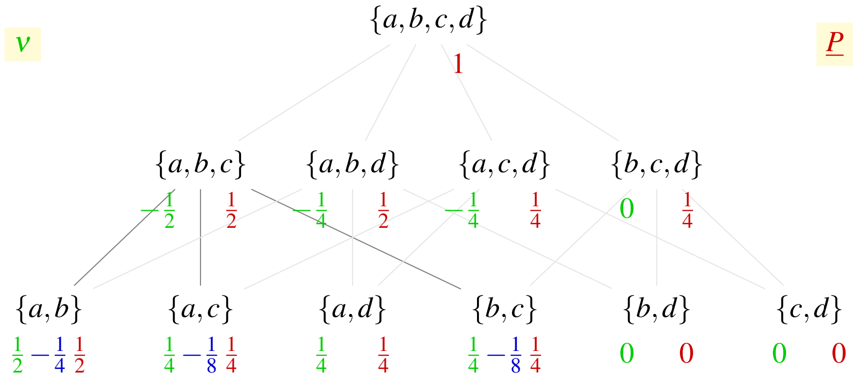
Iterative ***Minimal*** Rescaling Method



Minimal

Möbius transform
per cardinality

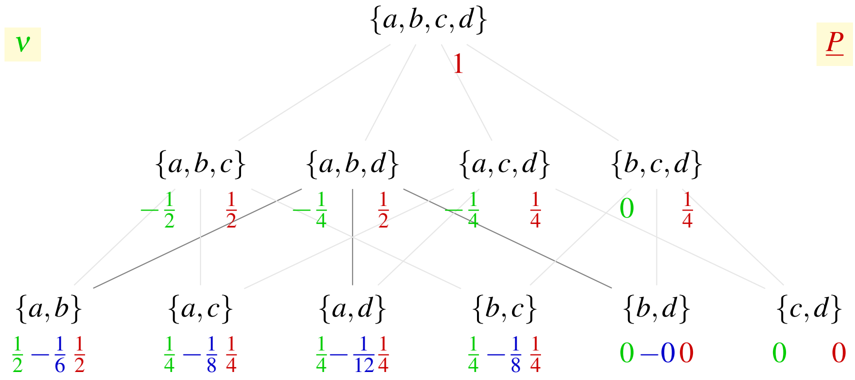
Iterative Rescaling Method



Minimal

Iterative Rescaling Method

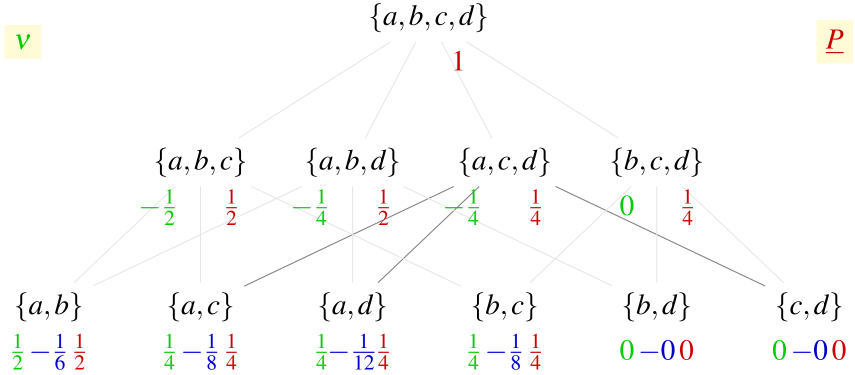
Rescale
when negative



Minimal

Iterative Rescaling Method

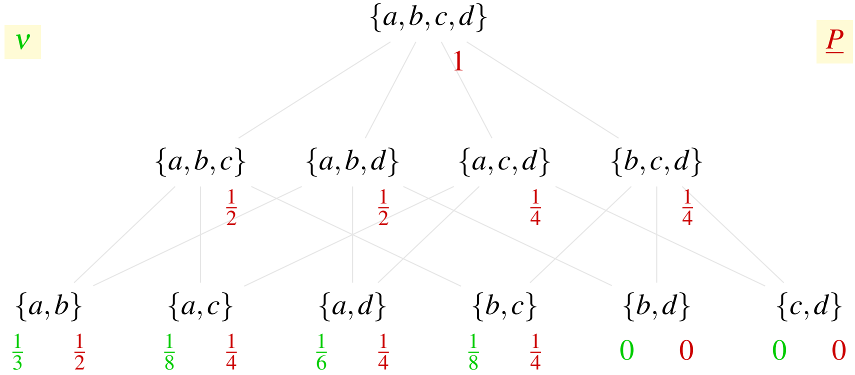
Rescale *minimally* when negative



Minimal

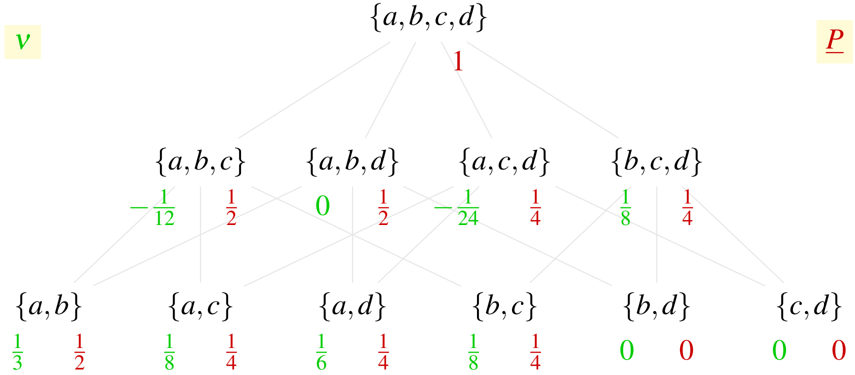
Iterative Rescaling Method

Rescale *minimally* when negative



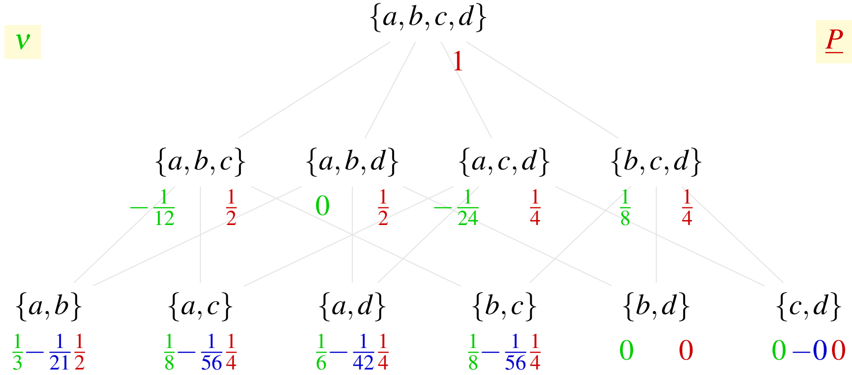
Minimal

Iterative Rescaling Method



Minimal

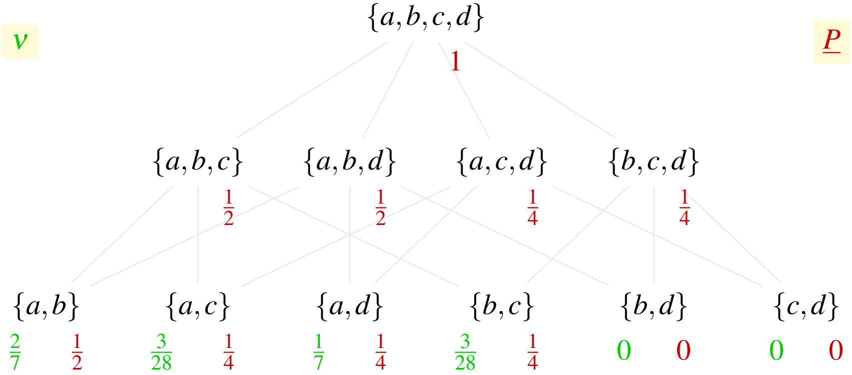
Möbius transform *per cardinality* | Iterative Rescaling Method



Minimal

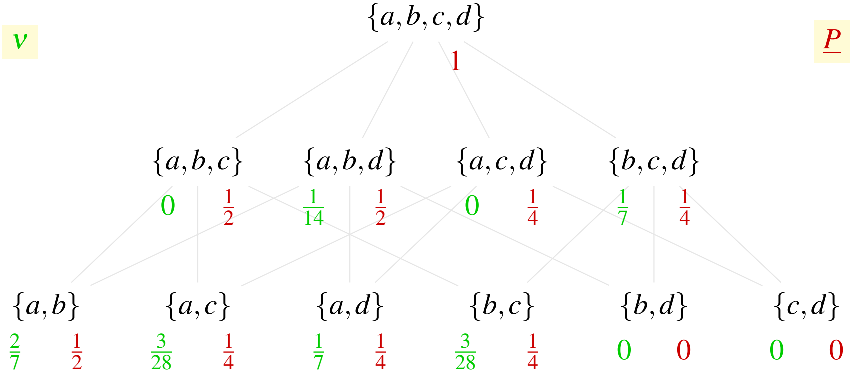
Iterative Rescaling Method

Rescale *minimally* when negative



Minimal

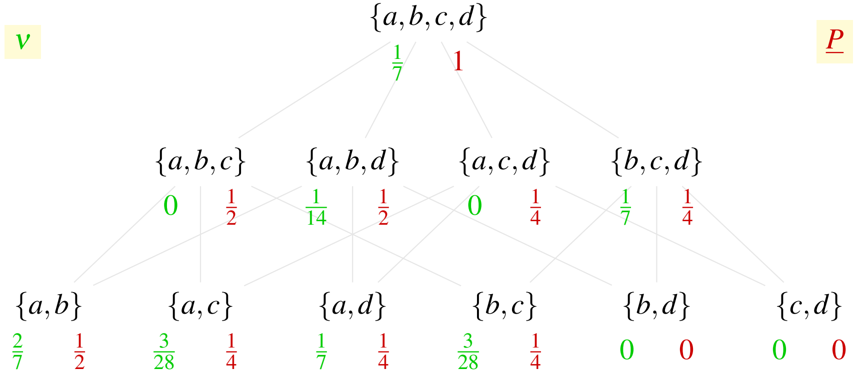
Iterative Rescaling Method



Minimal

Möbius transform *per cardinality*

Iterative Rescaling Method



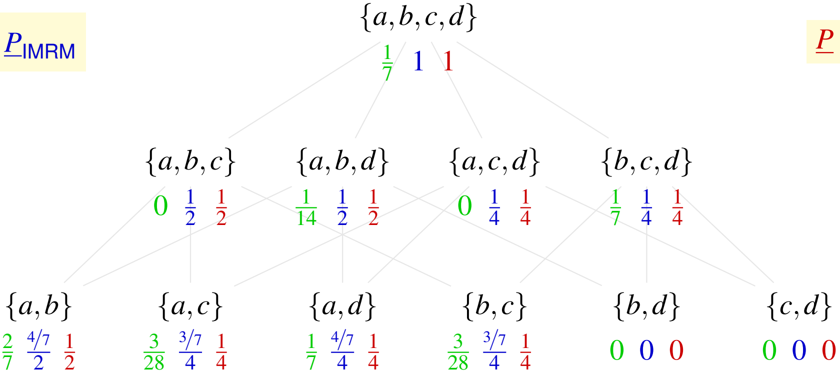
Minimal

Möbius transform
per cardinality

Iterative Rescaling Method

$\underline{P}_{\text{IMRM}}$

$\underline{P}$

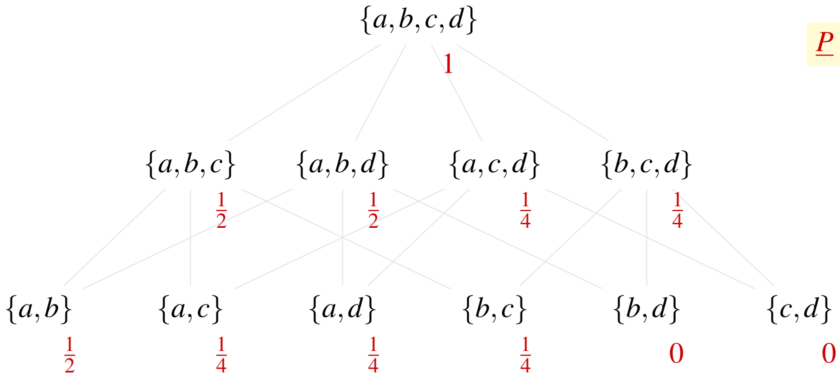


Minimal

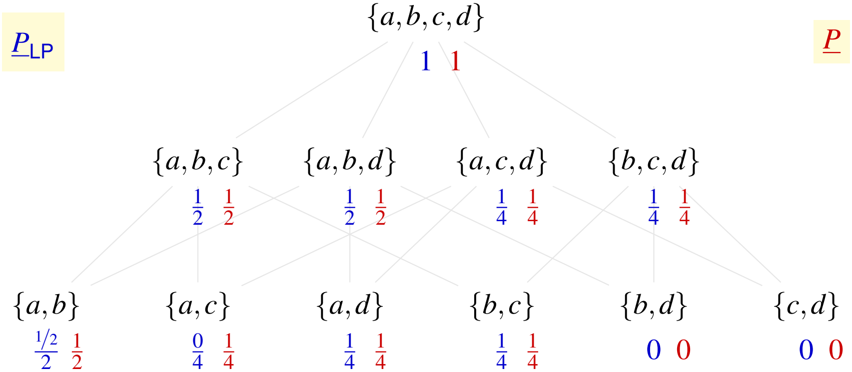
Iterative Rescaling Method

$\underline{P}_{\text{IMRM}}A = \sum_{B\subseteq A} \nu B$ Möbius inverse

$\underline{P}$



Optimization Approach

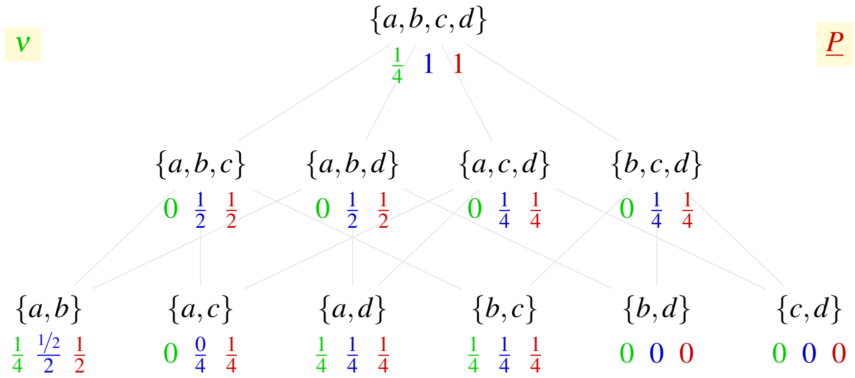


Linear Programming

minimize $\sum_{A\subseteq\Omega}|\underline{P}A - \underline{P}_{\mathrm{LP}}A|$

subject to $\underline{P}_{\mathrm{LP}} \leq \underline{P}$

$\underline{P}_{\mathrm{LP}}$ is ∞-monotone



Linear Programming

minimize $\sum_{A\subseteq\Omega}|\underline{P}A - \underline{P}_{\mathrm{LP}}A|$

subject to $\underline{P}_{\mathrm{LP}} \leq \underline{P}$

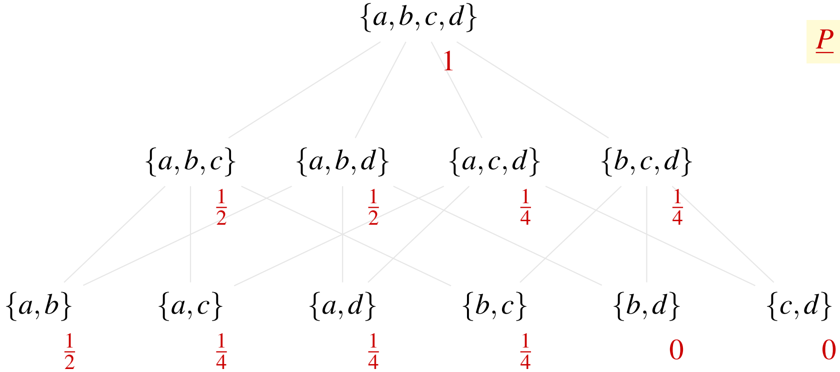
$\underline{P}_{\mathrm{LP}}$ is ∞-monotone

maximize $\sum_{B\subseteq\Omega} 2^{|\Omega\setminus B|}\nu B$

subject to $\forall_{A\subseteq\Omega}(\sum_{B\subseteq A}\nu B \leq \underline{P}A)$

$\nu \geq 0,\ \sum_{B\subseteq\Omega}\nu B = 1$

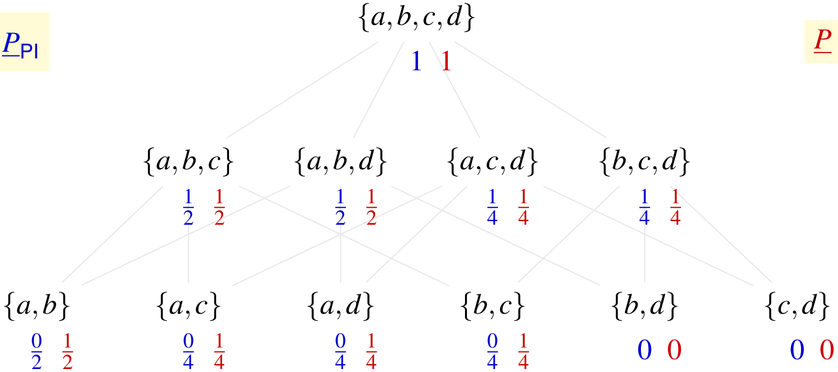
$\underline{P}$



Probability Interval Approach

$\underline{P}_{\mathrm{PI}}$

$\underline{P}$



Probability Interval Approach

Conclusions

From the paper:

- linear-imprecise decomposition is nice
- IMRM bests IRM at increased computational cost
- IMRM and LP have different strengths
- (PI is just lousy)

For the future:

- non-LP optimization approaches?
- generalize idea IMRM to less-than-complete monotonicity?