Supplementary Online Appendix to 'Is it Socially Efficient to Impose Job Search Requirements on Unemployed Benefit Claimants with Hyperbolic Preferences?" The Solution and Discussion for the Naive Agent

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In the main text we have only considered the case in which agents have sophisticated time preferences. In this online Appendix we analyze the behavior of agents who have naive time preferences. The literature contrasts two types of hyperbolic agents, a sophisticated and a naive one. They differ in the perception of how their respective future selves will behave. A sophisticated agent correctly realizes that her future selves will act exactly as the current self (discounting by $\beta\delta$), while a naive agent wrongly believes that her future selves will behave as an exponential agent (discounting by δ). Using the terminology of (Gruber and Köszegi, 2000, 2001) they both have a *self-control* problem, but only the naive agent has a *misperception* problem.

1 Perfect Monitoring

The optimization problem of the current self of a *naive* or a *sophisticated* agent coincides and can be formally represented by (2)-(4) in the main text. By contrast, the optimization problem of the future selves differs between the *naive* and *sophisticated* agent. While the sophisticated agent knows

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that she will continue to set her search effort in the future to the current level, a naive agent believes that she will set her future search effort to the same level as an exponential discounter would do. This means that the first-order condition for search effort, (12) in the main text, is not affected, but in the first-order condition of the reservation wage (11) the optimal search effort of the exponential agent σ_u^e replaces σ_u of the hyperbolic agent:

$$y_u + \frac{\delta\lambda\left(\sigma_u^e\right)}{1-\delta(1-q)} Q\left(x_u\right) = c\left(\sigma_u^e\right) + x_u, \qquad u \in \{b, z\} \tag{B.1}$$

where σ_u^e is determined by the first-order condition of search effort for the exponential agent, which is equivalent to (12) in the main text, but where $\beta = 1$:

$$\frac{\delta\lambda'\left(\sigma_u^e\right)}{1-\delta(1-q)} Q\left(x_u\right) + \mu_u^e = c'\left(\sigma_u^e\right) \quad \text{and} \quad \mu_u^e\left(\sigma_u^e - \bar{\sigma}\right) = 0, \qquad u \in \{b, z\} \tag{B.2}$$

where $\mu_b^e \geq 0$ is the Lagrange multiplier associated with the inequality constraint $\sigma_b^e \geq \bar{\sigma}$ and where $\mu_z^e = 0$, since job search is not constrained in the case of a sanction. Since search effort of an exponential agent is higher than that of a hyperbolic agent, this constraint is less likely to bind than the one in (12).

So, the solution for the naive agent is characterized by three first-order conditions instead of two: (B.1), (B.2) and (12) jointly determine the optimal solution $(\sigma_u, x_u, \sigma_u^e)$ for $u \in \{b, z\}$. Notice that we use the same notations as in the main text, but σ_u and x_u now designate the optimal solution for a naive agent instead of a sophisticated one. This convention is maintained throughout this Appendix. Observe that the naive agent sets her reservation wage at the level that the exponential agent sets it: $x_u = x_u^e$.

These first-order conditions can be represented by the following functions defined in the main text: $x = r(\sigma \mid b)$, $x = s(\sigma \mid 1)$, and $x = s(\sigma \mid \beta < 1)$. This allows to represent the solution of the naive agent graphically. The reservation wage of the unconstrained unemployed benefit claimant x_b is determined, as for an exponential agent, by the intersection between $x = r(\sigma \mid b)$, $x = s(\sigma \mid 1)$ at point A in Figure 1 below. The level of search effort is then set at point point B, the intersection of the horizontal line through x_b and $x = s(\sigma \mid \beta < 1)$. Similarly, the reservation wage and search effort of the sanctioned or non-complying agent are determined respectively by points C and D.

Let us now consider the behavior of the agent as the search requirement $\bar{\sigma}$ is raised. As long as $\bar{\sigma} < \sigma_b$, the agent does not change her behavior characterized by point B. If $\bar{\sigma}$ is set at a higher level than σ_b, she will at first comply and increase search effort accordingly. However, since a naive agent believes that she will act as an exponential agent in the future, she will not adjust her reservation wage as long as the requirement is set below the optimal search level of an exponential agent σ_b^e at point A. So, the optimal (σ_b, x_b) pair first follows the straight line BA. The reservation wage (and hence lifetime utility of the future selves) is lowered along the line passing through AHF only if $\bar{\sigma} > \sigma_b^e$.

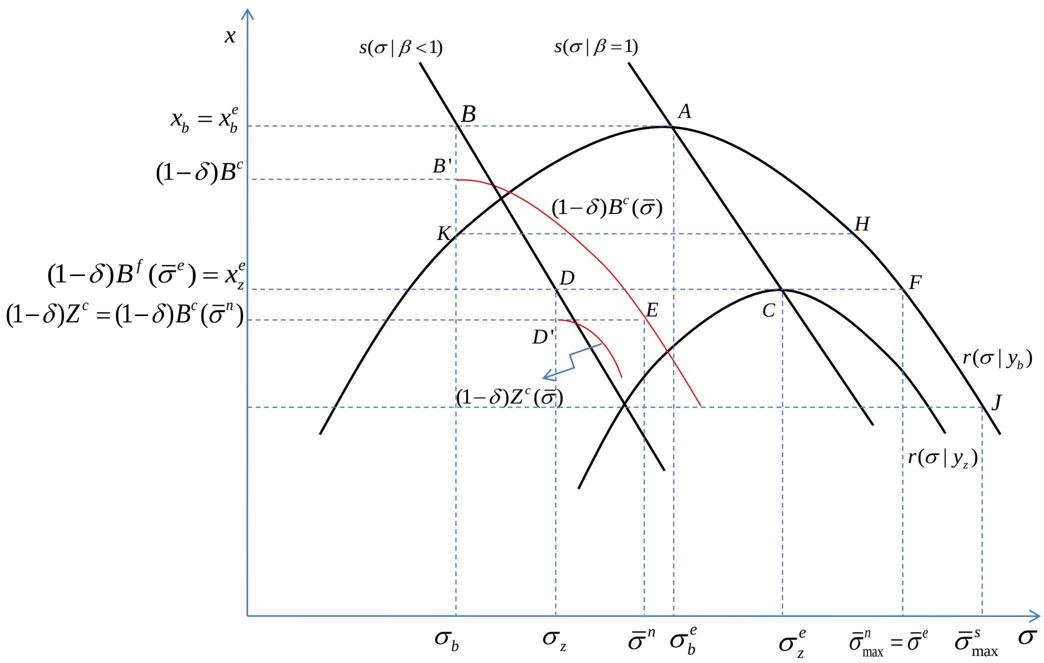

Figure 1: The Solution for the Naive Agent in Case of Perfect Monitoring. x = reservation wage; σ = realized search effort.

The decision to comply or not depends on the lifetime utility of the current self. This utility starts decreasing as soon as $\bar{\sigma}$ is set at a higher level than σ_b to the right of point B, because, as formally demonstrated in the proof of Proposition 1, the instantaneous cost of search increases while the reservation wage remains unaffected. The naive agent will stop complying as soon as the search requirement is raised above the effort level $\bar{\sigma}^n$ at which the current self is indifferent between complying or being sanctioned. The maximum search requirement $\bar{\sigma}^n$ verifies the following equation:

$$B^c(\bar{\sigma}^n) = Z^c \tag{B.3}$$

In Figure 1, this corresponds to the search intensity attained at point E.

Without being more precise about preferences and the search technology of the agent, we can say little about the exact level of $\bar{\sigma}^n$. Nevertheless, in Proposition 1 it is demonstrated that this level can be bracketed: $\bar{\sigma}^n \in (\sigma_z, \bar{\sigma}^n_{max})$, where $\bar{\sigma}^n_{max}$ is equal to the maximum search requirement $\bar{\sigma}^e$ before an exponential agent stops complying. In Figure 1, this corresponds to the search intensity attained at point F, where the reservation wage of the complier $r(\bar{\sigma} \mid y_b)$ is equal to that of a sanctioned exponential agent x_z^e. This means that the imposition of a search requirement leads to non-compliance at lower effort levels for naive hyperbolic agents than for exponential agents, as considered by Manning (2009) and Petrongolo (2009), since $\bar{\sigma}^n \in (\sigma_z, \bar{\sigma}^n_{max} = \bar{\sigma}^e) < \bar{\sigma}^e$, and than sophisticated hyperbolic agents, as considered in the main text. In the latter case the maximum search requirement that can be attained before an agent stops complying is equal to σ^s_{max} located on the abscissa of point J.

Proposition 1.

(i) The lifetime utility of the naive current self is unaffected by the search requirement for $\bar{\sigma}$ lower than her optimal free choice σ_b and is strictly decreasing in $\bar{\sigma}$ if it is higher.

(ii) The maximum search requirement $\bar{\sigma}$ at which a naive agent stops complying is not lower than the optimal search effort σ_z of a sanctioned naive hyperbolic agent and is strictly lower than the search effort $\bar{\sigma}^e$ at which an exponential agent stops complying.

Proof. See Appendix A. □

2 The Consequences of an Imperfect Monitoring Technology

We will not develop the complete analysis for a naive agent in the case of an imperfect monitoring technology, since the analysis is very similar to the case of a sophisticated agent. The main difference is that the long-run utility of the naive agent is constantly at the level of an agent with exponential time preferences because of her misperception problem. This means that the long-run utility of a naive agent is either constant or decreasing in σ, and never strictly increasing. Consequently, if in the benchmark case of an unbounded support of the measurement error the search requirement and, hence, the sanction probability is raised above zero, the long-run utility immediately strictly decreases, while for a sophisticated agent it initially increases. Proposition 5 in the main text differs therefore in this respect and we will provide a version of it, as well as its proof, in Proposition 2 below.

Proposition 2.

Assume that the sanction probability is expressed by (28) in the main text and that the support of measurement error is unbounded. Then, the optimal search effort for a naive agent σ_p strictly increases (resp., decreases) in $\bar{\sigma}$ for $\bar{\sigma} \to 0$ and, hence, $p(\bar{\sigma}/\sigma_p) \to 0$ (resp., $\bar{\sigma} \to +\infty$ and, hence, $p(\bar{\sigma}/\sigma_p) \to 1$). The reservation wage x_p is always decreasing in $\bar{\sigma}$. The Pareto frontier cannot be reached.

Proof. See Appendix B. □

APPENDIX

A Proof of Proposition 1

(i) *Proof.* Using (1), (2), (8) and (9) in the main text, and noting that the reservation wage of the naive agent is only affected by $\bar{\sigma}$ if $\bar{\sigma} > \sigma_b^e$, we can write:

$$B^c(\bar{\sigma}) = \max_{\sigma \geq \bar{\sigma}} y_b - c(\sigma) + \beta\delta \left\{ \frac{\lambda(\sigma)}{1-\delta(1-q)} Q\left[x_b^e(\bar{\sigma})\right] + \frac{x_b^e(\bar{\sigma})}{1-\delta} \right\} \tag{B.4}$$

where $x_b^e(\bar{\sigma}) \equiv 1_{\{\bar{\sigma} \leq \sigma_b^e\}} x_b^e + 1_{\{\bar{\sigma} > \sigma_b^e\}} r(\bar{\sigma} \mid y_b)$ and $1_{\{A\}} = 1$ if A is true and $1_{\{A\}} = 0$ otherwise. The search effort solving the maximization problem (B.4) is equal to σ_b if the latter is strictly higher than $\bar{\sigma}$ (with $x_b^e(\bar{\sigma}) = x_b^e$) and $\bar{\sigma}$ otherwise.

Partially differentiating $B^c(\bar{\sigma})$ in (B.4) with respect to $\bar{\sigma}$ is equal to zero if $\sigma_b > \bar{\sigma}$. Otherwise, using that $Q'\left[r(\sigma \mid y_u)\right] = -\bar{F}\left[r(\sigma \mid y_u)\right] r(\sigma \mid y_u)$:

$$\frac{\partial B^c(\bar{\sigma})}{\partial \bar{\sigma}} = S\left(\bar{\sigma}, x_b^e(\bar{\sigma}) \mid \beta\right) + 1_{\{\bar{\sigma} > \sigma_b^e\}} \beta\delta \frac{\left[(1-\delta)\left(1 - h\left[\bar{\sigma}, r(\bar{\sigma} \mid y_b)\right] + \delta q\right)\right]}{[1-\delta(1-q)](1-\delta)} \frac{\partial r(\bar{\sigma} \mid y_b)}{\partial \bar{\sigma}}$$

Since $S\left(\bar{\sigma}, x_b^e(\bar{\sigma}) \mid \beta\right) \lesseqqgtr 0$ iff $\bar{\sigma} \gtreqqless \sigma_b$ and since, by Proposition 1 in the main text, $\partial r(\bar{\sigma} \mid y_b)/\partial \bar{\sigma} < 0$ iff $\bar{\sigma} > \sigma_b^e$, $\partial B^c(\bar{\sigma})/\partial \bar{\sigma} \lesseqqgtr 0$ iff $\bar{\sigma} \gtreqqless \sigma_b$. In sum, $\partial B^c(\bar{\sigma})/\partial \bar{\sigma} = 0$ iff $\bar{\sigma} \leq \sigma_b$ and $\partial B^c(\bar{\sigma})/\partial \bar{\sigma} < 0$ iff $\bar{\sigma} > \sigma_b$. □

(ii) *Proof.* We prove (ii) by showing that for any $\bar{\sigma} \geq \sigma_b$ the difference between the expected lifetime utility of a complying and a non-complying current self must always be strictly greater for an agent discounting the future at an exponential rate ($\beta = 1$) than for an agent discounting the future at a hyperbolic rate ($\beta < 1$). Consequently the hyperbolic agent will always stop complying (when $B^c(\bar{\sigma}^n) = \tilde{Z}^c$) at a lower level of search effort than that of an exponential agent.

(1) Using (2), (8) and (9) of the main text, and restricting the analysis to the cases where $\bar{\sigma} \geq \sigma_b$, we obtain:

$$\forall \bar{\sigma} < \sigma_b^e : B^f(\bar{\sigma}) - B^c(\bar{\sigma}) = \delta(1-\beta) \left\{ \frac{\lambda(\bar{\sigma})}{[1-\delta(1-q)]} Q(x_b^e) + \frac{x_b^e}{(1-\delta)} \right\} \tag{B.5}$$

and

$$\forall \bar{\sigma} \geq \sigma_b^e : B^f(\bar{\sigma}) - B^c(\bar{\sigma}) = \delta(1-\beta) \left\{ \frac{\lambda(\bar{\sigma})}{[1-\delta(1-q)]} Q(r(\bar{\sigma}|y_b)) + \frac{r(\bar{\sigma}|y_b)}{(1-\delta)} \right\} \tag{B.6}$$

Similarly, using (3), (8) and (9) in the main text, we find

$$Z^f - Z^c = \delta(1-\beta) \left\{ \frac{\lambda(\sigma_z)}{[1-\delta(1-q)]} Q(x_z^e) + \frac{x_z^e}{(1-\delta)} \right\} \tag{B.7}$$

(2) Consider Definition (10) in the main text and assume that $x_1 < x_2$. Then we can rewrite $Q(x_1)$ as follows:

$$\begin{aligned} Q(x_1) &= \int_{x_1}^{x_2} (w - x_1)\, dF(w) + Q(x_2) + \bar{F}(x_2)\,(x_2 - x_1) \\ &= Q(x_2) + \bar{F}(x_1)\,(x_2 - x_1) - \{(x_2 - x_1) - E\,(w - x_1 | x_1 \leq w < x_2)\} \\ &\times \left[\bar{F}(x_1) - \bar{F}(x_2)\right] \end{aligned} \tag{B.8}$$

(3) Since $x_z^e < x_b^e$, we can use (10) in the main text and (B.8) to rewrite (B.7) as follows:

$$\begin{aligned} Z^f - Z^c &= \frac{\delta(1-\beta)}{[1-\delta(1-q)]}\Big\{\lambda(\sigma_z)Q(x_b^e) + h(\sigma_z, x_z^e)(x_b^e - x_z^e) + x_z^e\frac{1-\delta+\delta q}{1-\delta} \\ &- \lambda(\sigma_z)(x_b^e - x_z^e) + \lambda(\sigma_z)E(w - x_z^e | x_z^e \leq w < x_b^e)[\bar{F}(x_z^e) - \bar{F}(x_b^e)]\Big\} \end{aligned} \tag{B.9}$$

Subtracting (B.9) from (B.5) then yields for $\bar{\sigma} \in [\sigma_z, \sigma_b]$:

$$\begin{aligned} & B^f(\bar{\sigma}) - B^c(\bar{\sigma}) - Z^f + Z^c = \frac{\delta(1-\beta)}{[1-\delta(1-q)]}\Big\{Q(x_b^e)[\lambda(\bar{\sigma}) - \lambda(\sigma_z)] + (x_b^e - x_z^e) \\ & \times \frac{[(1-\delta)(1-h(\sigma_z, x_z^e)) + \delta q]}{(1-\delta)} + \lambda(\sigma_z)\,\{(x_b^e - x_z^e) - E(w - x_z^e)|x_z^e \leq w < x_b^e)\} \\ & \times \left[\bar{F}(x_z^e) - \bar{F}(x_b^e)\right]\Big\} > 0 \end{aligned} \tag{B.10}$$

Since $\forall \bar{\sigma} \in [\sigma_b^e, \bar{\sigma}_{max}^n] : r(\bar{\sigma}|y_b) > x_z^e$, we can derive using (B.6) a similar expression as (B.10) in which x_b^e is replaced by $r(\bar{\sigma}|y_b)$. Consequently,

$$\forall \bar{\sigma} \in [\sigma_z, \bar{\sigma}_{max}^n] : B^f(\bar{\sigma}) - Z^f > B^c(\bar{\sigma}) - Z^c \tag{B.11}$$

Since $B^f(\bar{\sigma}_{max}^n) = Z^f$, (B.11) implies that $B^c(\bar{\sigma}_{max}^n) < Z^c$. Because $B^c(\sigma_z) > Z^c$ and $B^c(\cdot)$ is a continuous function, it must be that $\sigma_z < \bar{\sigma}^n < \bar{\sigma}_{max}^n$. Finally, since the reservation wage of a naive agent (and hence lifetime utility of the future selves Z^f) is equal to that of an exponential agent ($x_z = x_z^e$), $B^f(\bar{\sigma}_{max}^n) = Z^f$ implies that $\bar{\sigma}_{max}^n$ is equal to the maximum search requirement $\bar{\sigma}^e$ at which an exponential agent stops complying.

□

B Proof of Proposition 2

Proof. Following arguments that are similar as those for the sophisticated agent in the case of measurement error, but adjusted, as in Section 1 of this Internet Appendix, the first-order conditions define a system of three equations in three unknowns: x, σ and σ^e. σ denotes the search effort of the naive agent, while σ^e refers to the effort of an exponential agent. Notice that because of the misperception problem the reservation wage of the naive and the exponential agent are equal: $x = x^e$. The system is thus

$$Ey + \frac{\delta\lambda(\sigma^e)Q(x)}{1-\delta(1-q)} - x - c(\sigma^e) - p\left(\frac{\bar{\sigma}}{\sigma^e}\right)\frac{\delta}{(1-\delta)}\left[1 - h\left(\sigma^e, x\right)\right](x - x_z) = 0 \tag{B.12}$$

$$\frac{\beta\delta\lambda'(\sigma)}{1-\delta(1-q)}Q(x) - \frac{\partial p(\bar{\sigma}/\sigma)}{\partial\sigma}\left\{y_b - y_z + \frac{\beta\delta}{(1-\delta)}\left[1-h\left(\sigma, x\right)\right](x-x_z)\right\}$$
$$+p\left(\frac{\bar{\sigma}}{\sigma}\right)\frac{\beta\delta}{(1-\delta)}\lambda'(\sigma)\bar{F}(x)(x-x_z) - c'(\sigma) = 0 \tag{B.13}$$

$$\frac{\delta\lambda'(\sigma^e)}{1-\delta(1-q)}Q(x) - \frac{\partial p(\bar{\sigma}/\sigma^e)}{\partial\sigma^e}\left\{y_b - y_z + \frac{\delta}{(1-\delta)}\left[1-h\left(\sigma^e, x\right)\right](x-x_z)\right\}$$
$$+p\left(\frac{\bar{\sigma}}{\sigma^e}\right)\frac{\delta}{(1-\delta)}\lambda'(\sigma^e)\bar{F}(x)(x-x_z) - c'(\sigma^e) = 0 \tag{B.14}$$

The first equation is the first-order condition of the reservation wage, Equation (31) in the main text, in which σ is replaced by σ^e. The second is the first-order condition of search for the naive agent, corresponding to (32) in the main text, but in which σ and x refer to the behavior of a naive agent instead of a sophisticated one. Finally, the third equation is the first-order condition of search effort for the exponential agent. This corresponds to equation (32) in the main text for which β is set to one and σ is replaced by σ^e.

Totally differentiating this system yields:

$$\begin{bmatrix} A_1 & A_2 & A_3 \\ A_4 & A_5 & A_6 \\ A_7 & A_8 & A_9 \end{bmatrix}\begin{bmatrix} dx \\ d\sigma \\ d\sigma^e \end{bmatrix} = \begin{bmatrix} A_{10} \\ A_{11} \\ A_{12} \end{bmatrix} d\bar{\sigma} \tag{B.15}$$

where, without recalling the arguments of $\lambda, c, p, f, \bar{F}, Q, h$ and their derivatives and without writing a subscript e to denote that a function is evaluated at σ^e

$$A_1 = -\frac{\delta\lambda_e\bar{F}}{1-\delta(1-q)} - 1 - p_e\frac{\delta}{1-\delta}\left[\lambda_e f(x-x_z) + 1 - h_e\right] < 0 \tag{B.16}$$

$$A_2 = 0 \tag{B.17}$$

$$
\begin{aligned}
A_3 &= \frac{\delta\lambda_e' Q}{1-\delta(1-q)} - c_e' - \frac{\partial p_e}{\partial \sigma^e}\left\{y_b - y_z + \frac{\delta(1-h)(x-x_z)}{1-\delta}\right\} + p_e\frac{\delta\lambda_e'\bar{F}(x-x_z)}{1-\delta} \\
&= S_p(\sigma^e, x \mid 1, \bar{\sigma}) && \text{(B.18)} \\
A_4 &= -\frac{\beta\delta\lambda'\bar{F}}{1-\delta(1-q)} - \frac{\partial p}{\partial \sigma}\frac{\beta\delta}{1-\delta}\left[\lambda f(x-x_z) + 1 - h\right] + p\frac{\beta\delta}{1-\delta}\lambda'\left[\bar{F} - f(x-x_z)\right] && \text{(B.19)} \\
A_5 &= \frac{\beta\delta\lambda'' Q}{1-\delta(1-q)} - \frac{\partial^2 p}{[\partial\sigma]^2}\left\{y_b - y_z + \frac{\beta\delta(1-h)(x-x_z)}{1-\delta}\right\} + 2\frac{\partial p}{\partial \sigma}\frac{\beta\delta\lambda'\bar{F}(x-x_z)}{1-\delta} \\
&+ p\frac{\beta\delta\lambda''\bar{F}(x-x_z)}{1-\delta} - c'' && \text{(B.20)} \\
A_6 &= 0 && \text{(B.21)} \\
A_7 &= -\frac{\delta\lambda_e'\bar{F}}{1-\delta(1-q)} - \frac{\partial p_e}{\partial \sigma^e}\frac{\delta\left[\lambda_e f(x-x_z) + 1 - h_e\right]}{1-\delta} + p_e\frac{\delta\lambda_e'\left[\bar{F} - f(x-x_z)\right]}{1-\delta} && \text{(B.22)} \\
A_8 &= 0 && \text{(B.23)} \\
A_9 &= \frac{\delta\lambda_e'' Q}{1-\delta(1-q)} - \frac{\partial^2 p_e}{[\partial\sigma^e]^2}\left\{y_b - y_z + \frac{\delta(1-h_e)(x-x_z)}{1-\delta}\right\} + 2\frac{\partial p_e}{\partial \sigma^e}\frac{\delta\lambda_e'\bar{F}(x-x_z)}{1-\delta} \\
&+ p_e\frac{\delta\lambda_e''\bar{F}(x-x_z)}{1-\delta} - c_e'' && \text{(B.24)} \\
A_{10} &= \frac{\partial p_e}{\partial \bar{\sigma}}\left\{y_b - y_z + \frac{\delta(1-h_e)(x-x_z)}{1-\delta}\right\} && \text{(B.25)} \\
A_{11} &= \frac{\partial^2 p}{\partial\sigma\partial\bar{\sigma}}\left\{y_b - y_z + \frac{\beta\delta(1-h)(x-x_z)}{1-\delta}\right\} - \frac{\partial p}{\partial \bar{\sigma}}\frac{\beta\delta\lambda'\bar{F}(x-x_z)}{1-\delta} && \text{(B.26)} \\
A_{12} &= \frac{\partial^2 p_e}{\partial\sigma^e\partial\bar{\sigma}}\left\{y_b - y_z + \frac{\delta(1-h_e)(x-x_z)}{1-\delta}\right\} - \frac{\partial p_e}{\partial \bar{\sigma}}\frac{\delta\lambda_e'\bar{F}(x-x_z)}{1-\delta} && \text{(B.27)}
\end{aligned}
$$

Solving system (B.12)-(B.14) yields

$$
\begin{bmatrix} \frac{\partial x}{\partial \bar{\sigma}} \\ \frac{\partial \sigma}{\partial \bar{\sigma}} \\ \frac{\partial \sigma^e}{\partial \bar{\sigma}} \end{bmatrix} = \frac{1}{D_n}\begin{bmatrix} A_5A_9 & 0 & -A_3A_5 \\ -A_4A_9 & (A_1A_9 - A_3A_7) & A_3A_4 \\ -A_5A_7 & 0 & A_1A_5 \end{bmatrix}\begin{bmatrix} A_{10} \\ A_{11} \\ A_{12} \end{bmatrix} \tag{B.28}
$$

where $D_n = A_5(A_1A_9 - A_3A_7)$.

In order to sign these partial derivatives, we use a couple of results of the proof of Proposition 5 in the Appendix of the main text. First, by substituting (A.25) into (B.25), we obtain that $A_{10} \geq 0$. Second, substituting (A.26) into (B.20) and (B.24) yields $A_5 < 0$ and $A_9 < 0$ if $g'\left[log\left(\frac{\bar{\sigma}}{\sigma}\right)\right]$ is positive or not too negative, an assumption we make throughout. Third, since $A_3 = S_p(\sigma^e, x \mid 1, \bar{\sigma})$, we have that $A_3 = 0$ if we evaluate it at the optimal choice (σ_b^e, x_b), since for a naive agent the reservation wage and search effort of the future selves are set at the optimum of an exponential agent ($x_b = x_b^e$). Observe that as a result, at the optimum, $D_n = A_1A_5A_9 < 0$. Finally, we insert

(A.25)-(A.27) in (B.19) and (B.26)[1] to observe that these have ambiguous signs:

$$A_4 = -\frac{\beta\delta}{(1-\delta)}\left\{\lambda'\bar{F}\frac{[(1-p)(1-\delta)-p\delta q]}{[1-\delta(1-q)]} + p\lambda' f(x-x_z) - \frac{g}{\sigma}\left[\lambda f(x-x_z)+(1-h)\right]\right\} \quad \text{(B.29)}$$

$$A_{11} = -\frac{g'}{\sigma\bar{\sigma}}\left\{y_b - y_z + \frac{\beta\delta(1-h)(x-x_z)}{1-\delta}\right\} - \frac{g}{\bar{\sigma}}\frac{\beta\delta\lambda'\bar{F}(x-x_z)}{1-\delta} \quad \text{(B.30)}$$

However, as in the proof of Proposition 5, we now show that these partial derivatives can be unambiguously signed in the limiting cases where p tends to zero or to one. We only consider the benchmark case in which the support of the measurement error is unbounded. This means that the following properties are satisfied: $\forall\sigma,\bar{\sigma}\in(0,+\infty) : g\left[log\left(\frac{\bar{\sigma}}{\sigma}\right)\right] > 0$, $\lim_{\varepsilon\to-\infty} g(\varepsilon) = \lim_{\varepsilon\to+\infty} g(\varepsilon) = 0$, $\lim_{\varepsilon\to-\infty} g'(\varepsilon) > 0$, and $\lim_{\varepsilon\to+\infty} g'(\varepsilon) < 0$.

Case 1: $\mathbf{p \to 0}$

Substituting $\lim_{\varepsilon\to-\infty} g(\varepsilon) = 0$ and $\lim_{\varepsilon\to-\infty} g'(\varepsilon) > 0$ in (B.29) and (B.30) yields

$$A_4 \to -\frac{\beta\delta\lambda'\bar{F}}{1-\delta(1-q)} < 0 \quad \text{(B.31)}$$

$$A_{11} \to -\frac{g'}{\sigma\bar{\sigma}}\left\{y_b - y_z + \frac{\beta\delta(1-h)(x-x_z)}{1-\delta}\right\} < 0 \quad \text{(B.32)}$$

Case 2: $\mathbf{p \to 1}$

Using $\lim_{\varepsilon\to+\infty} g'(\varepsilon) < 0$ and $x - x_z \to 0$ for $p \to 1$ allows to simplify equations (B.29) and (B.30) as follows:

$$A_4 \to \frac{\beta\delta}{(1-\delta)}\left\{\lambda'\bar{F}\frac{p\delta q}{[1-\delta(1-q)]} + \frac{g}{\sigma}(1-h)\right\} > 0 \quad \text{(B.33)}$$

$$A_{11} \to -\frac{g'}{\sigma\bar{\sigma}}\left\{y_b - y_z\right\} > 0 \quad \text{(B.34)}$$

where the last inequality follows from the fact that $\lim_{\varepsilon\to+\infty} g'(\varepsilon) < 0$ in the benchmark when the measurement error has an infinite support, and $g'(\bar{\varepsilon}) = 0$ in the case of a finite support.

To sum up, we found that $A_1 < 0$, $A_2 = 0$, $A_3 = 0$, $A_5 < 0$, $A_6 = 0$, $A_8 = 0$, $A_9 < 0$, $A_{10} \geq 0$ and $D_n < 0$, while $A_4 < 0$ and $A_{11} < 0$ if $p \to 0$ and $A_4 > 0$ and $A_{11} > 0$ if $p \to 1$. By continuity, there exists at least one $\tilde{p}_n \in (0,1)$ at which $A_4 = 0$ and at least one $\hat{p}_n \in (0,1)$ at which $A_{11} = 0$.

Inserting these results in (B.28) and evaluating these partial derivatives at the optimal solution $(\sigma_p, x_p, \sigma_p^e)$ yields:

$$\frac{\partial x_p}{\partial\bar{\sigma}} = \frac{A_5 A_9 A_{10}}{D_n} = \frac{A_{10}}{A_1} \quad \text{(B.35)}$$

$$\frac{\partial \sigma_p}{\partial\bar{\sigma}} = \frac{A_9(A_1 A_{11} - A_4 A_{10})}{D_n} = \frac{(A_1 A_{11} - A_4 A_{10})}{A_1 A_5} \quad \text{(B.36)}$$

[1]We do not consider A_7 and A_{12}, because we are not interested in the behavior of the future selves of a naive agent.

Consider first the case where $p \to 0$. Observe that since $\lim_{\varepsilon \to -\infty} g(\varepsilon) = 0$, $A_{10} \to 0$, so that in the limit $\frac{\partial x_p}{\partial \bar{\sigma}}$ tends to zero. However, for any finite ε, $g(\varepsilon) > 0$, so that $A_{10} > 0$, and, hence, $\frac{\partial x_p}{\partial \bar{\sigma}} < 0$. From (B.36) it is clear that $\frac{\partial \sigma_p}{\partial \bar{\sigma}} > 0$ in the case where $p \to 0$. Inserting the values of the partial derivatives for the case that $p \to 1$ yields again that $\frac{\partial x_p}{\partial \bar{\sigma}} < 0$ close to $p = 1$. Since $A_{10} \to 0$ if $p \to 1$, while A_{11} remains strictly negative, we obtain that $\frac{\partial \sigma_p}{\partial \bar{\sigma}} < 0$ sufficiently close to $p = 1$. □

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