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Note on the usage of the linear integrated speed-accuracy score (LISAS)[1]

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Liesefeld and Janczyk (in press) propose the balanced integration score (BIS) as the best measure so far to calculate integrated speed-accuracy scores. On the basis of artificial data generated on the basis of the diffusion model (Ratcliff, 1978), they claim that the measure yields better balanced results than any other measure, including LISAS. However, it can be shown mathematically that BIS and LISAS are linear transformations from each other if applied under the same conditions and restrictions (proof is provided in the Appendix to this note). It is important to note though, that in their applications of BIS Liesefeld and Janczyk calculate the integrated score per condition over the entire sample with standard deviations of speed and accuracy over all subjects in the sample, while LISAS is typically used with standard deviations calculated per subject and condition (Vandierendonck, 2017, 2018).

The methodology used by Liesefeld and Janczyk does not seem to distinguish between within-subject and between-subject conditions for calculation of the BIS score. Whether this is indeed the case or not, one should be aware that calculation of any integrated speed-accuracy score in a between-subjects design with the aim of controlling for speed-accuracy trade-off does not make any sense, because this would mean that in comparisons across conditions, one subject is considered to trade accuracy for the speed gain made by another subject and vice versa to trade speed for the accuracy gain of another subject. As subjects are not aware of each others trade-off strategies, the application of an integrated measure in between-subject comparisons only provides an illusory correction for trade-offs.

It is nevertheless interesting to see that the work of Liesefeld and Janczyk has clarified that the calculation of LISAS per condition per subject (in a within-subjects design) does not achieve a complete balancing of speed and accuracy. Indeed, it appears to calculation of LISAS with standard deviations estimated per subject per condition yields a weaker correction in conditions with higher error rates. A small example may clarify what goes wrong. Table 1 shows (artificial) data of 10 trials in a control condition and 10 trials in a more difficult experimental condition of a single subject. The table also shows the means and the standard deviations per condition as would be used for the calculation of LISAS. Calculation of LISAS with per condition standard deviations yields a score of 568 for the control condition and 684 for the experimental condition. If LISAS is calculated with pooled standard deviations (per subject over the two conditions), the LISAS amounts to 573 for the control condition and 708 for the experimental condition. What this example shows is that the calculation with per condition standard deviations yields underestimations, due to the

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fact that the accuracy means and standard deviations tend to be strongly correlated. The conclusion is simple, in order to avoid such underestimations, it is better to calculate LISAS on the basis of the standard deviations per subject over all the conditions available in the experiment. In view of what was discussed in the previous paragraph, calculation of LISAS on the entire sample is not a good approach because estimation of the standard deviations over several subjects calculates an integrated score that not only corrects for within-subjects trade-off but also for illusory between-subjects trade-offs.

Table 1. Fictional data illustrating incomplete balance when the standard deviations are estimated per subject and per condition.

Trial	Control		Experimental	
	RT	Acc	RT	Acc
1	506	0	623	0
2	569	1	672	0
3	545	0	614	1
4	602	0	659	0
5	519	0	681	1
6	587	0	634	0
7	538	0	695	0
8	551	0	728	1
9	572	0	677	0
10	593	0	641	0
Means	558.2	0.1	662.4	0.3
SD	31.91	0.32	35.26	0.48

Abbreviations used: Acc = accuracy and SD = standard deviation.

Conclusion. In order to achieve a balanced integrated speed-accuracy score, LISAS is still the best alternative around provided that the standard deviations are calculated per subject over all the conditions. Usage of BIS in the same way yields statistically, but not numerically, the same results. Calculation of any integrated score with standard deviations estimated over all subjects is to be avoided because it corrects for speed-accuracy trade-offs that do not exist.

Appendix

BIS is defined as

$$\text{BIS} = z_c - z_r \qquad (1)$$

where

$$z_c = \frac{\text{PC}_{ij} - \overline{\text{PC}}}{s_c} \qquad (2)$$

is the standard score of the proportion correct (PC_{ij}) responses of subject j in condition i, with s_c as standard deviation, and

$$z_r = \frac{\text{RT}_{ij} - \overline{\text{RT}}}{s_r} \qquad (3)$$

is the standard score of the correct responses (RT_{ij}) of subject j in condition i, with standard deviation s_r.

LISAS is defined as

$$\text{LISAS} = \text{RT}_{ij} + \text{PE}_{ij} \times \frac{s_r}{s_e} \qquad (4)$$

where PE is the proportion of errors, i.e., $\text{PE}_{ij} = (1 - \text{PC}_{ij})$, and due to the underlying binomial distribution, $s_e = s_c$.

Rewriting LISAS in terms of standardised scores, equation (4) can be rewritten as follows (using equations 2 and 3):

$$\text{LISAS} = (z_r \times s_r + \overline{\text{RT}}) + (z_e \times s_e + \overline{\text{PE}}) \times \frac{s_r}{s_e} \qquad (5)$$

and

$$\text{LISAS} = \frac{s_e}{s_e} \times [(z_r \times s_r + \overline{\text{RT}})] + (z_e \times s_e + \overline{\text{PE}}) \times \frac{s_r}{s_e} \quad (6)$$

working out the common denominator (s_e):

$$\text{LISAS} = \frac{1}{s_e} \times [z_r s_r s_e + \overline{\text{RT}} s_e + z_e s_e s_r + \overline{\text{PE}} s_r] \qquad (7)$$

and stripping off common terms

$$\text{LISAS} = z_r s_r + \overline{\text{RT}} + z_e s_r + \overline{\text{PE}} \frac{s_r}{s_e} \qquad (8)$$

$$\text{LISAS} = (z_r + z_e) s_r + \overline{\text{RT}} + \overline{\text{PE}} \frac{s_r}{s_e} \qquad (9)$$

which shows LIS as a linear function of BIS (considering that $z_r + z_e = z_r - z_c$), or

$$\text{LISAS} = s_r\left(\frac{\overline{\text{RT}}}{s_r} + \frac{\overline{\text{PE}}}{s_e} - \text{BIS}\right) \qquad (10)$$

Similarly:

$$\text{BIS} = \frac{\text{LISAS}}{s_r} - \frac{\overline{\text{RT}}}{s_r} - \frac{\overline{\text{PE}}}{s_e} \qquad (11)$$

This shows that for a linear integrated measure based on the sample standard deviations of RT and PE (or PC), this measure is a linear transformation of BIS, making the two measures statistically indistinguishable. The advantage of LISAS is that it expresses RT adapted for the occurrence of errors, while BIS is merely a standardised score not comparable between experiments. Also note that it does not matter whether the standard devation is or is not corrected to yield an unbiased estimate of the population standard deviation as LISAS takes the ratio of the two standard deviations, so that $\frac{\sqrt{n}}{\sqrt{n}} = \frac{\sqrt{n-1}}{\sqrt{n-1}} = 1$

References

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