

# Adiabatic passage and ensemble control of quantum systems

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## Abstract

This paper considers population transfer between eigenstates of a finite quantum ladder controlled by a classical electric field. Using an appropriate change of variables, we show that this setting can be set in the framework of adiabatic passage, which is known to facilitate ensemble control of quantum systems. Building on this insight, we present a mathematical proof of robustness for a control protocol—chirped pulse—practised by experimentalists to drive an ensemble of quantum systems from the ground state to the most excited state. We then propose new adiabatic control protocols using a single chirped and amplitude-shaped pulse, to robustly perform any permutation of eigenstate populations, on an ensemble of systems with unknown coupling strengths. These adiabatic control protocols are illustrated by simulations on a four-level ladder.

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

Population transfer from the eigenstate  $k$  to the eigenstate  $l$  of a quantum system refers to finding a control input such that the projection of the final system state on the eigenstate  $l$  of the free Hamiltonian has the same norm as the projection of the initial system state on the eigenstate  $k$ . Applications of population transfer range from population inversion [1], where  $k$  and  $l$  are the lowest and highest energy eigenstates, respectively, to quantum information processing [2–4], where logic gates would (selectively) permute the populations of several eigenstates. In many applications, including those mentioned, relative insensitivity to variations in system parameters is important for robustness issues.

In this paper, we show how control inputs designed on the basis of adiabatic passage can implement any given permutation of eigenstate populations for a finite anharmonic quantum ladder. The controls we use are chirped pulses [5] with appropriately modulated amplitudes and exploit the idea of eigenvalue crossing [6]. The ladder consists of a free Hamiltonian with approximately equidistant eigenvalues and

where the control input couples eigenstates associated with consecutive eigenvalues. We show that any control field satisfying a set of key properties achieves our target population transfer independently of the values of dipole moments coupling consecutive levels of the ladder, which is a striking robustness feature. This is a major difference with respect to early non-adiabatic approaches to molecular ladder dissociation using chirped pulses [5]. Adiabatic passage through eigenvalue crossings has also very recently been used to prove approximate controllability in finite time of an infinite dimensional quantum system [7].

In this sense, we achieve a specific form of *ensemble control*. Ensemble control in its most general form wants the same input to drive an ensemble of systems, with different values of some parameter  $p$ , from a given  $p$ -dependent initial state to a given  $p$ -dependent final state ([8], definition 1). Currently, solutions to this general problem are essentially restricted to two-level systems, achieving approximate ensemble control in finite time and exact ensemble control in infinite time [8–10]. They rely on accurate knowledge of laser–system coupling strengths and accurately tailored inputs, involving e.g. exact instantaneous ‘ $\pi$ -amplitude impulses’. In our setting, system parameters need not be exactly known and

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