Strategic flexibility in computational estimation for Chinese- and Canadian-educated adults

Chang Xu[1], Emma Wells[1], Jo-Anne LeFevre[1,2], and Ineke Imbo[3]

[1] Department of Psychology, Carleton University

[2] Institute of Cognitive Science, Carleton University

[3] Department of Experimental Psychology, Ghent University

Correspondence Address: Jo-Anne LeFevre

Institute of Cognitive Science, Carleton University

Room 2201, Dunton Tower, 1125 Colonel By Drive,

Ottawa, ON, Canada

Tel: 613-520-2600 ext. 2693; Fax: 613-520-3985

E-mail: jo-anne.lefevre@carleton.ca

Chang Xu

Department of Psychology, Carleton University

1125 Colonel By Drive,

Ottawa, ON, Canada

Phone (Fax) Number: 1-613-520-2600 x 4574

Email Address: chang_xu@carleton.ca

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Abstract

The purpose of the present study was to examine factors that influence strategic flexibility in computational estimation for Chinese- and Canadian-educated adults. Strategic flexibility was operationalized as the percentage of trials on which participants chose the problem-based procedure that best balanced proximity to the correct answer with simplification of the required calculation. For example, on 42 x 57, the optimal problem-based solution is 40 x 60 because *2400* is closer to the exact answer *2394* than is 40 x 50 or 50 x 60. In Experiment 1 (n = 50), where participants had free choice of estimation procedures, Chinese-educated participants were more likely to choose the optimal problem-based procedure (80% of trials) than Canadian-educated participants (50%). In Experiment 2 (n = 48), participants had to choose one of three solution procedures. They showed moderate strategic flexibility that was equal across groups (60%). In Experiment 3 (n = 50), participants were given the same three procedure choices as in Experiment 2 but different instructions and explicit feedback. When instructed to respond quickly, both groups showed moderate strategic flexibility as in Experiment 2 (60%). When instructed to respond as accurately as possible or to balance speed and accuracy, they showed very high strategic flexibility (greater than 90%). These findings suggest that solvers will show very different levels of strategic flexibility in response to instructions, feedback, and problem characteristics and that these factors interact with individual differences (e.g., arithmetic skills, nationality) to produce variable response patterns.

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Computational estimation is a skill that people use to solve arithmetic problems when exact calculation is not required and an approximate answer is sufficient (LeFevre, Greenham & Waheed, 1993; Sowder & Wheeler, 1989). Educators agree that estimation is an important mathematical skill that requires flexible application of a range of conceptual and procedural knowledge (Heinze, Star, & Verschaffel, 2009; Star, Rittle-Johnson, Lynch, & Perova, 2009). Despite the assumption that estimation involves simplifying problems to reduce computational demands, however, studies have shown that children and adults often perform better on problems requiring exact computation than on estimation problems (Hanson & Hogan, 2000; Hope & Sherrill, 1987; Liu, 2008). Estimation requires different knowledge than exact calculation and, in particular, requires that solvers must have a repertoire of solution procedures from which to select (Dowker, Flood, Griffiths, Harriss, & Hook, 1996; LeFevre et al., 1993; Star et al., 2009). According to Star et al. (2009), strategic flexibility requires "knowledge of multiple strategies and the ability to select the most appropriate strategy for a given problem and a given problem-solving goal" (p. 570). This definition suggests that both problem characteristics and individual characteristics in the form of goals and capabilities should be considered in evaluating strategic flexibility. Past research on computational estimation has not always allowed solvers to use their strategic repertoire, however, nor have individual goals and capabilities been considered. The purpose of the present research was to explore performance on computational estimation problems for two groups that varied in strategic repertoire and computational fluency (i.e., defined as the ability to apply arithmetic skills efficiently, appropriately, and flexibly; Baroody, Torbeyns, & Verschaffel, 2009). Our hypothesis was that task demands, individual characteristics, and problem factors would influence strategic flexibility and resulting estimation performance.

LeFevre et al. (1993) proposed that strategic processes in estimation are guided by two conceptual principles, simplicity and proximity. Individuals are assumed to adopt criteria for simplicity and proximity that reflect the characteristics of the problems and their own computational fluency, and to use those criteria strategically to select a solution procedure with which to solve the estimation problem. A focus on simplicity leads to choices of solutions that can be implemented quickly and accurately, whereas a focus on proximity leads to solutions that are close to the exact answer. Thus, for the problem 43 x 89 [=3827], choosing to solve 45 x 100 emphasizes simplicity whereas choosing to solve 40 x 90 emphasizes proximity. Thus, successful estimators should have good understanding of the conceptual knowledge necessary for estimation, efficient access to mathematical facts, and strong computational procedures (Dowker, 2005; Reys, Bestgen, Rybolt, & Wyatt, 1982).

We define an optimal problem-based solution in computational estimation as one that balances *simplicity* of computation and *proximity* to the exact answer. For a given task, for an individual, or even on a specific problem, the relative emphasis on these principles may vary and thus not all solutions will be optimal in relation to problem characteristics. Following previous research, however, in the present work we assume that it is possible to adopt a problem-based criterion to define an optimal solution procedure that represents a reasonable compromise between these principles. Specifically, for the two-digit by two-digit multiplication problems used in the present research, the optimal problem-based procedure involves rounding operands with unit digits less than five down to the nearest decade (e.g., 43 to 40) and rounding operands with unit digits greater than five up to the nearest decade (e.g., 47 to 50). Across problems, and independently of other factors that may influence procedure selection, this problem-based procedure results in solutions that are computationally manageable for most adults while maintaining a high degree of proximity to the exact answer (the precise degree of discrepancy varies depending on decade size and the unit value). Accordingly, although individuals may not always select the procedure that is defined as optimal for these problems, it nevertheless can be used as a benchmark solution that balances simplicity and proximity.

Note that, in the current research, we use the term *procedure* to refer to the sequence of processes involved in performing an estimation whereas *strategy* or *strategic* is used to refer to the process of selecting which procedure to implement (Bisanz & LeFevre, 1990). In the literature, the terms *procedure* and *strategy* are often used interchangeably but that can produce confusion. Accordingly, we use the construct of *strategic flexibility* to capture the possibility that different procedures might be selected for implementation, in response to problem-, individual- or task-related factors.

Because multiple factors may influence the criteria that individuals use to select procedures, a comprehensive model of estimation skill must consider how solvers weigh different factors in their solution choices. For example, individuals with more fluent procedural skills may emphasize different conceptual principles when selecting procedures (Star et al., 2009; Torbeyns, De Smedt, Peters, Ghesquiere, & Verschaffel, 2011). Procedure selection and estimation performance are also related to individuals' repertoire of estimation procedures (Star & Rittle-Johnson, 2009; Star et al., 2009). Star et al. (2009) found that students in grades 5 and 6 who had more knowledge of estimation strategies prior to an intervention showed less flexibility overall after the intervention than students who had relatively little prior knowledge. This finding seems counter-intuitive but the increased flexibility of the students with less prior knowledge occurred because they relied more on simplicity for procedure choices after the intervention, whereas those with more prior knowledge showed evidence of relying more on proximity and thus their greater conceptual knowledge was not reflected in their selection of procedures. Such findings suggest that it is important to assess individuals' repertoire of procedures because the availability of specific procedures can influence strategic choices and also interact with problem characteristics.

Much existing research on strategic flexibility in computational estimation has used the choice/no-choice method, where the available procedures were limited by the context of the experiment (e.g., Lemaire & Lecacheur, 2011; Imbo & LeFevre, 2009, 2011). The choice/no-choice methodology has also been used in many other studies of problem-solving processes where variability of procedure selection has been studied, such as numerosity estimation (Gandini, Lemaire, & Michel, 2009; Luwel, Foustana, Onghena, & Verschaffel, 2013; Verschaffel, De Corte, Lamote, & Dherdt, 1998), exact arithmetic (Siegler & Shrager, 1984; Siegler & Lemaire, 1997; Torbeyns et al., 2011), and linear functions (Acevedo Nistal, Van Dooren, & Verschaffel, 2012).

The choice/no-choice method has two phases. In the first phase (choice condition), participants solve problems by selecting a procedure from a limited set. In the second phase (no-choice conditions), they solve the same or similar problems in separate blocks, using a single procedure for all of the problems in the block. Participants are assumed to have the potential of being strategically flexible in the choice condition in that they know the designated procedures and can choose among them. In some studies, a measure of strategic flexibility termed *adaptivity* has been analyzed. *Person-based* adaptivity is measured by the degree to which each individual selects, in the choice condition, the better of the two procedures as determined by his or her performance on that problem or similar problems in the no-choice condition (e.g., Imbo & LeFevre, 2009; Lemaire & Callies, 2009; Torbeyns et al., 2011). In *problem-based* adaptivity, choice performance is evaluated relative to some external criterion for which solution provides the best answer – for example, in research on computational estimation, which procedure will give an approximate answer that is closest to the actual answer (Imbo & LeFevre, 2011; Lemaire & Lecacheur, 2011). Notably, because the choice/no-choice method requires that participants perform a no-choice condition for each of the procedures allowed in the choice condition, it is unwieldy if too many choice options are provided (Siegler & Lemaire, 1997). Further, the term "adaptivity" implies that individuals usually choose the procedure that maximizes performance in some way. However, even experts do not always choose the most maximally efficient procedure in their repertoire (Imbo & LeFevre, 2011; Star & Newton, 2009; Torbeyns et al., 2011). Accordingly, individual differences in skill, in procedure repertoire, and in inferred or assumed task goals may affect procedure selection processes and interactions between skill and procedure choices. In the present research, strategic flexibility (referred as "adaptivity" in other research; Imbo & LeFevre, 2011; Imbo et al., 2007; Lemaire & Lecacheur, 2011) was operationalized as the percentage of trials on which participants chose the rounding procedure that best balanced proximity to the correct answer with simplification of the required calculation. We termed this choice the optimal problem-based procedure.

Another complicating factor in understanding the results of computational estimation research that used the choice/no-choice design is that researchers have provided a limited set of procedures to participants that led to a consistent *mismatch* between the instructions ("choose the procedure that gives the closest estimation of the exact answer") and the set of permitted procedures. For example, in research with adults solving computational estimation problems using multiplication, all problems consisted of one operand with a unit digit smaller than five and one operand with a unit digit larger than five (e.g., 42 x 57). However, only two rounding procedures were allowed: rounding both unit operands down to the nearest decade (e.g., 42 x 57 ≈ 40 x 50) or rounding both operands up to the nearest decades (e.g., 42 x 57 ≈ 50 x 60; Imbo & LeFevre, 2011; Imbo, Duverne, & Lemaire, 2007). Thus, although the mixed rounding procedure (i.e., rounding one operand down and one operand up, e.g., 42 x 57 ≈ 40 x 60) was objectively the optimal problem-based procedure for all problems, participants were not allowed to choose this procedure. Preventing participants from choosing the optimal problem-based procedure may have influenced procedure choices in unexpected ways (Lemaire & Lecacheur, 2011).

These limitations of the choice/no-choice methodology may also have influenced the results when Chinese- and Belgian-educated participants were presented with computational estimation problems (Imbo & LeFevre, 2011). Imbo and LeFevre used the same problems and a similar procedure as Lemaire and Lecacheur (2002), such that participants were given a choice between either rounding both operands up or rounding both operands down. Participants were instructed to choose the best procedure, which was explained as the solution that "yielded the closest estimate of the exact answer" (p. 1296). Despite their superior computational fluency, the Chinese-educated participants were less likely than the Belgian participants to select the procedure that resulted in an answer that was closest to the exact answer and thus, the Chinese-educated individuals appeared to be less strategic than the Belgians. This finding is inconsistent with the assumption that high levels of mathematical skill are associated with strategic flexibility in computational estimation (Dowker, 2005; LeFevre et al., 1993; Star et al., 2009, cf. Torbeyns et al., 2011).

Several factors may account for the finding that the Chinese-educated individuals were judged as less strategic than the Belgians. Imbo and LeFevre (2011) suggested that the Chinese-educated individuals did not understand the requirement to estimate. Asian schooling emphasizes exact calculation (such as written algorithms and memorization of arithmetic facts) over approximate calculation (such as estimation and rounding procedures; Liu, 2008; Reys, Reys, Nohada, Ishida, & Yoshikawa, 1991; Yang, 2005; Zhang & Zhou, 2003). Alternatively, they may have been unsure how to choose the "best" procedure from two non-optimal ones[1] because the optimal problem-based procedure for these problems would be to round according to the size of the unit digit (Imbo & LeFevre, 2011; Lemaire & Lecacheur, 2011; Schunn & Reder, 2001; Verschaffel, Luwel, Torbeyns, & Doreen, 2009). For individuals who have a wider procedural repertoire of potential solution procedures, the limited set of choices may have interfered with strategic decisions. If participants were knowledgeable about various rounding procedures, and aware that rounding the operands according to the size of the unit digit was a reasonable way to adjust for rounding error, then they would presumably have had to suppress that solution (Schunn et al., 1995) and develop some alternative way of deciding which of the two available but non-optimal procedures would produce the answer closest to the exact answer. This potential limitation could have affected other studies as well, for example, reducing age differences in how often children selected the optimal problem-based procedure (as in Lemaire & Lecacheur, 2011), but the effects would be less obvious in those cases because expectations about group or individual performance would not be as strong.

Another limitation of previous studies is that, because participants were not given any specific information about how to decide which of the two procedures would produce an answer that was closer to the exact answer, they might decide that the best answer was one that was fast and easily computed. Using those criteria, rounding down will always be the best solution because rounding down minimizes mental processes and working memory demands. If participants chose to round both operands down on all problems, they would be considered strategic on 50% of trials. Other criteria might also be used that could depend on the individuals' knowledge, other problem characteristics, or cognitive limitations. Thus, solvers' interpretation of the instructions to produce the best answer may be an important factor in understanding patterns of strategic choices.

Present Research

In the present research we explored the issue of whether strategic flexibility was related to arithmetic fluency and repertoire of procedures by contrasting the performance of Chinese- and Canadian-educated adults. They solved computational estimation problems that were similar to those used in previous research (e.g., Imbo & LeFevre, 2011) and we manipulated problem characteristics, methodologies, and task demands. Our hypothesis was that strategic flexibility would depend on these factors, as well as on arithmetic fluency and the participants' repertoire of procedures.

Initially, we conducted a pilot study to rule out the possibility that Chinese-educated individuals were unwilling or unable to use estimation. Twenty participants (10 Chinese- and 10 Canadian-educated students) were asked to solve estimation problems such as 43 x 87. They were instructed to solve the problems in any way they chose (i.e., there was no restriction of procedures). They were told to estimate, but to choose what they considered the "best" solution. Problem types included all possibilities: both unit digits smaller than five, both unit digits greater than five, or a combination (e.g., 43 x 23).

Participants' self-reported procedures were recorded verbatim and afterwards categorized as shown in Table 1. There are three types of decade-rounding procedures (i.e., rounding both operands, either to the smaller or larger decade), and the optimal problem-based procedure is the one that is consistent with the magnitude of the unit digit on a given trial (e.g., 23 x 47 ≈ 20 x 50). The first important finding from this pilot work was that these decade-rounding procedures accounted for, in total, 43% and 68% of Chinese- and Canadian-educated participants performance, respectively, showing that both groups spontaneously selected these solutions on many trials. The second useful observation was that the mixed rounding procedure was a common response to these problems, used on 24% and 32% of trials for Chinese and Canadian participants, respectively, indicating that this procedure was familiar to both groups. Third, the data show that Chinese-educated participants used more complex and more computationally-demanding procedures than Canadian-educated participants: On 33% of trials, Chinese participants reported using alternative computationally complex procedures that involved rounding to numbers other than a decade, and/or adjusting the solution to compensate for rounding error either before or after the main computation. In contrast, Canadian participants very rarely reported these more complex procedures. Rounding the operands to the nearest decade reflects the conceptual principle of simplicity whereas rounding to some other number or adjusting the solution after rounding suggests a greater emphasis on proximity. Canadian participants were more likely to report the simplest procedure, truncation, which involves dropping the unit digit, multiplying the decade numbers (e.g., 27 x 36 as 2 x 3) and then adjusting for place value. They reported using truncation on 31% of trials, as compared to 11% of trials for Chinese participants. Use of truncation or of rounding-down on every trial suggests that the individual is emphasizing simplicity, because the main calculation is always based on the decade values that are shown and thus these strategies reduce working memory and other processing demands. Truncation and using the rounding-down procedure on all trials result in solutions that are distinguishable only by participants' self-reports. In summary, both of the two groups of participants used decade-rounding solutions, but the Chinese-educated individuals also used more computationally complex solutions whereas the Canadian-educated individuals were more likely to choose simpler solutions.

To capture these strategic differences, we categorized the participants in the pilot study according to which of the four procedure categories constituted the majority of their solution choices. As shown in Table 2, a substantial percentage of Chinese-educated participants (40%) chose complex alternative procedures on a majority of trials, whereas none of the Canadian-educated participants reported using such computationally-demanding procedures on more than a few trials. Notably, however, participants in both groups chose one of the three decade-rounding procedures on the majority of trials (or chose other more complex alternative procedures that involved rounding), indicating that their strategic repertoires included the relevant rounding procedures. In Table 2, participants who either used truncation or rounded down on the majority of trials were categorized together. Both of these solution choices suggest that participants are not considering the magnitudes of the operands when making procedure choices and thus were not responding strategically. Finally, two Chinese participants reported using exact calculation; one individual on all trials and the other on half of the trials. This pattern is consistent with previous findings suggesting that some Chinese-educated individuals prefer calculating exact answers to estimation.

The results of this pilot work showed clearly that Chinese- and Canadian-educated individuals sometimes interpret vague instructions differently. Asked to estimate, all of the Canadians used decade-rounding procedures or truncation. None of them reported any further manipulation of the answer to compensate for rounding error, nor did they choose other rounding solutions. In contrast, Chinese-educated individuals used a more varied repertoire of procedures than the Canadians. These findings suggested that the computationally more-skilled Chinese participants emphasized proximity when deciding how to select a procedure for solving the problem, whereas

Canadian participants favored simplicity. This pattern is consistent with the results reported by Liu (2008) for older versus younger Chinese children, by LeFevre et al. (1993) for adults versus children, by Lemaire and Lecacheur (2011) for older versus younger French children, and by Star et al. (2009) for more- versus less-skilled American children. In all cases, less-skilled solvers emphasized simplicity in their procedure selection, presumably in accord with their computational abilities, whereas more-skilled solvers used procedures that emphasized proximity. This behavior could be considered strategic across groups, in that a computationally complex procedure is a poor choice for individuals with limited computational abilities. It is clear that strategic performance is contextual in that it reflects both individual and problem characteristics. Thus, the pilot study supported the view that differences in the procedural repertoire (see also Gandini et al., 2009) and computational fluency (see also Torbeyns et al., 2011) might lead to different strategic behavior.

The pilot study also showed that the two groups interpreted the instructions to estimate and to produce the best answer somewhat differently and so the instructions for the three experiments were carefully constructed to ensure a more consistent interpretation of the task requirements across groups. The pilot study also suggested that the mixed rounding procedure was familiar to these groups. Thus, all three experiments included problems that varied in whether the unit digits were greater or less than five and participants were allowed (except in the no-choice conditions) to select from all three procedures. Finally, the instructions used in the three experiments emphasized that exact calculation was not allowed.

In Experiment 1, we allowed participants to choose any estimation procedure that they knew. We also instructed them to respond quickly and accurately, with an emphasis on speeded responding, to encourage the Chinese-educated individuals to estimate and hence to equate the task demands across groups. Because Chinese-educated participants are computationally more skilled, have a larger repertoire of procedures, and favor proximity to the exact answer in their estimates, we hypothesized that they would be more likely to choose the optimal problem-based procedure in Experiment 1. In Experiments 2 and 3, the instructions given to the participants further emphasized decade-rounding procedures by explicitly requiring participants to choose one of three decade-rounding procedures. In Experiment 2, we used the choice/no-choice methodology that has been used in previous research, but with all three possible procedures rather than just two and the wider range of problem types. In Experiment 3, we manipulated whether the instructions stressed speed or accuracy, because the no-choice condition in Experiment 2 showed that rounding both operands down produced the fastest performance. The results illustrate the importance of understanding both task demands and group differences in research on strategic flexibility.

To minimize differences in the task demands across groups, we controlled two factors. First, in all three experiments in the current research program, Canadian-educated participants were instructed in English by an English-speaking, Canadian-educated researcher and responded to the estimation problems in English; whereas the Chinese-educated participants were instructed in Chinese by a Chinese-English bilingual researcher and responded in Chinese. Second, the mixed decade-rounding procedure was a choice in all of the present experiments.

Experiment 1

Experiment 1 was a free choice paradigm with an instructional emphasis on the definition of estimation as simplifying the calculation to produce an approximate answer. Participants were free to choose from the repertoire of estimation procedures they knew, and were not given instructions as to which estimation procedure was correct or preferred. The duration of the presentation of the problems was manipulated to emphasize that estimation is a relatively fast process. To emphasize speeded responding, presentation duration was randomly varied across trials (2, 4, or 6 s) but participants were encouraged to respond even if the problem had already disappeared.

Method

Participants. In all three experiments participants were undergraduate students at a medium-sized Canadian university. All participants received a 1% bonus credit towards their introductory psychology course. Fifty undergraduate students were recruited for Experiment 1. Twenty-five participants (17 females) had completed elementary and secondary school in China. Twenty-five participants (14 females) had completed elementary and secondary school in Canada. The Chinese-educated participants (M = 22.0 years, SD = 2.27) were older than the Canadian-educated participants (M = 19.4 years, SD = 3.06), $t(48) = -3.41$, $p = .001$.

Materials. Eighty-four problems, plus six practice problems were created. All problems were two-digit x two-digit multiplication problems (e.g., 47 x 32). Operands with unit digits smaller than 5 are best rounded down (e.g., 32 → 30) and operands with unit digits larger than 5 are best rounded up (e.g., 47 → 50), resulting in four problem types: (1) round both operands down (DD), (2) round both operands up (UU), (3) round the first operand up and the second operand down (UD), and (4) round the first operand down and the second operand up (DU). Problem type was included to provide variability in the optimal procedure and retained in the analyses to account for variability due to different items. Because our hypotheses did not involve problem types and for simplicity of exposition, details of the effects of problem type are not discussed in the main text. Details of the analyses by problem type are shown in Appendix B.

The experimental problems were divided across four blocks of 21 problems. Each block contained 3 of the 4 possible problem types, and the division of problem types was counterbalanced across the 4 blocks. Within each block, problems were further divided across 3 durations: 2 s, 4 s, and 6 s. Seven problems were completed at each duration within a block. The order in which trials were presented was randomized within blocks. The durations were randomized to increase the difficulty of the task because the speed pressure imposed by the variable durations was predicted to encourage participants to respond more quickly.

All participants also completed the addition and subtraction/multiplication subsets from an arithmetic fluency test (French, Ekstrom & Price, 1963) to obtain a computational fluency score. The measure includes four pages of arithmetic problems; two pages with three-term addition problems and two pages with alternating rows of two-digit subtraction and two-digit multiplication problems. Total correct problems (maximum 240) were used as the index of computational fluency.

Procedure. Participants were tested individually in a quiet room over a 1-hour session. The two researchers involved in data collection observed one of each other's sessions in attempts to keep researcher methodology as similar as possible. Participants completed a paper-and-pencil practice session prior to the outset of the experimental trials. The researcher observed the participant's strategies and if the participant attempted to perform exact calculation he or she was instructed that the exact solution was not allowed. The aim was to ensure that none of the participants would use exact calculation during the experimental trials and that participants understood the concept of estimation. In the practice session, no Canadian-educated participants attempted to calculate the exact answers. In contrast, five Chinese-educated participants tried to calculate the exact answers to the problems. The experimenter reminded them not to calculate exact answers and suggested that they choose one estimation strategy from rounding, truncation or post-compensation to solve the practice problems. The additional instructions were effective in redirecting the Chinese-educated participants' solutions away from computing the exact answer.

Each trial began with a centered fixation (+) presented for 500ms, and a blank screen for 500ms, followed by the presentation of the centered multiplication problem. Participants were told to state an estimated solution to the multiplication problem as quickly and accurately as possible without computing the exact solution, and that the speed of their response was most important (see exact instructions in Appendix A). Speed was emphasized to discourage the Chinese-educated participants from using exact computation. Otherwise, they were free to use any solution procedure they knew.

A microphone detected the participants' verbal response and displayed confirmation of detection on the researcher's monitor. Following the multiplication problem a screen appeared (e.g., "How did you solve 42 x 67"). The problem that had just been solved was shown to minimize memory demands. Participants were told to accurately report everything they did to find the solution they reported. The researcher recorded the participant's response to the problem and the procedure that he or she used. The researcher marked trials on which interruptions occurred in the microphone detection of the participant's response. The only feedback participants received was on practice trials. If they calculated an exact answer, they were told to select an alternative procedure on subsequent trials.

Results

As described below, solution latencies, calculation errors, and strategic flexibility were analyzed in separate 2 (Group: Canadian vs. Chinese) by 3 (Duration Condition: 2 s vs. 4 s vs. 6 s) by 4 (Problem Type: DD vs. DU vs. UD vs. UU) mixed ANOVAs.

Computational fluency. Participants' computational fluency was measured in three different ways: arithmetic fluency test, solution latencies, and calculation errors.

Arithmetic Fluency Test. Analysis of the number of correct solutions confirmed that the Chinese-educated participants had higher arithmetic fluency than the Canadian-educated participants (97 vs. 53), $t(48) = -5.48$, $p < .001$, $d = 1.58$.

Solution Latencies. In total, 6.8% of trials were spoiled due to microphone failure. These invalid trials were excluded. Median solution times for each duration were calculated for each participant for all problems where latency was less than 15s, the percentage deviation from the exact answer (i.e., the difference between the exact and provided solution, divided by the exact solution and multiplied by 100) was less than 50% and the estimated calculation was correct. The Chinese-educated participants responded significantly faster than the Canadian-educated participants (2328 vs. 3890 ms), $F(1, 48) = 14.29$, $MSE = 25602482$, $p < .001$, $\eta_p^2 = .23$. Latencies varied with presentation duration, $F(2, 96) = 22.16$, $MSE = 1257454$, $p < .001$, $\eta_p^2 = .32$.

Participants responded faster in the 2 s duration (2791 ms) than in the 4 s duration (3144 ms), $F(1,48) = 24.15$, $p < .001$, and 6 s duration (3392 ms), $F(1,48) = 11.53$, $p = .001$. There was a significant interaction between presentation duration and group, $F(2, 96) = 8.60$, $MSE = 1257454$, $p = .002$, $\eta_p^2 = .15$. Latencies for Chinese-educated participants did not vary across durations, $F(1,48) = 1.79$, $p = .188$, whereas latencies for Canadian-educated participants were longer for the 4 s and 6 s durations than the 2 s duration, $F(1,48) = 33.56$, $p < .001$.

Calculation Errors. A solution was defined as a calculation error if the solution did not match the reported procedure. For example, if the participants reported the procedure for 63 x 74 as 60 x 70, the correct response would be 4200, whereas any other response would be considered a calculation error (e.g., 4800, 420). The Chinese-educated participants made fewer calculation errors than the Canadian-educated participants (2% vs. 19%), $F(1,48) = 23.04$, $MSE = 2079$, $p < .001$, $\eta_p^2 = .32$, demonstrating the superior computational fluency of the Chinese-educated participants compared to the Canadian-educated participants. No other effects were significant.

Selection of procedures. Participants' self-reported procedures were recorded verbatim and the same classifications were applied as in the pilot study (Table 1). As shown in Table 2, all but two (92%) of the Chinese-educated individuals used one of the decade-rounding procedures on a majority of trials. The performance of the Canadian-educated individuals was much more variable, although individuals were consistent in their choices: Just over half (54%) used one of the decade-rounding procedures on the majority of trials whereas most of the others (42%) used truncation or rounded down exclusively. One individual used a range of procedures. As a group, therefore, the

Chinese-educated individuals were more likely to select a procedure that was responsive to the problem characteristics.

Strategic flexibility. Strategic flexibility was measured as the percentage of trials on which the participant selected the optimal procedure based on problem characteristics. Specifically, each trial was coded as *strategic* if the participant chose the decade-rounding procedure that would give the most accurate answer for that problem, hence rounding each operand either up or down depending on whether the unit digit was larger or smaller than five. All remaining solution procedures that participants used were coded as *non-strategic*.

The Chinese-educated participants made more strategic procedure choices than the Canadian-educated participants (80% vs. 48%), $F(1, 48) = 10.00$, $MSE = 17287$, $p = .003$, $\eta_p^2 = .17$. The higher overall strategic procedure selection of the Chinese participants is in contrast to the findings of Imbo and LeFevre (2011) but consistent with the view that individuals with efficient calculation skills and good conceptual knowledge are strategic estimators (Dowker, 2005; LeFevre et al., 1993; Star et al., 2009). Strategic flexibility also varied with presentation durations, $F(2,96) = 3.28$, $MSE = 126$, $p = .045$, $\eta_p^2 = .06$. Participants were more strategic in the 2 s duration (65%) than in the 4 s duration (63%), $F(1,48) = 4.11$, $MSE = 122.12$, $p = .048$, and 6 s duration (62%), $F(1,48) = 7.20$, $MSE = 99.70$, $p = .010$. Strategic flexibility in the 4s and 6s durations did not differ, $F(1,48) = .13$, $MSE = 155.78$, $p = .726$. Thus, limiting problem presentation time encouraged strategic flexibility very moderately, but this pattern did not vary between groups.

Discussion

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Chinese participants had higher computational skill than Canadian participants, as measured by the arithmetic fluency test and by speed and accuracy of solving the estimation problems. Consistent with this high level of computational fluency, Chinese-educated participants were also more likely to choose the optimal problem-based procedure than the Canadian-educated participants. Chinese participants' consistent choice of the optimal problem-based procedure and their low error rates suggests that Chinese participants used both conceptual principles, balancing simplicity and proximity. Canadians, in contrast, were more likely to focus on the principle of simplicity, choosing procedures (e.g., truncating, round down) that were easy to implement. Despite choosing simpler procedures, they nevertheless made more calculation errors.

Thus, by all measures, the Chinese-educated participants showed superior performance compared to the Canadian-educated participants. Therefore, when selection of procedures in computational estimation was not restricted, individuals with better computational fluency were more strategic than individuals with less fluent calculation skills (cf. Imbo & LeFevre, 2011).

The results have implications for interpretation of experiments where the range of possible procedures is constrained to a non-optimal set, a situation that characterizes the majority of existing choice/no-choice studies on computational estimation (Imbo & LeFevre, 2011; Lemaire & Lecacheur, 2011). Without a pre-existing procedure to apply, individuals may be forced to guess which procedure to choose or to base their choices on factors that are not relevant for maximizing strategic flexibility. Therefore, for complex problem-solving situations, use of very limited choices in the choice/no-choice paradigm may have unintended consequences and thus undermine conclusions about strategic flexibility beyond the very limited experimental context.

In Experiment 2, we controlled the range of available procedures in order to determine whether the Canadian’s lower strategic flexibility reflected a lack of knowledge of appropriate rounding procedures. Experiment 2 also allowed a more direct comparison with previous research on computational estimation where the choice/no-choice approach was used (e.g., Imbo & LeFevre, 2011; Lemaire & Lecacheur, 2011), although participants were given the choice of three procedures rather than two, so the optimal problem-based procedure was always available on each trial. In Experiment 1, participants were more likely to choose the optimal problem-based procedure in the shortest presentation duration than for the longer durations, but the differences were small. Thus, in Experiment 2, the presentation duration was fixed at 3.5 seconds.

Experiment 2

In Experiment 2, we equated the set of procedures that were available to participants. Participants were given the choice of three procedures: round both operands down, round both operands up, or round one operand up and one down (all to the nearest decade). Because there were three rounding procedures in the choice condition, participants were asked subsequently to solve problems in three no-choice conditions. The option of choosing from among three procedures is in contrast to earlier research (Imbo et al., 2007; Imbo & LeFevre, 2011; Lemaire & Lecacheur, 2011) where the optimal procedure was not available for problems such as 42 x 58. It is also different than the situation in Experiment 1, where access to a larger repertoire of procedures might have influenced strategic processing. Presentation duration did not have a very strong influence on performance in Experiment 1, although participants tended to respond more quickly and strategically when problem durations were shorter. Accordingly, a moderate duration was chosen in the present experiment.

Method

Participants. Forty-eight participants were recruited for Experiment 2. Twenty-four participants (15 females) were Canadian-educated undergraduate students. Twenty-four participants (14 females) were Chinese-educated undergraduate students. The Chinese-educated participants (M = 20.9 years, SD = 1.54) were slightly older than the Canadian-educated participants (M = 19.6 years, SD = 2.36), $t(46) = -2.18$, $p = .035$.

Materials. Ninety-six problems, plus 20 practice problems were created. All problems were created using the same criteria as Experiment 1. Participants also completed the same test of arithmetic fluency as in Experiment 1.

Procedure. The procedure for Experiment 2 was very similar to Experiment 1 and only the differences are discussed here. In Experiment 2, participants were instructed to use one of three rounding procedures to solve the problems. The first procedure was to round both operands up to the nearest decade (e.g., 23 x 76 ≈ 30 x 80). The second procedure was to round both operands down to the nearest decade (e.g., 23 x 76 ≈ 20 x 70). The third procedure was a mixed procedure that involved rounding one operand up and rounding one operand down (e.g., 23 x 76 ≈ 20 x 80 or 30 x 70). To encourage all participants to adopt similar strategic criteria, the specificity of instructions was increased (see exact instructions in Appendix A). Problems were shown for 3.5 seconds but participants were expected to respond even if the operands had disappeared.

All participants completed the experimental trials under four different conditions. The first condition for all participants was the choice condition. Participants were instructed to choose from the three procedures (round up, round down, or mixed) to give the best estimation of the problem. The next three conditions were no-choice conditions and coincided with the three procedures: round down, round up and mixed. The order of the no-choice conditions was counterbalanced across participants. In both choice and no-choice conditions, participants were asked to report the procedure they used after each trial as in Experiment 1. Prior to each experimental condition participants completed a set of practice problems. Participants were given feedback during the practice problems on whether their implementation of the procedure was successful.

Results

As described below, solution latencies, implementation errors, calculation errors, and strategic flexibility in the choice condition were analyzed in 2 (Group: Canadian vs. Chinese) by 4 (Problem type: DD vs. UU vs. DU vs. UD) mixed ANOVAs.

Computational fluency. Computational fluency was measured in four different ways: arithmetic fluency test, solution latencies, implementation errors, and calculation errors.

Arithmetic Fluency Test. As in Experiment 1, the Chinese-educated participants were more fluent than the Canadian-educated participants (96 vs. 52), $t(46) = 6.90$, $p < .001$, $d = 2.05$.

Solution Latencies (No-choice conditions). In total, 5.9% of trials were invalid and removed from analyses of latencies. Chinese-educated participants responded more quickly than Canadian-educated participants (2047 ms vs. 3498 ms), $F(1, 46) = 42.88$, $MSE = 7.06E6$, $p < .001$, $\eta_p^2 = 0.48$. Latencies varied with Condition, $F(2, 92) = 90.57$, $MSE = 9.22E5$, $p < .001$, $\eta_p^2 = 0.78$. Participants responded more quickly in the round-down condition (2016 ms) than in the round-up (3073 ms) and mixed (3229 ms) conditions, $F(1, 46) = 108.74$, $MSE = 9.85E5$, $p < .001$ and $F(1, 46) = 148.77$, $MSE = 9.49E5$, $p < .001$, which did not differ, $F(1, 46) = 2.81$, $MSE = 8.33E5$, $p = .100$. This finding supports the view that if speed is the most important solution criterion, then the most strategic procedure will be to round down.

Implementation errors (No-choice conditions). A solution was defined as an implementation error in the no-choice condition if it did not correspond to the procedure that the participant was supposed to use in that condition. For example, in the round-down condition the correct procedure for 23 x 74 would be 20 x 70, whereas any other procedure would be an implementation error. The majority of the Chinese participants successfully used the required rounding procedure in which they were explicitly instructed whereas Canadian participants used the incorrect procedure on trials, such that Chinese-educated participants made fewer implementation errors than Canadian-educated participants (1% vs. 7%), $F(1, 46) = 5.20$, $MSE = 276$, $p = .027$, $\eta_p^2 = .10$. This finding shows that the Chinese-educated participants were more likely to execute the appropriate procedure in the no-choice condition than were Canadian-educated participants.

Calculation errors. As in Experiment 1, calculation errors were defined as the failure to correctly compute with the estimated operands for both no-choice and choice conditions.

No-choice condition. Calculation errors varied across conditions, $F(1, 46) = 4.61$, $MSE = 50$, $p = .012$, $\eta_p^2 = .09$. Errors were equally frequent in the round-down and round-up conditions (12% vs. 13%), $p = .403$, but were more frequent in the mixed condition (16%), $F(1, 46) = 8.54$, $MSE = 50$, $p = .005$, and $F(1, 46) = 4.13$, $MSE = 55$, $p = .048$. Calculation errors also varied across groups, $F(1, 46) = 52.06$, $MSE = 279$, $p < .001$, $\eta_p^2 = .53$. Chinese-educated participants made many fewer calculation errors than Canadian-educated participants (4% vs. 24%).

The interaction between condition and group was marginally significant, $F(2, 92) = 3.04$, $MSE = 50$, $p = .053$, $\eta_p^2 = .06$. Canadian-educated participants made more errors in the mixed condition (28%) than in the round-down condition (22%), $F(1, 46) = 7.40$, $MSE = 50.26$, $p = .009$, or the round-up condition (21%), $F(1, 46) = 9.44$, $MSE = 55.27$, $p = .003$, which did not differ, $F(1, 46) = .30$, $MSE = 43.83$, $p = .589$. In contrast, Chinese-educated participants were equally accurate across all conditions (2%, 5%, 4%; see Table C.1 in Appendix C for details).

Choice condition. Calculation errors varied across groups, $F(1, 46) = 34.70$, $MSE = 679$, $p < .001$, $\eta_p^2 = .43$. Chinese-educated participants made many fewer calculation errors than Canadian-educated participants (2% vs. 25%).

Strategic flexibility. All experimental trials were included in the analyses of strategic flexibility. A response was considered *strategic* if the participant chose the optimal problem-based procedure in the choice condition, regardless of the accuracy of the answer to that problem. For example, a response to 23 x 74 was considered *strategic* if participants reported their solution as “20 x 70” (rounding both operands down). There was no difference in strategic procedure selection across groups: Chinese-educated participants (61%) and Canadian-educated participants (61%) were equally likely to choose the optimal procedure when they were limited to three procedure choices and told to use the best solution procedure, $F(1, 46) < .001$, $MSE = 3363$, $p = .992$, $\eta_p^2 < .001$.

Discussion

The change in the experimental design from Experiment 1 to Experiment 2 equalized the frequency with which Chinese- and Canadian-educated participants selected the optimal procedure based on problem characteristics. Compared to Experiment 1, the strategic flexibility of the Canadian participants increased, whereas it decreased for Chinese-educated participants. Chinese-educated participants nevertheless responded more quickly and made many fewer errors in the no-choice conditions than the Canadian-educated participants.

There were two main differences between Experiment 1 and Experiment 2. First, in Experiment 1 participants were given free choice of procedures whereas in Experiment 2 the procedure selection was limited to three rounding procedures. Second, in Experiment 1, presentation durations (i.e., 2, 4, or 6 s) varied across problems whereas in Experiment 2 the presentation duration was fixed at 3.5 seconds. Even though the Chinese participants solved problems within 2 s in both experiments, the constrained procedure selection in Experiment 2 may have increased the difficulty of procedure selection for the Chinese participants, resulting in reduced strategic flexibility. Although this effect seems paradoxical, Beilock and De Caro (2007) have shown that pressure to respond according to experimenter-imposed constraints can have more negative effects on skilled individuals than on less-skilled individuals. Thus, the combination of increased presentation duration but more restricted procedure choices in Experiment 2 may have resulted in lowered strategic flexibility for the Chinese-educated participants (relative to Experiment 1) but increased strategic flexibility for Canadian-educated participants.

Another finding of interest was that, despite the provision of the optimal problem-based procedure as part of the choice condition, strategic processing in relation to problem characteristics was moderate when instructions emphasized speeded responding. The issue of why strategic processing is moderate even for highly skilled individuals in this paradigm has not been addressed directly before in the literature on computational estimation. Their performance in the no-choice condition shows that all participants were capable of executing the three procedures. Instructions to respond quickly but accurately should presumably have led to high levels of strategic performance.

This relatively modest level of strategic flexibility among skilled adults is not unique to the current research. Ten-year-old children in Lemaire and Lecacheur (2011) were strategic on 87% of problems when the unit operands were consistent with the best procedure (i.e., down-down or up-up problems in our terminology) but on only 64% of mixed problems. In contrast, when skilled adults were restricted to a choice of two procedures, Belgian participants in Imbo and LeFevre (2011) were strategic on 73% of problems whereas French participants in Imbo et al. (2007) were strategic about 63% of the time. Only for the Chinese participants in Experiment 1, where procedure choices were less restrictive, did strategic flexibility reach levels that are consistent with the capabilities of these skilled adults. Why are adults not more strategic, in general, on these problems when procedure choices are limited?

Estimation requires a person to make strategic decisions using the principles of proximity and simplicity (LeFevre et al., 1993). Responses are also expected to be relatively fast, which was emphasized in the present research by instructing speeded responding and limiting the presentation duration of the problem. In other experiments, participants in the choice condition were routinely asked to provide the *best* solution but they were not given explicit criteria for performance. “Best” could be interpreted in many ways, for example, the most accurate procedure (i.e., the one closest to the exact answer), the fastest procedure, the procedure that is easiest for the individual to execute, or the one that requires the least working memory. The results from the no-choice condition in Experiment 2 show that rounding-down is faster than rounding one or both operands up (regardless of problem characteristics) and so an individual who interpreted “best” as fastest might be more likely to chose to round down on most problems and thus show less strategic flexibility (as it was defined) than someone who interpreted “best” as the answer closest to the exact answer (Star & Newton, 2009; Newton, Star, & Lynch, 2010). To further explore this possibility, in Experiment 3 the instructions and the definition of ‘best estimate’ were manipulated explicitly. We hypothesized that the choice/no-choice conditions of Experiment 2, and of the other research using this methodology with computational estimation, may have led many participants to select procedures that favored relatively fast solutions over those that were strongly related to problem characteristics.

Experiment 3

Asking solvers to choose the 'best' procedure in a computational estimation task, as in Experiment 2 and previous studies, might have been interpreted as a suggestion to choose the fastest procedure (sacrificing accuracy) or the most accurate procedure (sacrificing speed). Thus, in Experiment 3 we explicitly manipulated the instructions and asked participants, in different conditions, to focus on speed, accuracy, or both. We hypothesized that the precise instructions and explicit feedback would allow individuals in both groups to develop criteria for procedure selection that would produce high levels of strategic flexibility.

Method

Participants. Fifty university students were recruited for Experiment 3. Twenty-six participants (16 females, mean age 20.3 years) were Canadian-educated undergraduate students. Twenty-four participants (19 females, mean age 23 years) were Chinese-educated undergraduate students. The Chinese-educated participants (M = 23.0 years, SD = 5.39) were marginally older than the Canadian-educated participants (M = 20.3, years, SD = 4.9), $t(47) = -1.82$, $p = .075$.

Materials. Seventy-two experimental problems, plus 15 practice problems were created using the same criteria as in Experiments 1 and 2. Participants also completed the same measure of arithmetic fluency as in Experiments 1 and 2.

Procedure. The sequence of tasks and experimental procedure were very similar to those in the previous experiments and only the differences are outlined here. The stimuli were divided into three problem sets, and counterbalanced across instructional conditions using a Latin square design. Problems were presented in random order within each condition for each participant. All participants completed the experimental trials in each of the three different conditions, the order of which was counterbalanced across participants using a balanced Latin square design.

The instructions in the three conditions were manipulated (exact instructions are in the Appendix A). In the Speed condition, participants were asked to focus on speed and to give an estimate quickly; in the Accuracy condition they were asked to focus on accuracy and to give an estimate close to the exact solution; and in the Balance condition they were asked to balance speed and accuracy. In all of these conditions participants were instructed to choose from one of the three procedures; round both operands up, round both down, or round one up and one down.

Participants received feedback on their adherence to the requirements after each trial, during the practice and experimental trials. They were given their response time for each problem in the Speed condition, information on if they chose the procedure that produced the answer closest to the exact answer in the Accuracy condition, and both types of information in the Balance condition.

Results

As described below, solution latencies, calculation errors, and strategic flexibility were analyzed in 2 (Group: Canadian vs. Chinese) by 3 (Instructional Condition: Speed vs. Accuracy vs. Balance) by 4 (Problem Type: DD vs. DU vs. UD vs. UU) mixed ANOVAs.

Computational fluency. Computational fluency was measured in the same way as in Experiment 1.

Arithmetic Fluency Test. As in Experiments 1 and 2, the Chinese-educated participants had higher arithmetic fluency scores than the Canadian-educated participants (104 vs. 65), $t(48) = -4.81$, $p < .001$, $d = 1.40$.

Solution Latencies. In total**,** 28 trials (0.8%) were missing response data and 260 trials (7.2%) had invalid response times. Latencies varied with instructional condition, $F(2, 96) = 44.02$, $MSE = 9.13E5$, $p < .001$, $\eta_p^2 = .48$. Participants were faster in the Speed condition (1898 ms) than in the Balance condition (2550 ms), $F(1, 46) = 54.41$, $MSE = 7.79E5$, $p < .001$. They were slower in the Accuracy condition (2759 ms) than in the Balance condition, $F(1, 46) = 4.67$, $MSE = 9.30E5$, $p = .035$. As in Experiments 1 and 2, Chinese-educated participants solved problems more quickly than Canadian-educated participants (1866 ms vs. 2939 ms), $F(1, 48) = 26.58$, $MSE = 6.49E6$, $p < .001$, $\eta_p^2 = .36$. The interaction between Group and Instructional Condition was also significant, $F(2, 96) = 5.61$, $MSE = 9.13E5$, $p = .005$, $\eta_p^2 = .11$. Chinese-educated participants were faster in the Speed condition (1539 ms) than in either in the Accuracy condition (2086 ms), $F(1, 48) = 13.96$, $MSE = 1.03E6$, $p < .001$, or the Balance condition (1975 ms), $F(1, 48) = 11.71$, $MSE = 779075$, $p = .001$*,* which did not differ, $F(1, 48) = .64$, $MSE = 9.31E5$, $p = .427$. The Canadian-educated participants were also faster in the Speed (2259 ms) condition than in either the Accuracy (3432 ms) condition, $F(1, 48) = 69.44$, $MSE = 1.03E6$, $p < .001$, or Balance condition (3127 ms), $F(1, 48) = 50.20$, $MSE = 7.79E5$, $p < .001$. However, they were also faster in the Balance condition than in the Accuracy condition, $F(1, 48) = 5.23$, $MSE = 9.31E5$, $p = .003$.

Calculation Errors. Errors varied with Instructional Condition, $F(2, 96) = 7.18$, $MSE = 195$, $p = .001$, $\eta_p^2 = .13$. Participants made more errors in the Speed condition

(18%) than in either the Accuracy condition (14%), $F(1, 46) = 7.51$, $MSE = 287$, $p = .008$, or the Balance condition (14%), $F(1, 46) = 10.09$, $MSE = 203$, $p = .002$. Chinese-educated participants made fewer errors than Canadian-educated participants (4% vs. 26%), $F(1, 48) = 13.95$, $MSE = 5100$, $p < .001$, $\eta_p^2 = .23$. Group interacted with Instructional Condition, $F(2, 96) = 5.99$, $MSE = 195$, $p = .004$, $\eta_p^2 = .11$. The Chinese participants were equally accurate across all conditions (5%, 5%, 4%; see Table C.2 in Appendix C for details); whereas the Canadian participants made significantly more errors in the Speed condition (32%) than in either the Accuracy condition (22%), $F(1, 46) = 15.69$, $MSE = 287$, $p < .001$, or the Balance condition (24%), $F(1, 46) = 16.46$, $MSE = 202$, $p < .001$.

Strategic flexibility. Strategic flexibility was measured as the same way as in Experiment 2. Strategic flexibility varied with instructional condition, $F(2, 96) = 69.43$, $MSE = 1152$, $p < .001$, $\eta_p^2 = .59$. Participants used the optimal problem-based procedure more frequently in the Accuracy (95%) and Balance (93%) conditions than in the Speed condition (59%), $F(1, 46) = 68.80$, $MSE = 1824$, and $F(1, 46) = 77.29$, $MSE = 1473$, *p*s < .001. These results, in combination with those from Experiment 2, suggest that many participants interpret the instructions to select the “best” procedure as the fastest one, resulting in about 60% strategic flexibility, on average. With more directed instructions to focus on proximity, and trial-by-trial feedback, strategic flexibility improved to near-perfect levels. Similar to Experiment 2, the effect of Group was not significant, $F(1, 48) = .06$, $MSE = 1277$, $p = .802$, $\eta_p^2 = .001$, indicating that Chinese-educated participants chose procedures equally strategically as did Canadian-educated participants. None of the interactions with Group reached significance, indicating that

Accuracy and Balance instructions influenced the procedure choices of Canadian- and Chinese-educated participants to a similar degree.

Discussion

In Experiment 3, instructional manipulations were used to examine whether interpretation of the instructions and subsequent implementation of estimation procedures could account for the moderate levels of strategic processing shown in Experiment 2 and in other research using similar methodologies. Consistent with this possibility, in the present experiment the combination of instructions and feedback resulted in higher strategic flexibility levels under accuracy or balanced (speed and accuracy) instructions than under speed instructions alone and there were no group differences in strategic flexibility, suggesting that both groups responded similarly to the explicit instructions. Group differences were still found in the computational fluency analyses, such that Chinese-educated participants showed better arithmetic skills than Canadian-educated participants. Arithmetic performance of the Canadian-educated participants was influenced by the instructions; that is, they answered faster but less accurately under speed instructions and slower but more accurately under accuracy instructions. In contrast, the Chinese-educated participants were fast and accurate in all conditions. These results support the view that individuals with stronger computational fluency show minimal effects of instructional manipulations on computation performance whereas individuals with weaker fluency are more likely to be influenced by speed (vs. accuracy) instructions (Smith-Chant & LeFevre, 2003).

Despite the group differences in computational fluency, we were able to equate the two groups' strategic flexibility and demonstrate the conditions under which close to perfect strategic flexibility was observed. The present results indicate that, for both groups, solvers are less strategic, that is, are less likely to choose the optimal problem-based procedure that produces the answer closest to the exact answer, when speed is emphasized.

General Discussion

In three experiments, Chinese-educated and Canadian-educated participants solved computational estimation problems. The Chinese participants were more computationally efficient (i.e., faster and more accurate) than the Canadian participants (Campbell & Xue, 2001; Imbo & LeFevre, 2009; LeFevre & Liu, 1997). In all three experiments, we examined problem-based strategic flexibility, defined as choosing the decade-rounding procedure that gives the approximate answer that is closest to the exact answer. In Experiment 1, Chinese-educated participants were more strategic than Canadian-educated individuals. When asked to respond quickly but choose the best procedure, they showed a balance between principles of simplicity and proximity, resulting in frequent selection of the optimal problem-based procedure. With exactly the same constraints, however, many Canadian-educated participants were much less likely to choose the optimal problem-based procedure: Instead, they focused on simplicity and often rounded both operands down or truncated them. These procedure choices were less strategic in relation to problem characteristics but were consistent with the pressure to respond relatively quickly.

In Experiment 2, restriction of the procedure choices to equate procedure repertoire across groups coupled with instructions that emphasized speeded responding resulted in equal and moderate strategic flexibility for both groups. Surprisingly, the strategic flexibility of the Chinese participants was lower in Experiment 2 than in Experiment 1. The reduced emphasis on proximity across experiments may have reflected the instructions to respond quickly. Accordingly, in Experiment 3 we provided the same procedure choices as in Experiment 2, but used instructions and feedback to bias speeded responding (to emphasize simplicity), accurate responding (to emphasize proximity), or to encourage a balance between speed and accuracy.

The results of Experiment 3 showed clearly that the combination of instructions and feedback is a central factor in the overall level of strategic flexibility observed in this paradigm. Instructions to respond quickly resulted in equal and moderate strategic processing across groups, which was very similar to the findings in Experiment 2. In contrast, instructions to respond accurately resulted in frequent choice of the optimal problem-based procedure by both groups, showing that the right combination of instructions and feedback can encourage maximum strategic flexibility in relation to problem characteristics.

Implications for Previous Research

There were several important results of the current research that help to explain or qualify previous findings. First, the present results provide some support for the conclusions drawn by Imbo and LeFevre (2011). Imbo and LeFevre explained the differences in strategic processing between Chinese- and Belgian-educated individuals by referring to group differences in educational approaches, with Asian schooling focusing more on exact calculation and less on approximate calculation compared to Belgian schooling. The results of Experiment 1 (and the pilot study) supported the view that Chinese-educated individuals have a strong tendency to emphasize proximity in computational estimation (see also Liu, 2008). Chinese-educated participants were more likely than Canadian-educated participants to choose procedures that potentially resulted in solutions closer to the exact solution when procedure choices were unconstrained. The results of all three experiments show, however, that Chinese-educated individuals do understand estimation and can use it when instructed, even though some may have a bias towards using exact calculation. In contrast, many Canadian-educated individuals were biased towards simplicity in that they often used truncation or rounded down on all problems, selecting procedures independently of problem characteristics.

We also found evidence for a related explanation of the group difference in strategic procedure selection observed by Imbo and LeFevre (2011). Recall that, in all previous experiments using the choice/no-choice paradigm with computational estimation, there was a mismatch between procedure choices available and the types of problems that were presented: Participants were not allowed to choose the mixed rounding procedure even though this procedure was clearly the *best* rounding procedure for the problems being presented. With children, forcing a choice between round-up and round-down on mixed problems leads to similarly moderate levels of adaptivity (e.g., ranging from 59 to 67% for third- to seventh-graders; Lemaire & Lecacheur, 2011). If the requirement to choose the best procedure from two inferior procedures affected Chinese-educated participants more, resolving the mismatch problem should stabilize performance across groups. Accordingly, in Experiment 2 where the optimal problem-based procedure was available for each problem type, Chinese- and Canadian-educated participants had equal levels of strategic flexibility.

Our research also supports the view that the Chinese-educated individuals were more affected by limited choice conditions than the Canadian-educated students. Experiment 1 showed that in a free choice paradigm, Chinese-educated participants chose the optimal problem-based procedure on about 80% of the problems, as compared to 46% for the Canadian participants. This finding suggests that many Asian participants in Imbo and LeFevre (2011) had to suppress their preferred procedure on all trials. Presumably, suppressing a familiar or preferred procedure requires working-memory resources, leaving fewer resources available for strategic procedure selection. Consistent with this interpretation, the Chinese-educated participants in Imbo and LeFevre showed greater working memory demands in the estimation task than the Belgians, a result that is otherwise quite difficult to interpret.

Similar observations have been made in other cognitive domains. For example, chess experts may not find the best solution to a chess problem because they find it difficult to suppress a familiar solution (Bilalic, McLeod, & Gobet, 2010) and doctors often miss important details about patients' symptoms that are inconsistent with their initial opinions (Groopman, 2007). Responses to limited procedure choices are thus not always an indication of low skill levels – they may also be indicative of strategic preferences associated with high expertise (Beilock & DeCaro, 2007). More specifically, studies of strategic flexibility in algebra problem solving indicate that knowledge of multiple procedures does not always lead to flexible use of procedures among experts (Newton & Star, 2009) or among those struggling with the task (Newton et al., 2010). Accordingly, failure to implement a particular procedure or choice of a sub-optimal procedure does not necessarily indicate that an individual has low strategic flexibility.

Another important finding in the present research was that selection rates of the optimal problem-based procedure differed across the limited-choice conditions, with higher strategic flexibility rates (more than 90%) in the Accuracy and Balance conditions in Experiment 3, and lower strategic flexibility rates (around 60%) in the Speed condition in Experiment 3 and in Experiment 2. These results indicate that providing explicit instructions and feedback can greatly influence participants' procedure choices, and hence offers interesting implications for educational settings (see also Luwel, Foustana, Papadatos, & Verschaffel, 2011; Newton et al., 2010). More generally, the results suggest that solvers' criteria for strategic choices may be influenced by a range of factors. Finally, these results also suggest that findings from the choice/no-choice paradigm, where implemented with a limited selection of non-optimal procedures, should be interpreted cautiously. Comparisons across experiments may not be warranted even when instructions and problem sets are equated.

Effects of Nationality

Characteristics of the participants in the present research may have influenced the pattern of results. First, the Chinese-educated participants had better computational arithmetic skills than the Canadian-educated participants across all measures. These group differences in computational fluency were consistent with previous research where Chinese-educated adults were more efficient when solving arithmetic problems compared to those educated in North American or European countries, whether the problems were simple (e.g., 3 x 7; Campbell & Xue, 2001; LeFevre & Liu, 1997) or more complex (e.g., 34 + 27; Imbo & LeFevre, 2009, 2010). Chinese children are also typically more skilled at arithmetic than North American children (e.g., Miller, Smith, Zhu, & Zhang, 2005; Siegler & Mu, 2008; Stevenson, Chen, & Lee, 1993), so these differences are not solely a function of differential selection of students who choose to attend university in Canada. These differences in computational fluency are usually linked to a much greater focus on practice of arithmetic procedures in Chinese education, but there are other group differences in educational experiences that might be relevant, such as the greater emphasis on proximity in computational estimation (Cantlon & Brannon, 2007; Liu, 2008). One implication of the group differences in computational fluency is that Chinese-educated individuals presumably require less computational effort to solve the problems, and, as a result, they may have an advantage in strategic processing because their greater computational fluency may have resulted in more available working memory resources compared to the Canadian-educated participants.

A second group difference was that the Chinese-educated participants were slightly older than the Canadian-educated participants. In studies where there were large age ranges (e.g., greater than 30 years), younger participants had faster, more accurate, and more adaptive performance than older participants on computational estimation (Lemaire, Arnaud, & Lecacheur, 2004). However, the age differences between groups in the present study (ranging from 1 to 3 years) were much smaller than in previous research and thus are unlikely to be the main source of differences in estimation performance across groups.

Limitations

The present research has several limitations. First, the results may not generalize to comparisons across other groups, in that Chinese-educated individuals who are studying in Canada may be different than individuals who stay in China or than individuals with other backgrounds. This limitation does not compromise the main conclusion, however, which is that for complex cognitive tasks, it is important to consider individual differences that may influence strategic choices. Second, the results should be compared cautiously to other research on computational estimation using the choice/no-choice paradigm because in previous studies, only mixed problems were used and only two procedure choices were allowed. Our contention is that the present methodology is a more valid approach to understanding strategic flexibility than that used in previous research. The three rounding procedures used in the present research cover a wider range of the procedural repertoire than the two choices used in previous studies. Further research is needed to explore the strengths and weaknesses associated with using variants of the choice/no-choice approach in complex tasks (cf. Luwel et al., 2009).

Implications for Future Research

Strategic flexibility is a central question in current research on problem solving, particularly for complex tasks (Ionescu, 2012) and is an important educational goal in mathematics (Star et al., 2009; Newton et al., 2010). The present results suggest that researchers need to consider the effects of instructions and prior knowledge on the measurement of strategic flexibility. The influence of instructions was very apparent throughout these three experiments. Participants were asked to complete very similar tasks with varying task demands that resulted in completely different results for strategic flexibility. Hence, a first recommendation is that researchers need to ensure equivalent interpretation of instructions when testing group differences. Even when two groups receive exactly the same instructions, their interpretation of the task requirements can differ and these differences in interpretation may influence their subsequent strategic processes.

Our second recommendation is that researchers need to establish the validity of the indices of strategic processes that they report. Assessment of strategic flexibility requires that participants can make a real choice from among different possible solutions that vary in how useful they are to fulfilling task goals. In the real world, flexible selection of procedures is assumed to be relevant in complex problem-solving situations, such as air traffic control, health decisions, or stock-market selections and typically, the number of choices that are available are not artificially constrained. In experimental paradigms, however, there exist many different measures of strategic flexibility (cf. Luwel et al., 2009) and all measures do not necessarily assess the same construct. For example, when testing problem-based strategic flexibility (as in the present research), participants should be given a choice of procedures that is consistent with their knowledge of the domain. The issue of how to best define and measure strategic flexibility must be considered whenever complex problem-solving behaviors are being explored.

Conclusions

In a computational estimation task where the participants are familiar with the available procedures, selection of procedures will be influenced by instructions, preferences for simplicity of calculation, individual differences in computational fluency, and problem characteristics. The findings are informative for interpreting differences in strategic processing in experimental and educational settings (Heinze et al., 2009). They support models of strategic choice that have criteria for selection of procedures that vary with individual differences in factors such as preferences, knowledge, and problem characteristics (e.g., Siegler & Araya, 2005).

Footnotes

1. There is a simple procedure for deciding which of the round up and round down procedures will be optimal on mixed problems. If the unit digits sum to 10 or less, then rounding down gives a closer solution (e.g., 34 x 76), whereas if the unit digits sum to 12 or more, then rounding up gives a closer solution (e.g., 34 x 78). [With unit digits that sum to 11, the optimal procedure is not consistent across individual items.] However, the Chinese participants in Imbo and LeFevre (2011) were probably not familiar with this procedure. None of the participants in the current research ever reported using it whereas Belgians may be taught this procedure when they learn estimation.

Appendix A

Instructions used in each experiment

Experiment 1

In today's experiment you are going to be answering double-digit (e.g. 21 x 78) multiplication problems, using estimation. You may use any strategy you know to get the most accurate estimation of the solution as quickly as possible. Please remember to ESTIMATE. The speed of your response is most important. Exact calculation is not allowed.

When you estimate the answer to an arithmetic problem you are obtaining an answer that is close to the answer you would obtain using exact calculation. Estimation is beneficial because it allows you to come up with an answer faster by using easier strategies.

Experiment 2

In today's experiment you are going to be answering double-digit (e.g. 25 x 75) multiplication problems, using estimation. You are going to be using three different estimation strategies:

1) Round both operands up (Ex: 25 x 75 rounded to 30 x 80)
2) Round both operands down (Ex: 25 x 75 rounded to 20 x 70)
3) Round one operand up and one operand down (Ex: 25 x 75 rounded to 20 x 80)

The problem will remain on the screen for 3.5 seconds. I want you to try to respond before the problem disappears. Speak your answer out loud into the microphone as quickly as possible, without making too many mistakes. Remember, you are to ESTIMATE the best solution using any of the three rounding strategies.

Choice condition: You may choose any of the three strategies. Remember, you are to ESTIMATE the best solution using any of the three strategies. Try to respond quickly and accurately.

No choice condition[1]: You must use the ROUND UP/DOWN/OPPOSITE strategy on all trials. You MUST round up/down/opposite both operands for each trial. Try to respond quickly and accurately.

Experiment 3

In today's experiment you are going to be answering double-digit multiplication problems (e.g. 25 x 75), using estimation. We are interested in how you choose strategies[2]. Different strategies can be used to accomplish different goals. The main goals of estimation are being fast, being accurate and balancing speed and accuracy.

Speed condition: In this condition I want you to choose the strategy for each individual problem that will give you a solution the quickest. You will be given feedback on how quickly you responded.

Accuracy condition: In this condition I want you to choose the strategy for each individual problem that will give you the solution that is closest to the exact answer. Remember that you are not allowed to give the exact solution and that you must choose from the three rounding strategies. You will be given feedback on selecting the strategy that will give you a solution that is closest to the exact answer.

Balanced condition: In this condition I want you to choose the strategy for each individual problem that will give you a solution that is close to the exact answer very

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[1] Three conditions were used in the no-choice condition: round down, round up and round opposite. #

[2] The three strategies that the experimenter provided to participants were exactly the same as in Experiment 2.

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quickly. Try to find a balance between speed and accuracy. You will be given feedback on both your speed and the accuracy of your strategy selection.

Appendix B: Details of effects of problem type

Table B.1

Significant effects of problem type on strategic flexibility and solution latencies in Experiment 1

	$F(3,144)$	η_p^2	Detailed description
Strategic flexibility			
Problem type	9.38**	.16	Participants were most likely to select the optimal problem-based procedure on DD problems (69%) and least likely on UU problems (56%). They were equally likely to select the optimal problem-based procedure on DU (63%) and UD (64%) problems, $F(1,48) = .99$, $p = .326$.
Solution latencies			
Problem type	24.68**	.34	Participants were fastest on DD problems (2775 ms) and slowest on UU problems (3391 ms). They were equally fast on DU (3115 ms) and UD problems (3155 ms), $F(1,48) = .52$, $p = .476$.
Problem type x Group	3.46*	.07	Chinese were faster on DD problems than DU, UD, and UU problems, $F(1,48) = 27.17$, $p < .001$. Canadians were faster on DD and DU problems than UD and UU problems, $F(1,48) = 34.92$, $p < .001$. No difference was observed either between DD and DU, $F(1,48) = 2.40$, $p = .128$, or between UD and UU, $F(1,48) = 1.77$, $p = .190$.
Problem type x Duration	6.44**	.04	Latencies were similar on DD problems, $F(1,48) = .17$, $p = .686$. Latencies increased as the duration increased for DU, $F(1,48) = 27.27$, $p < .001$, and UD problems, $F(1,48) = 29.56$, $p < .001$. On UU problems, participants were faster for the 2 s than the 4 s duration, $F(1,48) = 17.68$, $p < .001$; however, the latencies for 4 s and 6 s did not differ from each other, $F(1,48) = .25$, $p = .622$.

* $p < .05$; ** $p < .01$

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Table B.2

Significant effects of problem type on strategic flexibility, implementation errors, and solution latencies in Experiment 2

	$F(3,138)$	η_p^2	Detailed description
Strategic flexibility (choice condition)			
Problem type	51.23**	.53	Participants were more likely to select the optimal problem-based procedure on DD (97%) problems and least likely on DU (39%) problems. The UD (54%) and UU (54%) problems did not differ, $F(1,46) = .01$, $p = .933$.
Implementation errors (no-choice condition)			
Problem type x condition	2.37*	.05	See interpretations in “Problem type x Condition x Group” below
Problem type x condition x Group	2.31*	.05	Two separate 4 (Type) x 3 (Condition) ANOVAs were conducted on Chinese- and Canadian-educated participants. Chinese: They made equally few implementation errors across all problem types for all conditions, $F(6, 138) = 1.69$, $p = .129$. Canadians: Implementation errors varied with conditions and problem types, $F(6, 138) = 2.41$, $p = .030$. In the round-down condition, errors did not differ between problem types, $F(3, 69) = .12$, $p = .946$. In the round-up condition, DU problems had fewer errors than the other problem types, $F(1,46) = 10.41$, $p = .002$. In the mixed condition, DD and UD problems had fewer errors than UU and DU problems, $F(1,46) = 9.56$, $p = .003$.

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Solution latencies (no-choice condition)			
Problem type x condition	6.08**	.12	See interpretations in "Problem type x Condition x Group" below
Problem type x condition x group	3.94**	.08	Two separate 4 (Type) x 3 (Condition) ANOVAs were conducted on Chinese- and Canadian-educated participants. Chinese: Latencies varied with conditions and types, $F(6, 138) = 3.07$, $p = .008$. In the round-down and round-up conditions, Chinese solved all problem types equally quickly, $F(3, 69) = 1.33$, $p = .271$, and $F(3, 69) = .89$, $p = .453$. In the mixed condition, they solved DU and UD problems more quickly than DD and UU problems, $F(1,46) = 6.00$, $p = .018$. Canadians: Latencies varied with conditions and types, $F(6, 138) = 5.42$, $p < .001$. In the round-down condition, Canadians solved all problem types equally quickly, $F(3, 69) = 1.04$, $p = .381$. In the round-up condition, they solved UU and UD problems more quickly than DD and DU problems, $F(1,46) = 21.69$, $p < .001$. In the mixed condition, they solved DU problems more quickly than the other three problem, $F(1,46) = 12.95$, $p < .001$.

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Table B.3

Significant effects of problem type on strategic flexibility and solution latencies in Experiment 3

	F(3,144)	η_p^2	Detailed description
Strategic flexibility			
Problem type	71.83**	.60	Participants were more likely to select the optimal problem-based procedure on DD problems (99%) than on UD (77%), UU (77%) and DU (76%) problems.
Problem type x condition	39.92**	.45	Participants were equally likely to select the optimal problem-based procedure on DD problems across the three conditions. They were less likely to select the optimal problem-based procedure on DU, $F(1, 48) = 69.25$, UD, $F(1, 48) = 48.51$, and UU $F(1, 48) = 79.73$, problems in the Speed condition than in the Accuracy and Balanced conditions, ps < .001.
Solution latencies			
Problem type	48.24**	.50	Participants were fastest on DD (2119 ms) problems. They were equally fast on DU (2391 ms) and UD (2427 ms) problems, $F(1, 48) = .46$, $p = .499$. Participants were slower on UU (2673 ms) problems than on DU, $F(1, 48) = 37.93$, and UD problems, $F(1, 48) = 33.78$, $ps < .001$.
Problem type x Condition	2.29*	.05	The difference across the four problem types (DD vs. DU, UD, and UU) was smaller in the Speed condition $F(1, 48) = 22.65$, than in the Accuracy condition, $F(1, 48) = 49.08$, or the Balance condition, $F(1, 48) = 45.11$, $ps < .001$.

Note: DD = round both operands down; DU = round the first operand down and the second one up; UD = round the first operand up and the second one down; UU = round both operands up.

Appendix C: Details of effects for non-significant pairwise comparisons in Experiments 2 and 3

Table C. 1

Calculation errors (No-choice) across all conditions for Chinese-educated participants in Experiment 2

Pairwise Comparisons	*F*(1, 46)	*MSE*	*p*
Round-down and Round-up Conditions	3.02	43.83	.089
Round-down and Mixed Conditions	2.00	50.26	.164
Round-up and Mixed Conditions	.04	55.47	.842

Table C. 2

Calculation errors across all conditions for Chinese-educated participants in Experiment 3

Pairwise Comparisons	*F*(1, 48)	*MSE*	*p*
Speed and Accuracy Conditions	< .001	286.90	1.00
Speed and Balanced Conditions	.26	202.80	.615
Round-up and Mixed Conditions	.55	95.51	.464

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Table 1

Percentage of reported procedures by each nationality group in the Pilot Study and Experiment 1

Procedure Category	Procedure Descriptions	Examples of self-reports 23 x 76	Percentage of Trials (by group)			
			Pilot study		Experiment 1	
			Chinese	Canadian	Chinese	Canadian
Decade-rounding	(a) Round both operands down to the nearest decades (both down)	(a) 20 x 70 = 1400	9	19	27	15
	(b) Round both operands up to the nearest decades (both up)	(b) 30 x 80 = 2400	10	17	17	11
	(c) Round one operand down and the other operand up (mixed rounding)	(c) 20 x 80 = 1600 *or* 30 x 70 = 2100	24	32	42	25
Complex alternatives	(a) Round to the 5s unit, or (b) Round just one operand, or (c) Round, then adjust solution with either addition or subtraction	(a) 25 x 80 (b) 23 x 80 (c) 25 x 80 - 200	33	1	6	5
Truncation or Rounded-down	Truncate the operands and adjust the place value of the solution by (a) adding zeros, multiplying by 100 or (b) appending the product of the unit digits	(a) 2 x 7 x 100; (b) 2 x 7 = 14; 3 x 6 = 18, solution = 1418	11	31	8	42
Exact calculation	Compute the exact answer	23 x 76 = 1748	14	0	0	0

Table 2

Participants categorized according to the procedure they used on the majority of trials (numbers represent the percentages of individuals in each group) in the pilot study and Experiment 1

Procedure Category	Definition	Pilot Study Chinese (n = 10)	Pilot Study Canadian (n = 10)	Experiment 1 Chinese (n = 25)	Experiment 1 Canadian[1] (n = 24)
Decade-rounding	Used on ≥ 85% of the problems	30	70	92	54
Complex alternatives	Used on ≥75% of the problems	40	0	4	4
Truncation or Rounded-down	Used on ≥ 85% of the problems	10	30	4	42
Exact calculation	Used on ≥ 50% of the problems	20	0	0	0

Note. These numbers are slightly different than the ones in the Results section in Experiment 1, because the latter represents the percentage of trials that participants solved using the optimal problem-based procedure.

[1] One Canadian reported a mixture of truncation, complex rounding, and guessing and thus could not be categorized.

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